Spring 2007

Sutcliffe Development Theory

Ref: Carlson, T.N., Mid-Latitude Weather Systems, 1991, Harper-Collins, 507 pp (see Ch 8.1)

Write vorticity equation at two levels, 1000 and 500 hPa, and then subtract the equations to get the relative divergence. Ignore friction, twisting and the vertical advection of vorticity.

$$\frac{\partial \xi_5}{\partial t} = -\vec{V}_5 \cdot \nabla (\xi_5 + f) - \underbrace{(\xi_5 + f)}_{\approx f_0} \nabla \cdot \vec{V}_5$$

$$\frac{\partial \xi_0}{\partial t} = -\vec{V}_0 \cdot \nabla (\xi_0 + f) - \underbrace{(\xi_0 + f)}_{\approx f_0} \nabla \cdot \vec{V}_0$$

Now subtract to get:

$$\frac{\partial \xi_5}{\partial t} - \frac{\partial \xi_0}{\partial t} = -\vec{V}_5 \cdot \nabla (\xi_5 + f) + \vec{V}_0 \cdot \nabla (\xi_0 + f) - f_0 (\nabla \cdot \vec{V}_5 - \nabla \cdot \vec{V}_0)$$

Rewriting:

$$\underbrace{f_0 (\nabla \cdot \vec{V}_5 - \nabla \cdot \vec{V}_0)}_{\text{Relative Divergence}} = -\frac{\partial}{\partial t}(\xi_5 - \xi_0) - \vec{V}_5 \cdot \nabla (\xi_5 + f) + \vec{V}_0 \cdot \nabla (\xi_0 + f) \qquad (8.1)$$

Eq. 8.1 is Carlson (8.1) but note sign errors in book

Now,

$$(\zeta_s - \zeta_0) = \frac{g}{f_0} \nabla_p^2 (Z_s - Z_0) = \frac{g}{f_0} \nabla_p^2 h \quad (8.2)$$

and with $\frac{\partial h}{\partial t} \sim -\bar{V} \cdot \nabla_p h$ (8.3)

we have: $$\frac{\partial}{\partial t}(\zeta_s - \zeta_0) = \frac{g}{f_0} \nabla_p^2 \frac{\partial h}{\partial t} = \frac{g}{f_0} \nabla_p^2 (-\bar{V} \cdot \nabla_p h)$$

$$= -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\bar{u} v_T - \bar{v} u_T) \quad (8.4a)$$

$\bar{(\;)}$ = layer-mean quantity where $u_T = -\frac{g}{f_0}\frac{\partial h}{\partial y}$, $v_T = -\frac{g}{f_0}\frac{\partial h}{\partial x}$

Expanding (8.4a) and neglecting deformation terms

$$\frac{\partial}{\partial t}(\zeta_5 - \zeta_0) = \frac{g}{f_0}\nabla_p^2(-\bar{V} \cdot \nabla_p h) = \left(\bar{u}\frac{\partial}{\partial x} + \bar{v}\frac{\partial}{\partial y}\right)\zeta_T + \left(\bar{u}\frac{\partial}{\partial y} - \bar{v}\frac{\partial}{\partial x}\right)(\nabla_p \cdot \nabla T)$$
$$- \left(u_T\frac{\partial}{\partial x} + v_T\frac{\partial}{\partial y}\right)\bar{\zeta} - \left(u_T\frac{\partial}{\partial y} - v_T\frac{\partial}{\partial x}\right)\nabla_p \cdot \bar{V} \quad (8.4b)$$

Assume and equivalent barotropic atmosphere $\Longrightarrow$ (8.4b) simplifies to:
(better: straight-line hodograph)

$$\frac{\partial}{\partial t}(\zeta_5 - \zeta_0) = \frac{g}{f_0}\nabla_p^2 \frac{\partial h}{\partial t} = -\vec{V}_0 \cdot \nabla_p \zeta_s + \vec{V}_s \cdot \nabla_p \zeta_0$$

(8.5)

Using (8.5) to eliminate $-\frac{\partial}{\partial t}(\zeta_s - \zeta_0)$ in (8.1) yields:

$$f_0(\nabla \cdot \vec{V}_s - \nabla \cdot \vec{V}_0) = -\vec{V}_s \cdot \nabla_p \zeta_0 - \vec{V}_s \cdot \nabla(\zeta_s + f)$$
$$+ \vec{V}_0 \cdot \nabla_p \zeta_s + \vec{V}_0 \cdot \nabla(\zeta_0 + f)$$

or $$f_0(\nabla \cdot \vec{V}_s - \nabla \cdot \vec{V}_0) = -\vec{V}_s \cdot \nabla_p(\zeta_0 + \zeta_s + f)$$
$$+ \vec{V}_0 \cdot \nabla(\zeta_0 + \zeta_s + f) \quad (8.6)$$

Consolidating:

$$f_0 \left(\nabla_p \cdot \vec{V}_S - \nabla_p \cdot \vec{V}_0 \right) = - \left(\vec{V}_S - \vec{V}_0 \right) \cdot \nabla_p \left(\zeta_0 + \zeta_S + f \right) \quad (8.6)$$

But: $\vec{V}_T = \vec{V}_S - \vec{V}_0$ and $\zeta_T = \zeta_S - \zeta_0$

Eliminate $\zeta_S = \zeta_T + \zeta_0$ from (8.6) to yield

$$f_0 \left(\nabla_p \cdot \vec{V}_S - \nabla_p \cdot \vec{V}_0 \right) = - \vec{V}_T \cdot \nabla_p \left(\zeta_T + 2\zeta_0 + f \right)$$

If we assume that $\nabla_p \cdot \vec{V}_S \sim 0$ (level of non divergence)

we have:

$$- \nabla_p \cdot \vec{V}_0 = - \frac{2}{f_0} \vec{V}_T \cdot \nabla_p \zeta_0 - \frac{1}{f_0} \vec{V}_T \cdot \nabla_p \zeta_T - \frac{1}{f_0} \vec{V}_T \cdot \nabla_p f \quad (8.7)$$