Equivalence of the QG Height Tendency and Potential Vorticity Equations

QG Height Tendency Eq:

$$\underbrace{\left(\nabla^2 + \frac{f_o^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\chi}_{A} = \underbrace{-f_o \vec{V}_g \cdot \nabla\left(\frac{1}{f_o}\nabla^2\Phi + f\right)}_{B} + \underbrace{\frac{f_o^2}{\sigma}\frac{\partial}{\partial p}\left(-\vec{V}_g \cdot \nabla \frac{\partial \Phi}{\partial p}\right)}_{C}$$

Rewriting C as : $\underbrace{-\frac{f_o^2}{\sigma}\vec{V}_g \cdot \nabla \frac{\partial^2 \Phi}{\partial p^2}}_{C.1} \underbrace{-\frac{f_o^2}{\sigma}\frac{\partial \vec{V}_g}{\partial p} \cdot \nabla \frac{\partial \Phi}{\partial p}}_{C.2}$

But, $f_o \frac{\partial \vec{V}_g}{\partial p} = \hat{k} \times \nabla\left(\frac{\partial \Phi}{\partial p}\right)$ which is $\perp$ to $\nabla\left(\frac{\partial \Phi}{\partial p}\right)$

$$\Rightarrow C_2 = 0$$

$$\therefore A = B + C1 = -f_o \vec{V}_g \cdot \nabla\left(\frac{1}{f_o}\nabla^2\Phi + f\right) - \frac{f_o^2}{\sigma}\vec{V}_g \cdot \nabla \frac{\partial^2 \Phi}{\partial p^2}$$

or $$A = -f_o \vec{V}_g \cdot \nabla\left[\frac{1}{f_o}\nabla^2\Phi + f + \frac{f_o}{\sigma}\frac{\partial^2 \Phi}{\partial p^2}\right]$$

Term A can be rewritten as:

$$\left(\nabla^2 + \frac{f_o^2}{\sigma}\frac{\partial^2}{\partial p^2}\right)\frac{\partial \Phi}{\partial t} = \nabla^2 \frac{\partial \Phi}{\partial t} + \frac{f_o^2}{\sigma}\frac{\partial^2}{\partial p^2}\left(\frac{\partial \Phi}{\partial t}\right)$$

$$= \frac{\partial}{\partial t}\nabla^2\Phi + \frac{\partial}{\partial t}\left(\frac{f_o^2}{\sigma}\frac{\partial^2 \Phi}{\partial p^2}\right)$$

$$\chi \equiv \partial \Phi / \partial t$$

Simplify previous expression

$$(A - B - C1)/f_0 = 0 \qquad \text{or}$$

$$\left(\frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla\right)\left\{\frac{1}{f_0}\nabla^2\Phi + f + \frac{f_0}{\sigma}\frac{\partial^2\Phi}{\partial p^2}\right\} = 0$$

or $$\frac{dq}{dt} = 0$$

where $$q \equiv QGPV \equiv \underbrace{\frac{1}{f_0}\nabla^2\Phi + f}_{\substack{\text{absolute}\\\text{vorticity}\\\text{(geostrophic)}}} + \underbrace{\frac{f_0}{\sigma}\frac{\partial^2\Phi}{\partial p^2}}_{\text{stability}}$$

Synoptic application:

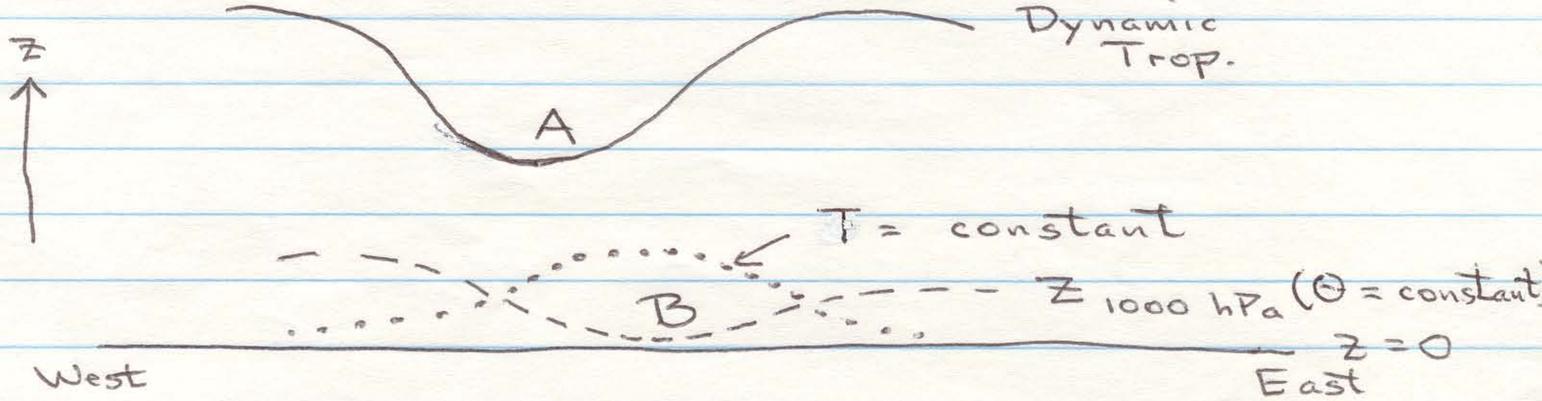
Height fall (rise) regions on isobaric surfaces are associated with positive (negative) QGPV advection

Use of QGPV avoids the possibly cancelling effects of vorticity advection and differential thermal advection in the QG height tendency equation.

Dynamic Tropopause View of Extratropical Cyclogenesis

Ref: Morgan and Nielsen-Gammon, 1998: Using Tropopause Maps to Diagnose Midlatitude Weather Systems. Mon. Wea. Rev., 126, 2555-2579.

(Nielsen-Gammon, 2001: A Visualization of the Global Dynamic Tropopause. Bulletin of the AMS, 82, 1151-1167.)



A : upper-level disturbance (PV anomaly)

B : lower-level " " "

(manifest as a warm area near the surface)

Magnitude of interaction depends upon A/B proximity

"Coupling Potential" (CP), $CP \equiv H_R / H$

H = tropopause height $\quad H_R = \dfrac{f l}{N}$ = Rossby Depth

$l \equiv$ length scale $\equiv 1/k$; $N \equiv \left\{ (g/\theta)(\partial \theta_{ref} / \partial z) \right\}^{1/2}$

For a QG atmosphere (to 1st approximation):

$CP \propto 1/H$ ("sucking" potential)