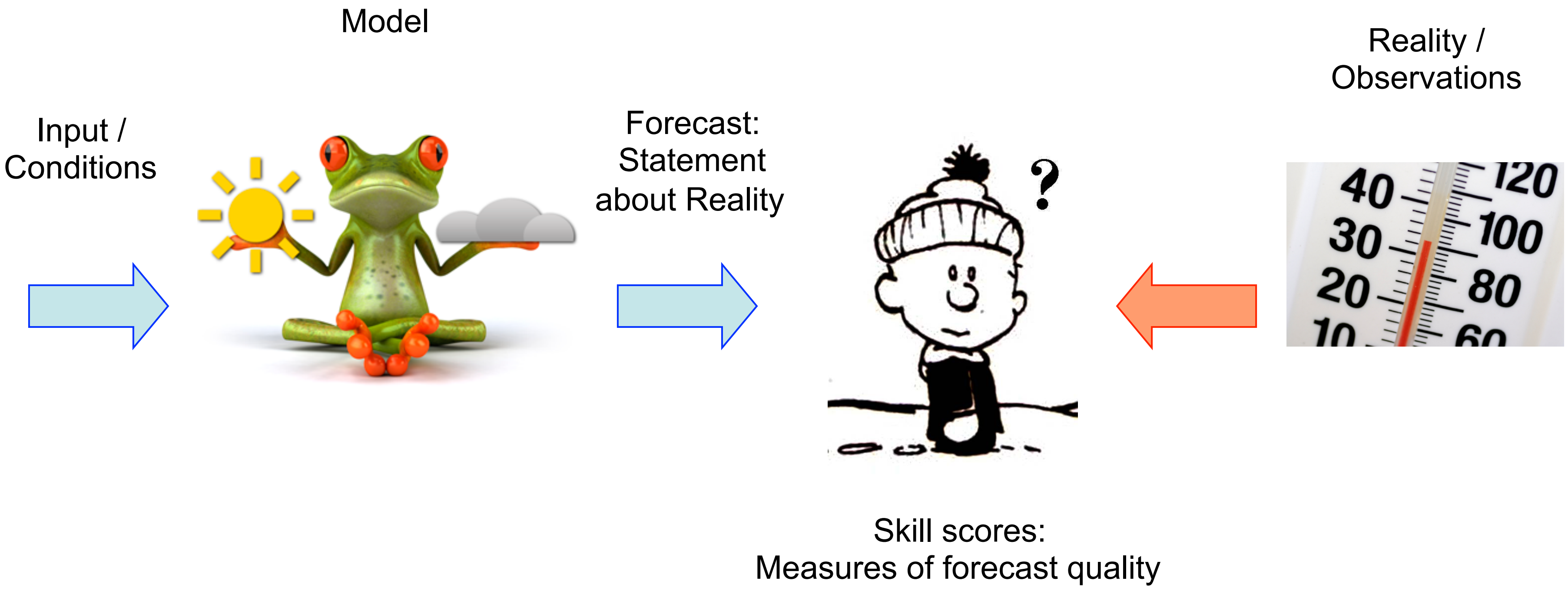
Section 5:
Forecast Evaluation and Skill Scores

What is Forecast Evaluation ?

- **Assessing the quality / error structure of forecasts by comparison to independent observations**



Model

Input / Conditions

Forecast: Statement about Reality

Reality / Observations

Skill scores: Measures of forecast quality

“Forecasts”

- **Weather Forecast**

  How accurate are temperature forecasts one day ahead?

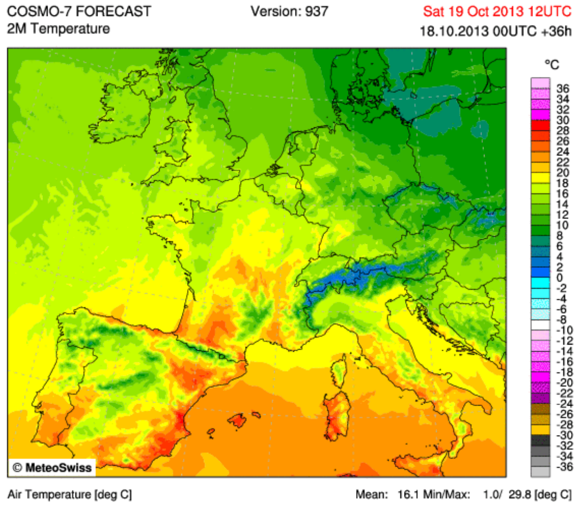
- **Simulations of Climate**

  Reproduce the distribution of mean summer precipitation in Europe?

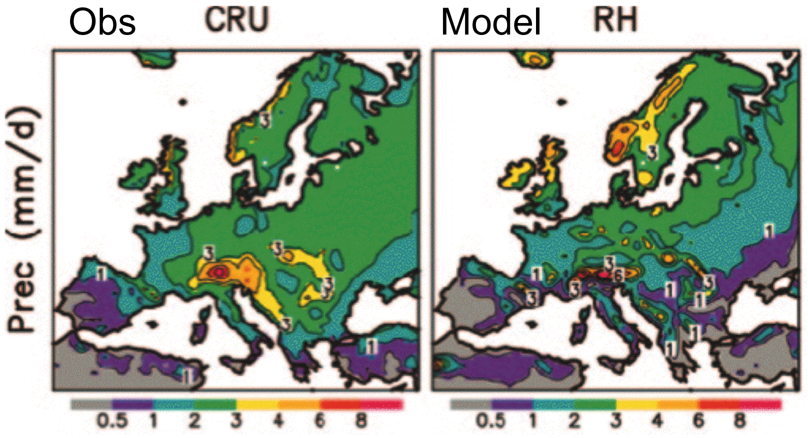
- **Spatial analysis**

  Estimate precipitation at a non-instrumented site from observations in the neighbourhood?

- **Remote sensing, …**



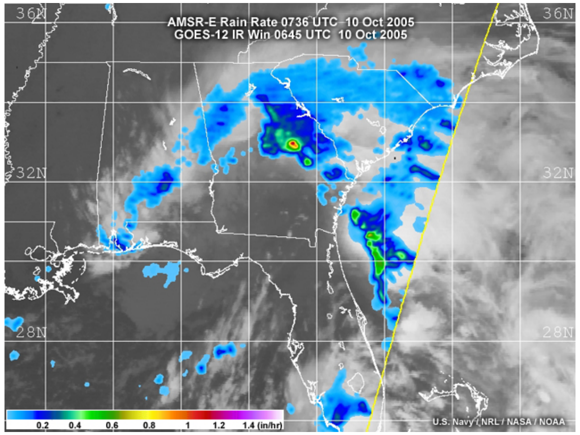
www.meteoswiss.ch



Räisänen et al. 2004

Observations

- **Generic for “measure of reality”**
- **The chosen Reference**
- **In practice:**
    - In-situ measurements
    - Indirect estimates of “reality”: re-analyses, remote sensing
- **Important:**
    - Role of observation errors for your evaluation?
    - Are observations and model independent?



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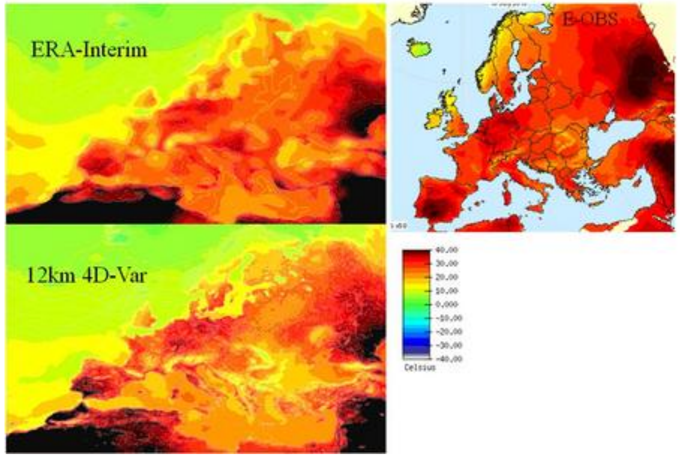


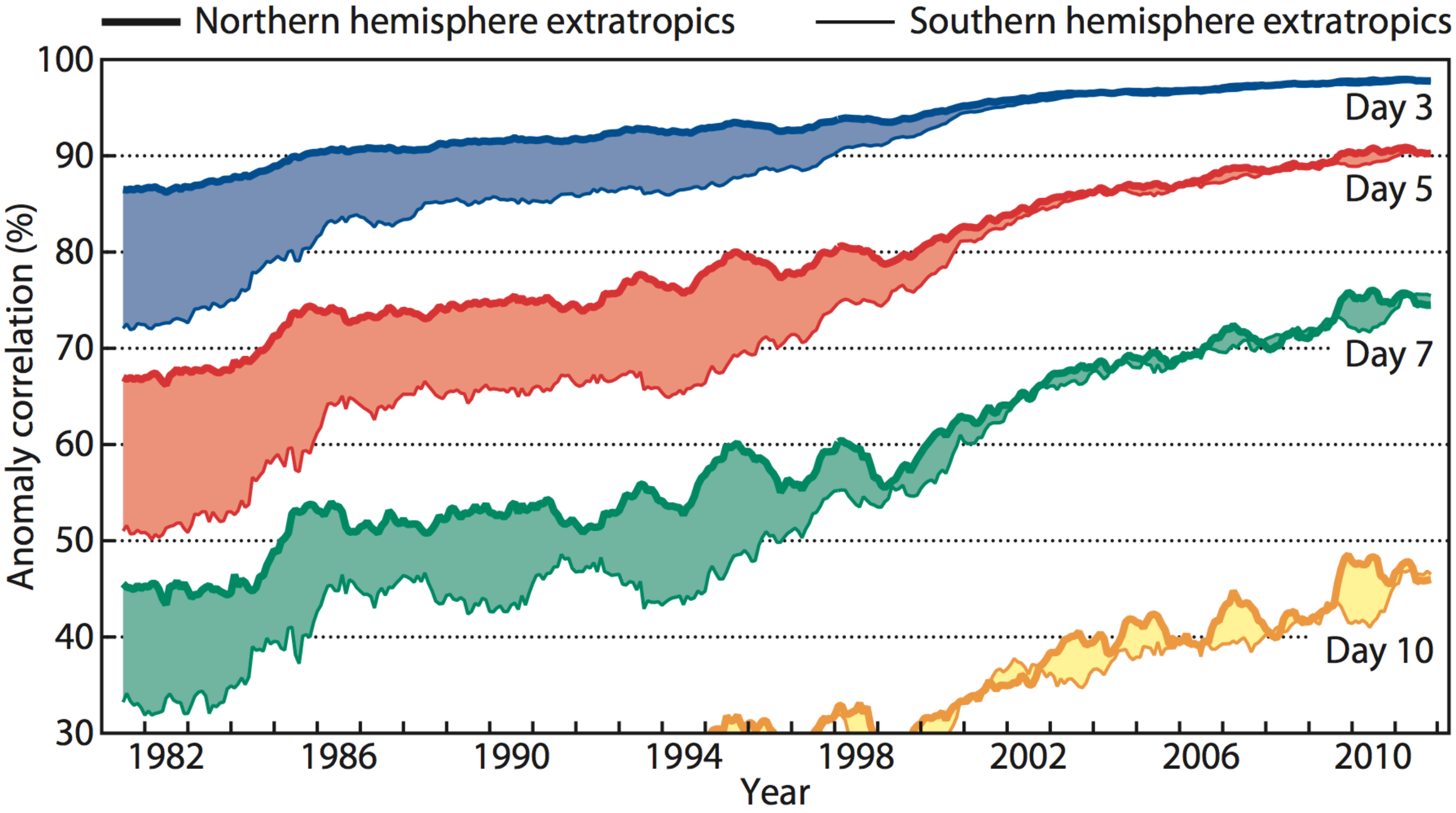
www.euro4m.eu

Why Forecast Evaluation?

- **Learn how to properly use / interpret forecast**
  - E.g. the issuing of a public flood warning depends on the frequency with which the forecast produces false alarms
- **Learn how and where to improve forecast**
  - E.g. by comparison of forecast quality for different model parametrizations
- **Justify investments made into models, instruments**
  - E.g. launching of new weather satellites depends on the expected improvement of weather forecasts (pay-back on investment)

ECMWF MR-Forecast

Anomaly correlation of 500 hPa Geopotential



ECMWF 2012

Forecasts

- **Continuous:**
    - real value, e.g. temperature in Zürich
- **Categorial:**
    - values in discrete classes (e.g. cold, normal or warm) or events (e.g. a tornado tomorrow).

Type

- **Deterministic:**
    - a single number, e.g. the expected temperature tomorrow
- **Probabilistic:**
    - probabilities, e.g. the prob. of rain tomorrow
    - expresses the degree of forecast uncertainty

Nature

Outline

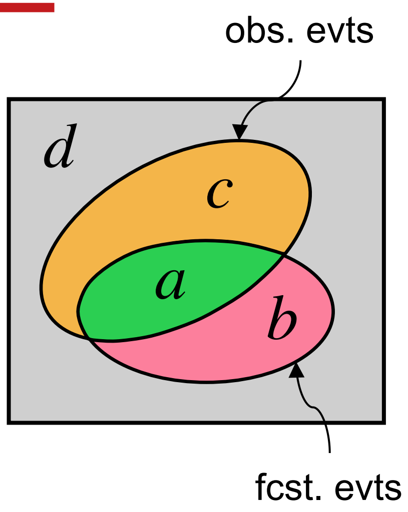
- **Deterministic categorial forecasts**
- **Deterministic continuous forecasts**
- **Probability forecasts**
- **Evaluation based on economic value**

- **Material based on:**
  - Wilks 2005, Chap 7, (von Storch & Zwiers 1999, Chap 18)
  - Richardson 2000, Wilks 2001
  - Web-Site of WWRP/WGNE WG Forecast Verification Research: http://www.cawcr.gov.au/projects/verification/

Section 5: Forecast Evaluation and Skill Scores

Deterministic Categorial Forecasts

Contingency Table



- **Binary Forecasts**
  - *Y* = {yes, no}, e.g. events: tomorrow it will (will not) rain
  - simplest categorial case

- **Contingency Table**
  - Distribution (Y,O)

		Observation		
		yes	no	Marginal of Fcst
Forecast	yes	a hits	b false alarms	$a+b$ yes fcsts
	no	c misses	d correct rejects	$c+d$ no fcsts
Marginal of Obs		$a+c$ yes obs	$b+d$ no obs	N total fcsts

Finley Tornado Forecasts 1884

U.S. Army forecasts of tornado occurrence
east of the Rockies, based on synoptic information

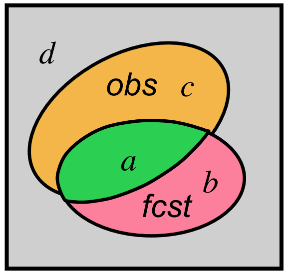
		Tornados Observed		
		yes	no	
Tornados forecasted	yes	*28*	*72*	*100*
	no	*23*	*2680*	*2703*
		51	*2752*	*2803*

Oldest known tornado photograph taken near Howard, South Dakota, on August 28th, 1884.
Courtesty of NOAA Photo Library

www.photolib.noaa.gov

Galway 1985

Simple Scores



- **Bias score:**

$$B = \frac{a+b}{a+c} = \frac{\text{forecasted events}}{\text{observed events}}$$

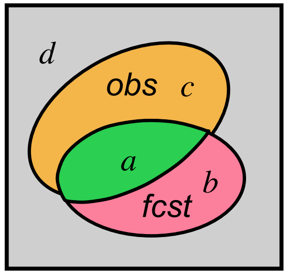
  - $B = 1$ unbiased, $B < 1$ underforecast, $B > 1$ overforecast
  - depends on marginals only, does not measure ‘correspondence’

- **Probability of detection (hit rate):**

$$POD = \frac{a}{a+c} = \frac{\text{hits}}{\text{observed events}}$$

  - Fraction of all observed events correctly forecasted
  - $0 \leq POD \leq 1$, best score: $POD = 1$, best score ≠ perfect fcst
  - Focus on events. No penalty for false alarms.

Simple Scores



- **False alarm ratio:**

$$FAR = \frac{b}{a+b} = \frac{\text{false alarms}}{\text{forecasted events}}$$

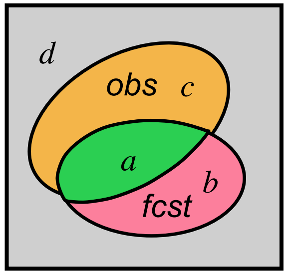
  - Fraction of forecasted events that were false alarms
  - $0 \leq FAR \leq 1$, best score: $FAR = 0$, best score ≠ perfect fcst

- **Probability of false detection (false alarm rate):**

$$POFD = \frac{b}{b+d} = \frac{\text{false alarms}}{\text{non-events}}$$

  - Fraction of all non-events when forecast predicted an event
  - $0 \leq POFD \leq 1$, best score: $POFD = 0$, best score ≠ perfect fc

Simple Scores



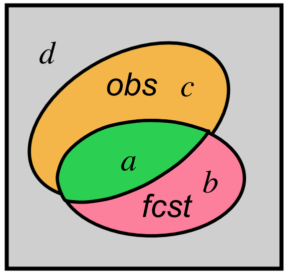
- **Accuracy (fraction correct):**

$$ACC = \frac{a+d}{N} = \frac{\text{correct forecasts}}{\text{all forecasts}}$$

  - Fraction of all forecasts that were correct
  - $0 \leq ACC \leq 1$, best score: $ACC = 1$, best score = perfect fcst
  - Events and non-events treated symmetrically
  - For rare events the score is dominated by non-events

  - Finley tornado forecast:
    - ACC = (28+2680)/2803 = 0.96 (!)
    - But: POD = 28/51 = 0.54 and FAR = 0.72 (!)

Simple Scores



- **Threat score (Critical Success Index):**

$$TS = CSI = \frac{a}{a+b+c} = \frac{\text{hits}}{\text{all forecasted or observed events}}$$

  - Fraction of all forecasted or observed events that were correct
  - $0 \le TS \le 1$, best score: $TS = 1$, best score = perfect fcst
  - Asymmetric between events and non-events.

  - Finley tornado forecast:
    - TS = 28/(28+72+23) = 0.23

Limitations of Simple Scores

- **How large is a “good” score?**
- **Best score not necessarily perfect forecast!**
- **Hedging (“Playing”) a score:**
  - Example: Modify Finley’s Forecast --> constant forecast

		Observed	
		yes	no
Forecasted	yes	~~*28*~~ *0*	~~*72*~~ *0*
	no	~~*23*~~ *51*	~~*2680*~~ *2752*

Finley: ACC = 0.96
Constant: ACC = 0.98 (!)

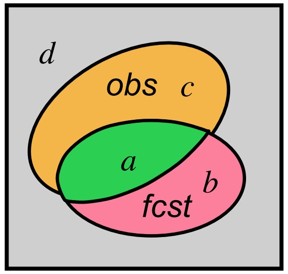
Generic Form of a Skill Score

$$SS = \frac{A - A_{ref}}{A_{perf} - A_{ref}}$$

A	accuracy score, e.g. ACC or TS
A_{ref}	accuracy of reference forecast, e.g. random
A_{perf}	accuracy of perfect forecast

$SS = 1$	perfect forecast
$SS > 0$	skillful, better than reference
$SS < 0$	less skillful than reference

Heidke Skill Score



- **Generic Score with …**

  … ACC as A and random forecast as reference

$$A = \left(\frac{a+d}{N}\right) \qquad A_{perf} = 1$$

$$A_{ref} = \left(\frac{(a+b)}{N}\right)\cdot\left(\frac{(a+c)}{N}\right)+\left(\frac{(d+c)}{N}\right)\cdot\left(\frac{(d+b)}{N}\right)$$

- **Heidke Skill Score**

$$HSS = \frac{ad-bc}{\left((a+c)\cdot(c+d)+(a+b)\cdot(b+d)\right)/2}$$

$$-\infty < HSS \le 1, \quad HSS \le 0 \;\; \text{no skill}$$

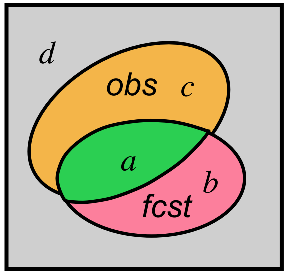
HSS for Finley Forecast

- **HSS**
    - for Finley forecast: HSS=0.355
    - for constant forecast: HSS=0.0
    - note, ACC is large even for random forecast:

$$ACC_{random} = \left(\frac{28+72}{2803}\right)\cdot\left(\frac{28+23}{2803}\right)+\left(\frac{2680+23}{2803}\right)\cdot\left(\frac{2680+72}{2803}\right)=0.947$$

- **HSS (generic form of skill scores) compensates for high random ACC, when events are very rare.**

Hanssen-Kuipers Discriminant



- **Similar to HSS but unbiased ACC in denominator**

$$SS = \frac{ACC - ACC_{random}}{1 - ACC_{unbiased\ random}}$$

$$ACC_{unbiased\ random} = \frac{(a+c)^2 + (b+d)^2}{N^2}$$

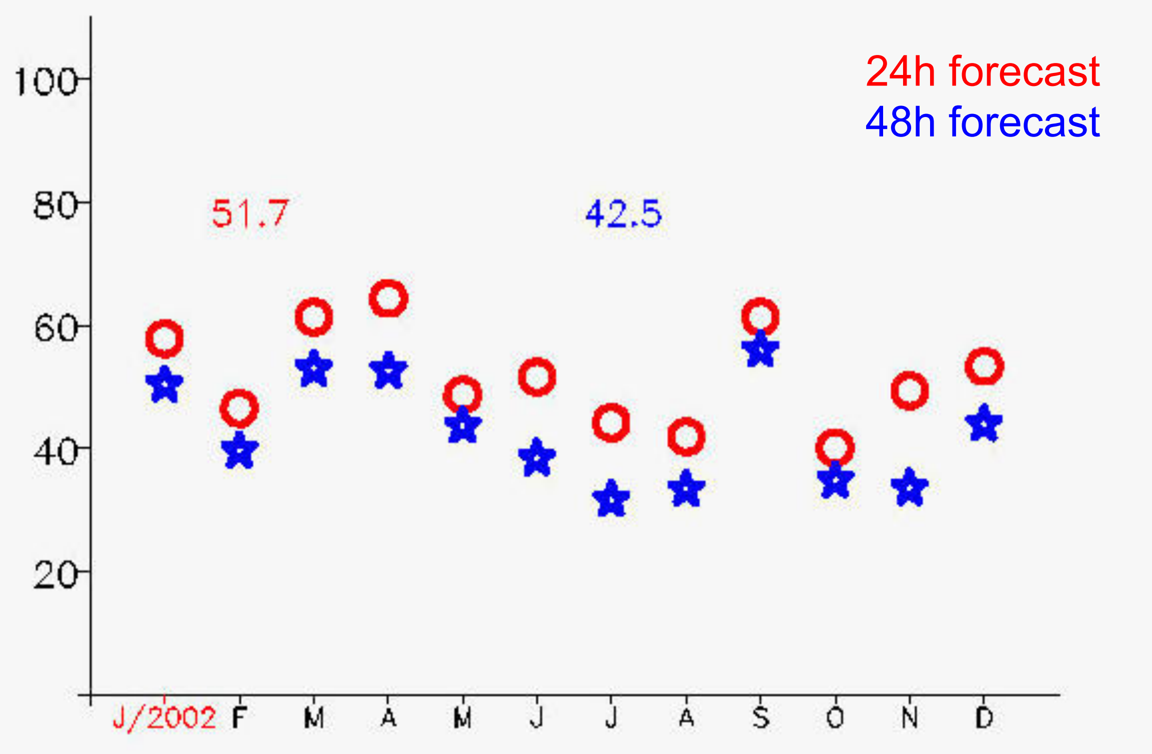
- **Hanssen-Kuipers (also True Skill Statistic, Pierce Skill Score)**

$$HK = \frac{ad - bc}{(a+c)\cdot(b+d)} = POD - POFD$$

  - $-1 \le HK \le 1$, $HK \le 0$ no skill,
  - for unbiased forecasts: $HK = HSS$
  - $HK(\text{Finley}) = 0.523$, $HK(\text{constant}) = 0.0$

Example

Hanssen-Kuipers Score (in %)
for daily precipitation occurrence (P>1 mm)



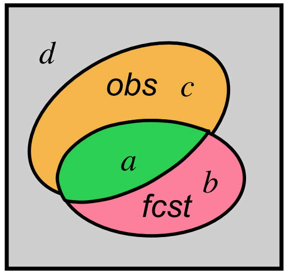
LokalModell: Operational NWP model of DWD in 2002, dx = 7 km)

Evaluation for all grid points in Germany for year 2002

Skill varies between seasons: E.g. 24h fcst in summer is less accurate than 48h fcst in winter.

U. Damrath (DWD)

Equitable Threat Score



- **Equitable Threat Score (also Gilbert Skill Score)**
    - Use TS (CSI) for A in generic form, random forecast as reference

$$ETS = \frac{a/(a+b+c) - a_{ref}/(a+b+c)}{1 - a_{ref}/(a+b+c)} = \frac{a - a_{ref}}{a - a_{ref} + b + c}$$

$$a_{ref} = (a+c)\cdot(a+b)/N$$

-
    - $-1/3 \leq ETS \leq 1$, $ETS \leq 0$ no skill,
    - $ETS(\text{Finley}) = 0.216$, $ETS(\text{constant}) = 0$
    - Unlike with HSS and HK, with ETS focus is on events only

Skill Scores Differ …

- **… in the relative importance of systematic and random errors**
  - E.g. artificially biasing a forecast decreases HK linearly
    but less than linearly for HSS
- **… in the relative role of events and non-events**
  - ETS values only events <--> HSS, HK value both
- **… in their behaviour for rare events**
  - Most skill scores tend to approach 0 for more and more rare events

- **There is no single best recommendation!**

Uncertainty in Scores

- **You've got 30 event forecasts.**
  You obtain HSS=0.2.
  Not too bad but …

- **… what is the probability that such a score is obtained by chance?**

Further Remarks

- **Sampling uncertainty**
  - Accuracy of skill scores decreases with sample size
  - Scores for forecasts of very rare events may be difficult to determine accurately.
  - Use resampling methods to quantify skill uncertainty.

- **Multi-category skill scores:**
  - 2x2 Table --> kxk Table
  - Extend classical scores to multi-category case.
  - E.g. ACC is sum of diagonal table elements divided by total forecasts.
  - Ordered multi-category case: introduce weights to penalize for elements more far off the diagonal. (Gerrity 1992, see Wilks p. 274)

Section 5: Forecast Evaluation and Skill Scores

Deterministic Continuous Forecasts

Notation

- **Sample, forecast-observation pairs (real valued)**

$$\{y_i, o_i\}, \quad i = 1..N$$

- **Sample means**

$$\overline{y} = \frac{1}{N}\sum_i y_i, \quad \overline{o} = \frac{1}{N}\sum_i o_i$$

- **Sample variance**

$$s_y^2 = \frac{1}{N}\sum_i \left(y_i - \overline{y}\right)^2, \quad s_o^2 = \frac{1}{N}\sum_i \left(o_i - \overline{o}\right)^2$$

Example Data

Charles Doswell

- **24-h forecasts of T-max Oklahoma City**
- **Comparison of:**
  - NWS: Human forecast
  - NGM, LFM: Numerical model forecasts with MOS
  - PER: Persistence forecast
- **Here**
  - 2 summers (1993/4, N=182)

Brooks & Doswell 1996

Simple Error Scores

- **Bias (mean error, systematic error):**
    - additive, multiplicative

$$B_{add} = \overline{y} - \overline{o}, \quad B_{mult} = \overline{y} / \overline{o}$$

- **Mean absolute error:**
    - Mean of absolute deviations from obs

$$MAE = \frac{1}{N} \sum_{i} \left| y_i - o_i \right|$$

$$\underline{0} \leq \left\{ \begin{array}{c} MAE \\ MSE \\ RMSE \end{array} \right\} < \infty$$

- **Mean squared error (MSE), root MSE (RMSE):**

$$MSE = \frac{1}{N} \sum_{i} \left(y_i - o_i \right)^2, \quad RMSE = \sqrt{MSE}$$

- Sensitive to outliers, dominated by large deviations
- Favors forecasts avoiding large deviations from the mean

Simple Error Scores

- **Root means squared fraction (RMSF):**

$$RMSF = \exp\left(\sqrt{\frac{1}{N}\sum_i \left[\log\left(\frac{y_i}{o_i}\right)\right]^2}\right)$$

  - similar to RMSE but for multiplicative errors
  - “average multiplicative error”
  - meaningful for rainfall, wind speed, visibility, … (>0 !)
  - *log* insures that multiplicative under- / overestimates are equally penalized.
  - perfect forecast: RMSF = 1

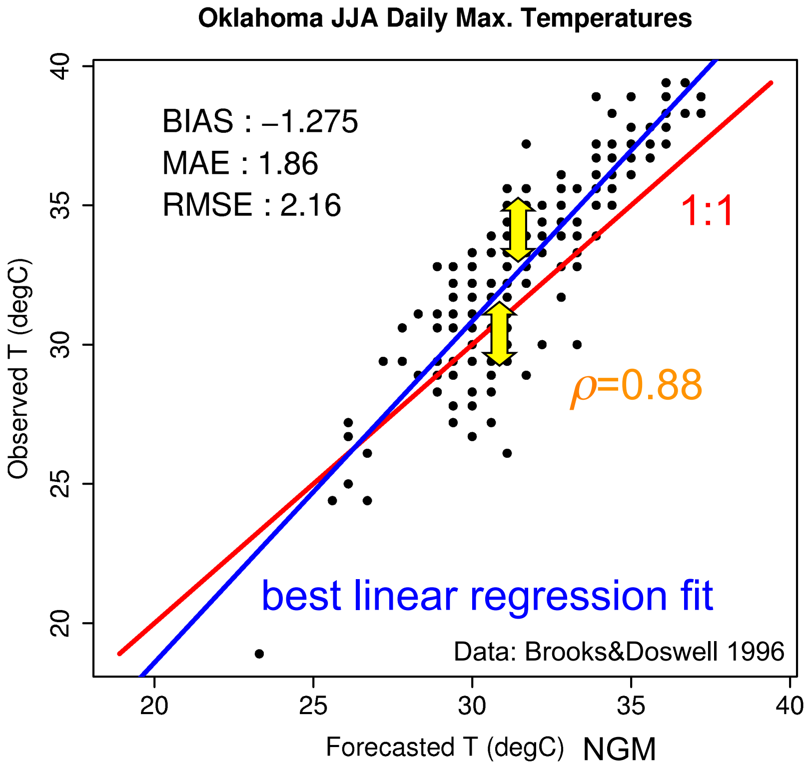
Golding 1998

Correlation Skill Score

- **Linear correlation coeff.**

$$\rho = \frac{\frac{1}{N}\sum_{i}^{N}(y_i - \overline{y})\cdot(o_i - \overline{o})}{s_y \cdot s_o}$$

  - $-1 \le \rho \le 1$, $\rho = 1$ best score
  - A measure of random error (scatter around best fit)
  - Insensitive to biases and errors in variance
  - ρ^2: fraction of variance in obs explained by "best" linear model
  - ρ measures potential skill (see also later)



Linear Regression:

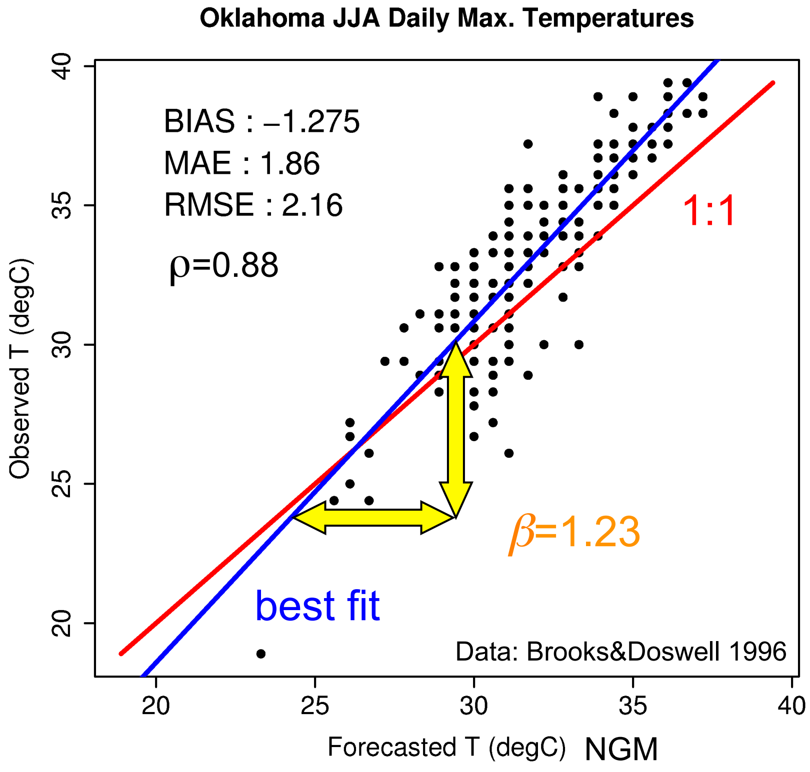
$$o_i = \beta \cdot y_i + a + \varepsilon_i$$

Conditional Bias

- **Linear regression slope**

$$\beta = \frac{s_o}{s_y} \cdot \rho$$

  - $\beta = 1$ best score
  - Deviations of β from 1 measure conditional bias
  - $\beta > 1$: Large (small) values tend to be under- (over-) estimated (unless compensated by absolute bias).
  - β is a function of correlation and fraction of variances



Linear Regression:

$$o_i = \beta \cdot y_i + a + \varepsilon_i$$

Decomposition of RMSE

- **RMSE’ (debiased RMSE)**

$$RMSE^2 = \left(\overline{y} - \overline{o}\right)^2 + s_y^2 + s_o^2 - 2 s_y s_o \rho$$

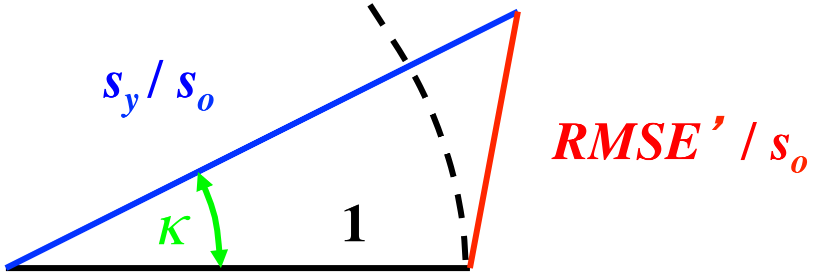
$$\Rightarrow \quad \frac{RMSE'^2}{s_o^2} = \frac{RMSE^2 - B^2}{s_o^2} = 1 + \frac{s_y^2}{s_o^2} - 2\frac{s_y}{s_o}\rho$$

relative error
in variance

degree of
correspondence

- **Geometric interpretation (cosine triangle theorem):**

$$\cos \kappa = \rho$$



Taylor 2001

Derivation

$$RMSE^2 = \frac{1}{N}\sum\left(y_i - o_i\right)^2 = \frac{1}{N}\sum\left(\left(y_i - \overline{y}\right) - \left(o_i - \overline{o}\right) + \left(\overline{y} - \overline{o}\right)\right)^2$$

$$= \frac{1}{N}\sum\left(\left(y_i - \overline{y}\right) - \left(o_i - \overline{o}\right)\right)^2 + \frac{1}{N}\sum\left(\overline{y} - \overline{o}\right)^2$$

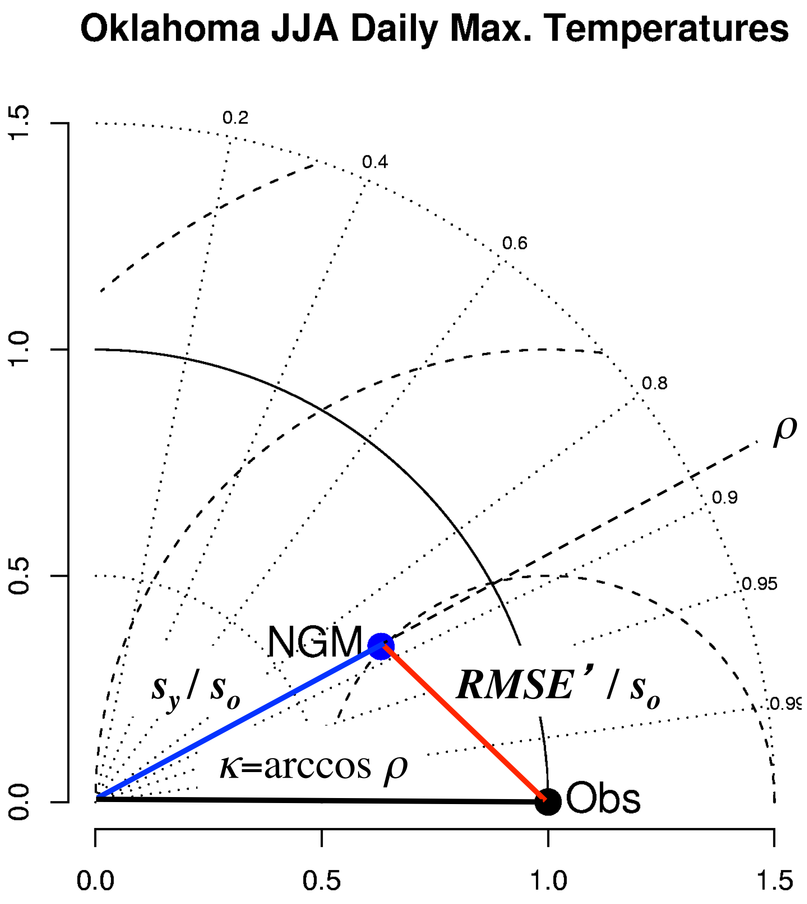
$$= s_y^2 + s_o^2 - 2 s_y s_o \rho + B^2$$

$$RMSE^2 - B^2 = s_y^2 + s_o^2 - 2 s_y s_o \rho$$

Taylor Diagram

- **Visualisation of forecast performance by three related scores in one graph.**

- **Ideal for:**
  - Comparing several forecast models,
  - Comparing to a reference forecast
  - Comparing to several observation datasets.
  - Assessing skill uncertainty e.g. by ensembles.



Taylor 2001

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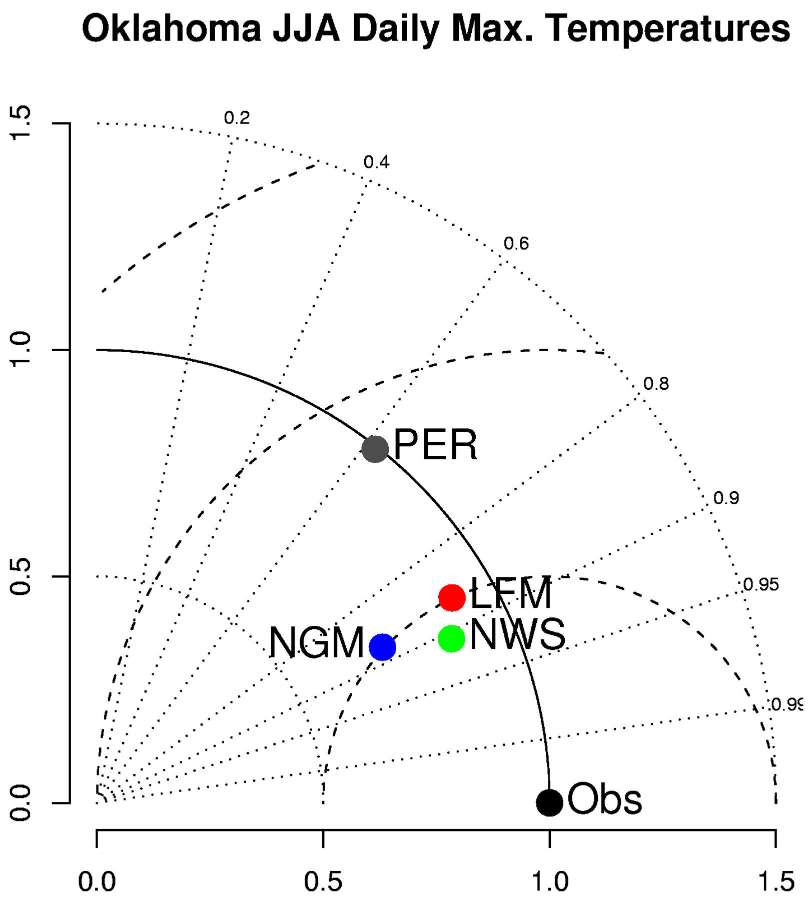
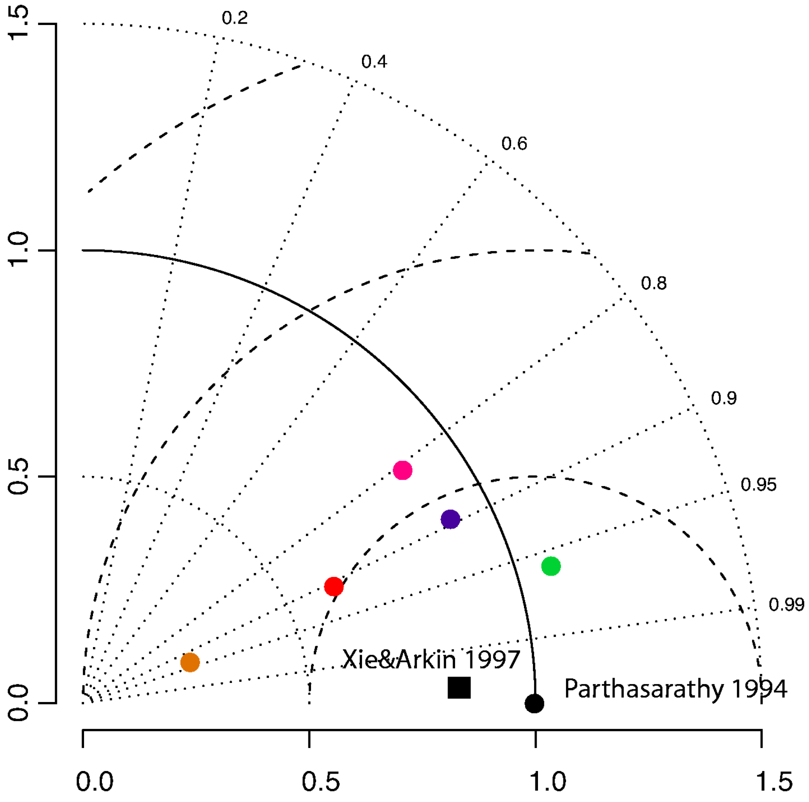


NWS: human forecaster
NGM, LFM: numerical models
PER: persistence forecast

Taylor 2001

Quiz

- **How will the points change with another obs. reference?**



Indian Monsoon in global climate models (AMIP Models) (from Taylor 2001)

Reduction of Variance

$$SS = \frac{MSE - MSE_{clim}}{MSE_{perfect} - MSE_{clim}} = 1 - \frac{MSE}{MSE_{clim}} = 1 - \frac{\frac{1}{N}\sum\left(y_i - o_i\right)^2}{s_o^2}$$

- also called *Brier score* or *Nash-Sutcliffe Efficiency* (Hydrology)
- generic form of skill score with $A = MSE$ and climatological forecast as reference.
- value range: $-\infty < SS \leq 1$
- perfect forecast: $SS = 1$
- climatology forecast: $SS = 0$
- random forecast with same variance and mean like observations: $SS = -1$
- sensitive to biases and errors in variance
- Always: $SS \leq \rho^2$ (see later)
- Oklahoma Temperature Forecast (NGM): $SS = 0.607$ ($\rho^2 = 0.77$)

Murphy-Epstein Decomposition

- **Decomposition of SS (Reduction of Variance)**

$$\frac{MSE}{MSE_{clim}} = \frac{RMSE^2}{s_o^2} = \frac{\left(\bar{y}-\bar{o}\right)^2}{s_o^2} + 1 + \underbrace{\frac{s_y^2}{s_o^2} - 2\frac{s_y}{s_o}\rho}_{\left(\rho-\frac{s_y}{s_o}\right)^2-\rho^2}$$

(see previously Taylor diagram)

$$\Rightarrow \quad SS = 1 - \frac{MSE}{MSE_{clim}} = \rho^2 - \underbrace{\left(\rho - \frac{s_y}{s_o}\right)^2}_{\left[\frac{s_y}{s_o}(\beta-1)\right]^2} - \frac{\left(\bar{y}-\bar{o}\right)^2}{s_o^2}$$

linear correspondence
"maximum explained variance"

penalty for
absolute bias

penalty for
conditional bias

Murphy & Epstein 1989

Murphy-Epstein Decomposition

- **Implications**
  - $SS = \rho^2$ only for absolute and conditionally unbiased forecasts. I.e. ρ^2 is a measure of potential skill.
  - A non-perfect forcast ($\rho^2 < 1$) can only be conditionally unbiased if $s_y < s_o$, i.e. if variance is underestimated.
  - Conditional bias can be minimized by setting $s_y/s_\mathrm{o} = \rho$,
    i.e. SS can be “played”!
  - Among forecasts with the same ρ and the same absolute bias, SS (and $RMSE$) favors those with small conditional bias, i.e. too smooth forecasts.
  - Forecasts with “good variance” are generally handicaped.

Oklahoma Temperatures

Model	ρ^2	(Conditional bias)^2	(Absolute bias)^2	SS	
NWS	0.824	0.002	0.000	0.822	human forecast
NGM	0.771	0.026	0.138	0.607	
LFM	0.750	0.002	0.000	0.748	
PER	0.382	0.141	0.000	0.241	persistence forecast

$\beta<1$, because $s_y=s_o$

Summary

- **Correlation is a measure of potential skill only.**
- **A thorough assessment of forecast quality requires consideration of several skill scores.**
- **Frequently used scores favor *smooth* forecasts. It is difficult to demonstrate skill of high variability forecasts.**
- **Use creative graphics (such as the Taylor diagram) to visualize several skill measures.**