"Easy" Q-Vectors after Sanders and Hoskins (1990)

$$\left(\sigma \nabla^2 + f_0^2 \frac{\partial^2}{\partial p^2}\right)\omega = -2\nabla\cdot\vec{Q} \qquad (\beta = 0)$$

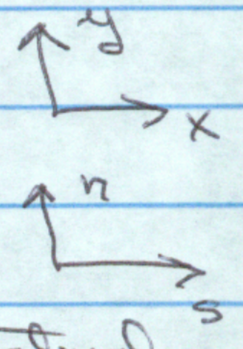
$\alpha = -\partial\Phi/\partial p$

$$\vec{Q} \equiv \left[\frac{\partial \vec{V}_g}{\partial x}\cdot\nabla\left(\frac{\partial\Phi}{\partial p}\right), \frac{\partial \vec{V}_g}{\partial y}\cdot\nabla\left(\frac{\partial\Phi}{\partial p}\right)\right] \qquad p\alpha = RT$$

But, $\nabla(\partial\Phi/\partial p) = -\frac{R}{p}\nabla T$

T_1 ---------
T_2 ---------
T_3 ---------

$T_3 > T_2 > T_1$



natural coordinates

orient s-axis along isotherms

Orient x-axis along isotherms

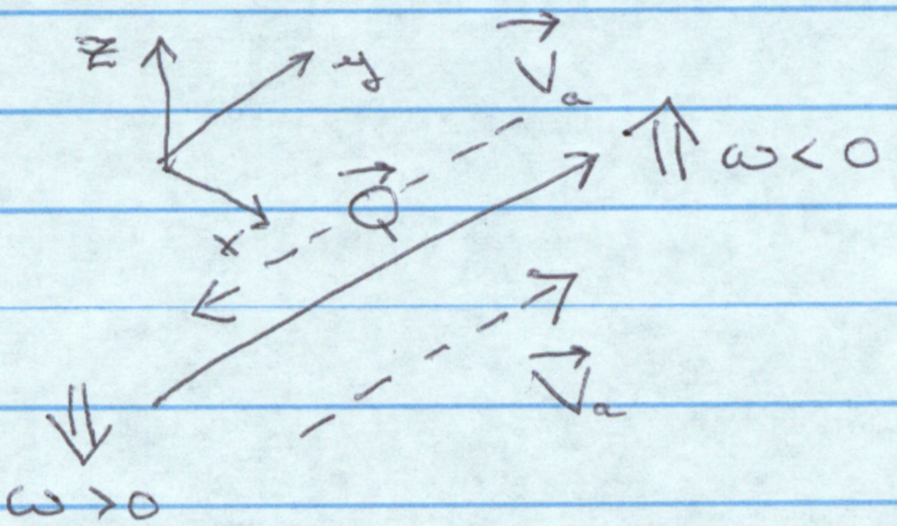
$$\therefore \nabla(\partial\Phi/\partial p) = -\frac{R}{p}(\partial T/\partial y)\hat{j}$$

and $$\vec{Q} = -\frac{R}{p}\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial v_g}{\partial x}\hat{i} + \frac{\partial u_g}{\partial y}\hat{j}\right)$$

with f = constant, $\nabla\cdot\vec{V}_g = 0 \Rightarrow \frac{\partial v_g}{\partial y} = -\frac{\partial u_g}{\partial x}$

or $$\vec{Q} = -\frac{R}{p}\left(\frac{\partial T}{\partial y}\right)\left(\frac{\partial v_g}{\partial x}\hat{i} - \frac{\partial u_g}{\partial y}\hat{j}\right) = \frac{-R}{p}\left|\frac{\partial T}{\partial y}\right| \hat{k}\times\frac{\partial \vec{V}_g}{\partial x}$$

or in natural coordinates: $$\vec{Q} = -\frac{R}{p}\left|\frac{\partial T}{\partial n}\right| \hat{k}\times\frac{\partial \vec{V}_g}{\partial s}$$



$$Q = \frac{R}{\sigma p}\left(\frac{p}{p_0}\right)^{\kappa} \frac{D_g}{Dt}(\nabla_p\theta)$$

$$\vec{F} = \frac{\sigma p}{R}\frac{1}{|\nabla_p\theta|}(\nabla_p\theta\cdot\vec{Q})$$

1. Measure change of $\vec{V}_g$ along an isotherm (cold air on left in NH)
2. Rotate vector change of $\frac{\partial \vec{V}_g}{\partial s}$ 90° clockwise to the right