Objective Limits on Forecasting Skill of Rare Events

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ABSTRACT

A method for determining baselines of skill for the purpose of the verification of rare-event forecasts is described and examples are presented to illustrate the sensitivity to parameter choices. These “practically perfect” forecasts are designed to resemble a forecast that is consistent with that which a forecaster would make given perfect knowledge of the events beforehand. The Storm Prediction Center’s convective outlook slight risk areas are evaluated over the period from 1973 to 2011 using practically perfect forecasts to define the maximum values of the critical success index that a forecaster could reasonably achieve given the constraints of the forecast, as well as the minimum values of the critical success index that are considered the baseline for skillful forecasts. Based on these upper and lower bounds, the relative skill of convective outlook areas shows little to no skill until the mid-1990s, after which this value increases steadily. The annual frequency of skillful daily forecasts continues to increase from the beginning of the period of study, and the annual cycle shows maxima of the frequency of skillful daily forecasts occurring in May and June.

1. Introduction

Forecasting rare, severe weather events is challenging. Equally challenging, however, is the problem of developing verification procedures that can be meaningful from the standpoint of forecasters, forecast users, and the forecasting organization. A wide range of difficulties arises within this context, from collecting good observations of the phenomena in question to conveying information from the verification in a meaningful way.

In his essay on the nature of goodness in weather forecasting, Murphy (1993) defines “quality” (type 2 goodness) as the degree of correspondence between forecasts and events. Ten aspects of quality are defined, with “accuracy” and “skill” of particular interest when considering the forecasting of rare events. Accuracy is described as the average correspondence between individual pairs of forecasts and observations, while skill is described as the accuracy of forecasts relative to the accuracy of a forecast produced by a standard of reference.¹ So, while knowing the accuracy of a forecast is helpful, it may be more significant to know whether or not a forecast was skillful (and the degree of skill) in order to better understand the value added by a forecaster. For instance, a forecast that is accurate within 1.0° is more skillful when compared to a guidance forecast that is accurate within 5.0° than a guidance forecast that is accurate within 1.5°.

In this paper we will focus on one particular problem associated with the verification of rare-event forecasts: development of appropriate baselines for skill given that forecast difficulty varies from situation to situation. Efforts to identify “no skill” baselines date back to Gilbert (1884) and have focused primarily on the use of

¹ Note that there may be a large number of metrics used to describe accuracy and skill depending upon the particular forecasting situation.
faced by Brown et al. (1997) and Ebert et al. (2004), particularly in the form of guidance products such as the SPC's categorical convective outlooks issued by the Storm Prediction Center (SPC), the problem of verifying rare events takes on additional complexity. Much like the challenges faced by Brown et al. (1997) and Ebert et al. (2004), almost all of the observations come in from volunteer spotters, so that there is no regular temporal and spatial order to the observations. The existence of a forecast that an event is likely to occur could increase the likelihood that observers will be present to collect observations compared to a forecast of no event. Further, outlook products are issued with the explicit expectation that there will be “false alarms” (parts of the forecast for which there are no events) and “missed detections” (events which are not included in the forecast). Thus, the expected range of values of the probability of detection (POD) or false alarm rate (FAR), for example, does not run from 0 to 1 in practice. Here, we will discuss the concept of a “practically” perfect (PP) forecast (Brooks et al. 1998; Davis and Carr 2000) and apply it to the SPC’s categorical convective outlook products, specifically their “slight risk” areas, which we will refer to henceforth using the broad term “outlooks.” The dataset of outlooks used herein is described in Hitchens and Brooks (2012), with the addition of data from 2011.

By practically perfect, we mean a forecast that is consistent with that which a forecaster would make given perfect knowledge of the reported events beforehand and the operational constraints associated with the forecasting system. If, as in the case of outlooks, there are explicit or implicit limits on the size of the product (e.g., outlooks are rarely smaller than 50 000 km²) or if the forecaster has the goal of having a minimum number of reports within a forecast area before a product should be issued, then there will be false alarms and missed detections associated with the PP forecast. The PP forecast can then be used to estimate the maximum score, and as will be shown, also the minimum score that a forecaster could reasonably obtain. In general, that range will be much smaller than the absolute minimum and maximum, but will provide a range over which forecast performance can be judged. Note that such a concept is not limited to any particular score that can be derived from a set of verification data, nor is it limited to dichotomous (yes–no) forecasts. It can be applied to any forecast measure and to probabilistic forecasts easily.

For the verification of severe thunderstorm forecasts, particularly in the form of guidance products such as the convective outlooks issued by the Storm Prediction Center (SPC), the problem of verifying rare events takes on additional complexity. Much like the challenges faced by Brown et al. (1997) and Ebert et al. (2004),
2. Practically perfect forecasts

To develop the PP forecast, we will begin with the reports of events, as recorded at the SPC. Reports of severe weather are put on a grid with each grid box representing an area approximately 80 km $\times$ 80 km, roughly equivalent to the area associated with SPC forecast definitions of the probability of an event occurring within 25 miles (mi) of a point. For now, we will consider all severe weather reports as equal and look at only whether a box has had an event or not. (The methodology could be extended to consider the intensity and number of reports, but we will limit the procedure to the simplest case.) The PP forecast is then created by smoothing the events using nonparametric density estimation with a two-dimensional Gaussian kernel (Silverman 1986).
Specifically, at each grid point in the domain, the PP forecast value $f$ is given by

$$f = \sum_{n=1}^{N} \frac{1}{2\pi\sigma^2} \exp \left[ -\frac{1}{2} \left( \frac{d_n}{\sigma} \right)^2 \right],$$

where $d_n$ is the distance from the forecast grid point to the $n$th location that had an event occur, $N$ is the total number of grid points with events, and $\sigma$ is a weighting function that can be interpreted as the confidence one has in the location of the forecast event. Increasing $\sigma$ is equivalent to increasing the spatial uncertainty associated with the forecast as one would do with increasing lead time of the forecast. That is, within the context of severe weather forecasting, very small $\sigma$ can be thought of as being associated with the warning stage, while larger $\sigma$ is associated with the watch or convective outlook stages.

The field of $f$ gives an artificial forecast that is as good as could be expected for a forecaster knowing the locations of events with a confidence level associated with $\sigma$; it gives the probability that an event occurs in a given grid box. To illustrate the impact of the choice of the $\sigma$ value (0.75, 1.5, or 3.0), we examine events representing the two extremes on the spectrum of severe weather forecast scenarios: a large “outbreak” (case 1; Figs. 1a–e) and a relatively small number of reports with little spatial concentration (case 2; Figs. 1f–j). There are 1045 severe weather reports covering 209 grid boxes in case 1, and 73 reports covering 18 grid boxes in case 2. For both cases the PP forecast size decreases with increasing probability threshold values, with the rate of decrease being larger at higher $\sigma$ values (Fig. 2). Concentrating on each $\sigma$ value, for 0.75 the size of the 0.01 or greater probability area for case 1 (case 2) is 1.79 (1.27) million km$^2$, and reduces to no area at a probability of 94% (57%). Likewise, for a $\sigma$ value of 1.5 (3.0), the size of the 0.01 probability area is 3.04 (4.76) million km$^2$ for case 1, and 2.60 (4.00) million km$^2$ for case 2. The probability threshold at which it reduces to no area for case 1 is 100% (84%) and is 28% (17%) for case 2.

The choice of the value of $\sigma$ should at least in part be made with the goal of emulating the range of sizes made by forecasters. A comparison of quantiles of outlook size and PP forecast size from 1982–91 and 2002–11 (Fig. 3) reveals a shift from outlooks being more closely associated in size to PP forecasts at the 5% threshold to PP forecasts at the 10% threshold. Forecasts from the mid-1990s were purposely excluded due to apparent changes in forecasting philosophies that significantly impacted the size of outlook areas (Hitchens and Brooks 2012). Quantiles of the first two $\sigma$ values (0.75 and 1.5) correspond well to the line $y = x$, with noticeable curvature toward the $y$ axis at lower quantiles, suggesting that there is greater variability among outlook sizes at these quantiles. The quantiles for $\sigma = 3.0$ are sharply curved toward the $x$ axis at lower quantiles, indicating more dispersion among PP forecasts at these quantiles and, in general, show much less correspondence to the line $y = x$ compared to the other $\sigma$ values.

By considering each increasing PP forecast probability value as an individual threshold, we can convert the range of probabilities into a set of dichotomous (yes or no) forecasts of severe weather. This allows for the development of a $2 \times 2$ contingency table for each probability threshold (Table 1) and the calculation of standard measures of performance [see Doswell et al. (1990) for a complete description of the measures used herein]. For each probability threshold the critical success index (CSI), a performance measure that compares the number of correct forecasts to the union of all forecasts and observations, is calculated. The value of the CSI at a
probability threshold of 0% (always forecast yes) is equal to the areal coverage of the event. This represents one estimate of the lower bound on expected performance, which the forecaster could attain by forecasting that the event would occur everywhere. A slightly greater lower bound can be found by noting that there is a large drop in CSI from a probability threshold of 1% to a threshold of 0%, and considering the value that CSI approaches as the probability threshold approaches zero. The 2% and 1% CSI values are used to linearly extrapolate this adjusted lower bound. In the two cases described above the CSI values at the 0% threshold are 0.06 and 0.02 for cases 1 and 2, respectively, while the values for the adjusted lower bound are 0.29 and 0.08 (for $\sigma = 1.5$).
As $s$ increases, the maximum value of CSI generally decreases (Fig. 4). Assuming that a value of $s$ can be found that produces forecasts that “look” like real forecasts (e.g., similar areal coverage), the practical maximum value of CSI for that situation, given the nature of the forecast, can be obtained. Thus, the simple artificial forecast can be used to estimate the upper and lower bounds on the performance measure. Based on the size comparison above, $s = 3.0$ would not be a suitable choice, and of the remaining two values, $s = 1.5$ better represents the outlooks issued by the SPC. For example, the maximum CSI value attained using $s = 0.75$ in case 1 (case 2) is 0.90 (0.84), while for $s = 1.5$ the maximum is 0.78 (0.31). Not only does $s = 1.5$ provide a more reasonable upper bound, especially in the case of an event that is less concentrated (e.g., Fig. 4b), but it also compares better visually to the SPC’s outlooks (Fig. 1). Thus, $s = 1.5$ will be used hereafter when PP values are calculated. Using this approach to define practical lower and upper bounds for severe weather forecasts, we can determine the skill of a forecast by calculating its relative position between the two bounds. In our two cases the relative skill of the first case is 0.71 and 0.13 for the second, providing increases from their CSI values of 0.64 and 0.11.

### 3. Application to categorical forecasts

Expanding this analysis to the entire digital dataset of SPC outlooks (1973–2011), it is seen from a 365-day running mean$^3$ that these forecasts show little to no skill until 1987, after which the relative skill remains above zero (Fig. 5). Further, until the mid-1990s the relative skill shows little variation, ranging between −0.03 and 0.06, but beginning in 1995 there is a steady increase until the end of the period of record, with a maximum relative skill of 0.25 in 2010. This corresponds with the improvements in POD and frequency of hits (FOH) observed in Hitchens and Brooks (2012) for the same forecasts. A number of factors could have contributed to this increase in relative skill, including advancements in numerical weather prediction, changes in forecasting philosophies within the SPC, improved physical understanding of severe thunderstorms, and the completion of upgrades to the national radar network. It is impossible to determine the relative importance of these, or other, factors, particularly considering that we cannot put strong limits on the interannual variability of the difficulty of the forecasts. Through the first 25 yr of data the CSI value for

### Table 1. A 2×2 contingency table for forecasts and observations.

<table>
<thead>
<tr>
<th></th>
<th>Observed yes</th>
<th>Observed no</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forecast yes</td>
<td>$a$</td>
<td>$b$</td>
<td>$a+b$</td>
</tr>
<tr>
<td>Forecast no</td>
<td>$c$</td>
<td>$d$</td>
<td>$c+d$</td>
</tr>
<tr>
<td>Sum</td>
<td>$a+c$</td>
<td>$b+d$</td>
<td>$n$</td>
</tr>
</tbody>
</table>

Quantities of interest: probability of detection (POD) = $a/(a+c)$, frequency of hits (FOH) = $a/(a+b)$, critical success index (CSI) = $a/(a+b+c)$, and bias = $(a+b)/(a+c)$.

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$^3$ In this study 365-day running means are computed by constructing a 2×2 table that sums all 365 forecasts centered on each day. In the case of maximum CSI from PP forecasts, the 2×2 table associated with each day’s maximum CSI value is used in the construction of the table for the 365-day period.
both the upper and lower bounds increased steadily, by \( \sim 0.10 \) for the upper bound and \( \sim 0.04 \) for the lower, but from 1998 onward neither displayed a noticeable trend. Likewise, during the first two decades the CSI of the outlooks increased at nearly the same rate as the lower bound, but after 1995 it increased at a higher rate than the lower bound, accounting for the increase in relative skill over that time.

The SPC outlooks and the practical upper and lower bounds can be further analyzed using a performance diagram of annual mean values to examine additional measures related to these forecasts (Fig. 6). As expected, the lower bound has a POD of 1.0 since these PP forecast areas include all reports, but these large areal extents limit the FOH to 0.05–0.11. Of more interest is the upper bound, which shows a moderate increase in POD through time (0.50–0.69), but exhibits the most improvement in FOH (0.38–0.65). While the POD of the outlooks increased significantly during the first 20 yr of the study period, it has not displayed a steady improvement in the remaining years, but only trails the upper bound by 0.12 for 2011. Although the FOH values of the outlooks have been increasing since the mid-1990s, the best value (0.26 in 2011) has not yet reached the lowest value of the upper bound (0.38 in 1977) and trails the FOH of 2011 (0.63) by 0.37. The recent improvement in the performance of the outlooks, as well as the potential for continued improvement, can be attributed to the placement of outlook areas (i.e., correctly forecasting as many events as possible while seeking to minimize outlook size).

Another useful application of the upper and lower bounds is for investigating how often individual outlooks are skillful (i.e., positive, nonzero values of relative skill). Throughout the period of record the annual frequency of skillful forecasts has increased steadily from 22% in 1973 to 75% in 2011 when considering all forecast days,\(^4\) with a maximum of 78% during 2008 (Fig. 7). These frequencies are slightly higher when considering only days with both an outlook and at least one observation, with a maximum of 86% during 2009. The annual cycle of skillful forecast frequency for all days (Fig. 8) shows a peak in May (55%) during the 10-yr period covering 1982–91, and for 2002–11 the peak frequency occurs in June (86%), with a distinct secondary peak in November (69%). For both decades January is the month with the lowest frequency of skillful forecasts (23% and 54%, respectively). It should be noted that the entirety of the frequencies from the latter period are above 50%, while only two (April and May) from the prior period exceed this number. Frequencies in the first decade are impacted by a larger proportion of false alarms and missed events during the cool season (October–February) compared to the latter decade, while the proportions of these events were similar in both decades for the remaining months. The 90% confidence interval for each decade is constructed from 101 trials with random samples without replacement sized at half the number of events for a particular month. These confidence intervals suggest that the frequencies from each decade are well separated, with larger intervals during the cool season in both periods.

Over the same two 10-yr periods, the frequency of the relative skill values for all days is calculated (Fig. 9) illustrating an increase in skillful forecasts from 44% to 73%. This increase is the result of forecasts that showed small negative (0.0 to \( -0.2 \)) to small positive (\( \pm 0.1 \)) skill during 1982–91 instead showing more skill in the moderate to high positive values (0.2–0.6). From a forecasting perspective this change in the distribution of relative skill is partly caused by the reduction of false alarms by 58% (a reduction in frequency of 0.03), which are considered to have zero relative skill, as well as increases in the skill of forecasts with relative skill values of at least \( -0.2 \) by

\(^4\) The term “all forecast days” includes days when an outlook was issued and no reports were recorded (“false alarm”), and days when no outlook was issued but reports were recorded (“missed events”). In the latter scenario the area of the upper bound must be at least as large as the smallest regular outlook area (~64 000 km²).
making better forecasts (namely increasing FOH). Another interesting change observed in Fig. 9 is the increase in the frequency of forecasts with relative skill values between −0.2 and −0.3. This increase (0.0228) is entirely the result of a rise in missed events (up 49%), contributing 0.0235 to the frequency of the relative skill values in that range. The discrepancy is likely due to slight improvements in forecasts with accompanying observations (i.e., forecasts that were not “false alarms”) between the two decades. Some of these “missed events” are rather substantial, with 46% having at least 20 grid boxes containing reports, and 5% with at least 50 grid boxes. However, the majority of these apparent misses are forecast by the SPC as “see text”; locations where a threat of severe weather exists, but that threat is not sufficient to issue a slight risk. These forecasts were first issued publicly beginning in 1999, and their spatial extents are not explicitly defined.

As demonstrated in the performance diagram, improvements in CSI (and relative skill) are influenced more by improvements in FOH. This relationship is supported in a plot of FOH values as a function of relative skill (Fig. 10), with a 0.77 coefficient of determination ($R^2$). For relative skill values from just below 0.0 to
approximately 0.2 there appears to exist a well-defined minimum threshold of FOH values. In contrast, there is no significant relationship between the size of the outlook areas or the observation areas and relative skill (Fig. 11). Although no relationship exists with the size of outlook areas ($R^2 = 0.001$), there is a tendency for outlook sizes related to more extreme relative skill values to be smaller relative to some outlooks with relative skill values near zero. Outlooks with smaller areas are more likely to have extreme relative skill values since the FOH plays such a large role in the calculation; a small-sized outlook that misses a very localized cluster of reported events will have an FOH and POD of zero, while in the same scenario, if the small-sized outlook is well placed, it will have a large FOH and POD. Larger outlooks are less likely to have greater values of FOH due to a higher likelihood of false alarms. On the other hand, the size of the observation areas ($R^2 = 0.22$) is relatively uniform across the range of relative skill values, suggesting that forecasters do not seem to perform better (or worse) based on the areal extent of an event.

4. Concluding remarks

The primary objective of this paper was to develop objective, practical baselines for forecasts of rare events. This is accomplished by using practically perfect forecasts to identify a “no skill” lower bound and a practical upper bound of CSI values for any particular forecast. The position of the CSI value of a forecast relative to the values attained from a PP forecast indicates the skill of that forecast. The choice of $\sigma$ in the calculation of PP forecasts plays an important role, and can be effectively used to simulate the level of uncertainty inherent to a particular forecast (e.g., convective outlooks versus severe weather watches). For the purpose of evaluating the SPC’s convective outlooks, $\sigma = 1.5$ was chosen based on a comparison of the sizes of PP forecasts and outlooks, as well as the range of CSI values for the PP forecasts.

Analysis of convective outlooks issued from 1973 to 2011 using PP forecasts for CSI baselines revealed that
these forecasts showed little to no relative skill for two decades, but after 1995 the relative skill of the outlooks has shown steady improvement. The timing of this increase in relative skill is not unexpected since Hitchens and Brooks (2012) found that FOH values for these forecasts began to increase at a similar point in time. Additionally, the annual frequency of skillful forecasts has continued to increase, with at least 50% of forecasts each year since 1995 showing some skill.

We plan to extend the analyses using PP forecasts to the full suite of convective outlook products—the “day 2” and “day 3” forecasts valid for the same 24-h period examined in this study, and the various updates to the 0600 UTC “day 1” outlook—and the probabilistic convective outlooks that were introduced in the early 2000s. Future work will also explore approaches to include the number and intensity of events within the PP forecast framework.

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