# Taylor Diagram Primer

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Taylor diagrams (Taylor, 2001) provide a way of graphically summarizing how closely a pattern (or a set of patterns) matches observations. The similarity between two patterns is quantified in terms of their correlation, their centered root-mean-square difference and the amplitude of their variations (represented by their standard deviations). These diagrams are especially useful in evaluating multiple aspects of complex models or in gauging the relative skill of many different models (e.g., IPCC, 2001).

Figure 1 is a sample Taylor diagram which shows how it can be used to summarize the relative skill with which several global climate models simulate the spatial pattern of annual mean precipitation. Statistics for eight models were computed, and a letter was assigned to each model considered. The position of each letter appearing on the plot quantifies how closely that model's simulated precipitation pattern matches observations. Consider model F, for example. Its

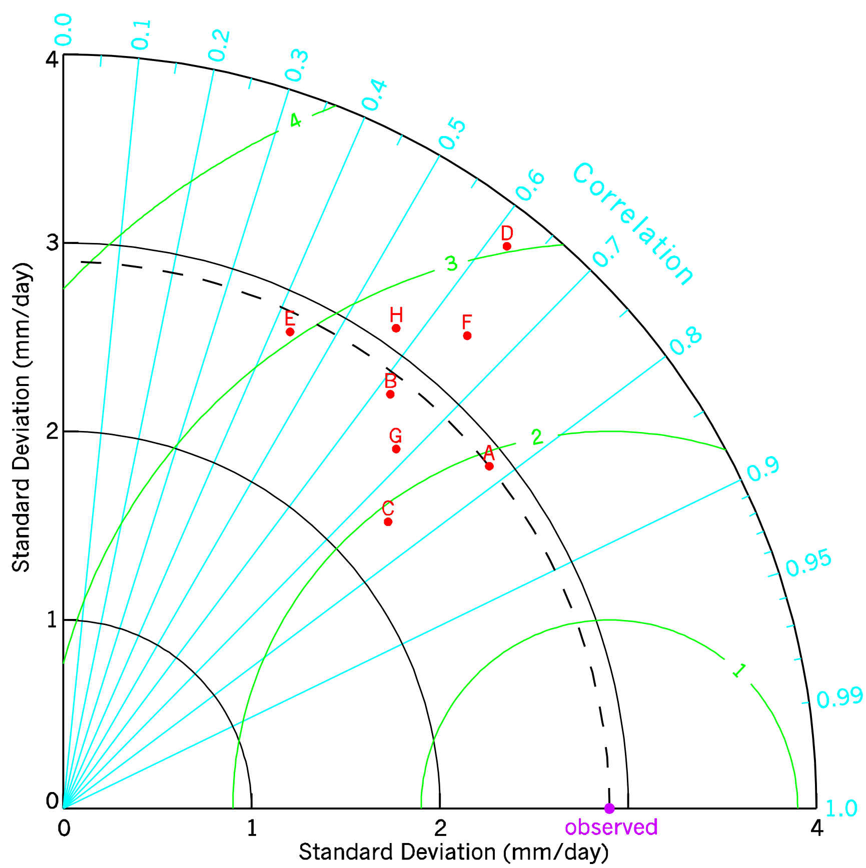


Figure 1: Sample Taylor diagram displaying a statistical comparison with observations of eight model estimates of the global pattern of annual mean precipitation.

pattern correlation with observations is about 0.65. The centered root-mean-square (RMS) difference between the simulated and observed patterns is proportional to the distance to the point on the x-axis identified as "observed." The green contours indicate the RMS values and it can be seen that in the case of model F the centered RMS error is about 2.6 mm/day. The standard deviation of the simulated pattern is proportional to the radial distance from the origin. For model F the standard deviation of the simulated field (about 3.3 mm/day) is clearly greater than the observed standard deviation which is indicated by the dashed arc at the observed value of 2.9 mm/day.

The relative merits of various models can be inferred from figure 1. Simulated patterns that agree well with observations will lie nearest the point marked "observed" on the x-axis. These models will have relatively high correlation and low RMS errors. Models lying on the dashed arc will have the correct standard deviation (which indicates that the pattern variations are of the right amplitude). In figure 1 it can be seen that models A and C generally agree best with observations, each with about the same RMS error. Model A, however, has a slightly higher correlation with observations and has the same standard deviation as the observed, whereas model C has too little spatial variability (with a standard deviation of 2.3 mm/day compared to the observed value of 2.9 mm/day). Of the poorer performing models, model E has a low pattern correlation, while model D has variations that are much larger than observed, in both cases resulting in a relatively large (~3 mm/day) centered RMS error in the precipitation fields. Note also that although models D and B have about the same correlation with observations, model B simulates the amplitude of the variations (i.e., the standard deviation) much better than model D, and this results in a smaller RMS error.

In general, the Taylor diagram characterizes the statistical relationship between two fields, a "test" field (often representing a field simulated by a model) and a "reference" field (usually representing “truth”, based on observations). Note that the means of the fields are subtracted out before computing their second-order statistics, so the diagram does not provide information about overall biases, but solely characterizes the *centered* pattern error.

The reason that each point in the two-dimensional space of the Taylor diagram can represent three different statistics simultaneously (i.e., the centered RMS difference, the correlation, and the standard deviation) is that these statistics are related by the following formula:

$$E'^2 = \sigma_f{}^2 + \sigma_r{}^2 - 2\sigma_f \sigma_r R,$$

where R is the correlation coefficient between the test and reference fields, $E'$ is the centered RMS difference between the fields, and $\sigma_f^2$ and $\sigma_r^2$ are the variances of the test and reference fields, respectively. (The formulas for calculating these second order statistics are provided at the end of this document.) The construction of the diagram (with the correlation given by the cosine of the azimuthal angle) is based on the similarity of the above equation and the Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\phi$$

There are several minor variations on the diagram that have been found useful for various purposes (see, Taylor, 2001). For example,

- The diagram can be extended to a second "quadrant" (to the left) to allow for negative correlations.
- The statistics can be normalized (and non-dimensionalized), dividing both the RMS difference and the standard deviation of the "test" field by the standard deviation of the observations. In this case the "observed" point is plotted on the x-axis at unit distance from the origin. This makes it possible to plot statistics for different fields (with different units) on the same plot.
- The isolines drawn on the sample plot above are often omitted to make it easier to see the plotted points.
- When comparing fields simulated by two different versions of a model, the two points on the graph representing those fields are often connected by an arrow to indicate more clearly whether or not the model is moving toward "truth," as defined by observations.

Some sample diagrams are available here.

**Further notes:**

Given a "test" field (*f*) and a reference field (*r*), the formulas for calculating the correlation coefficient (*R*), the centered RMS difference (*E'*), and the standard deviations of the "test" field ($\sigma_f$) and the reference field ($\sigma_r$) are given below:

$$R = \frac{\frac{1}{N}\sum_{n=1}^{N}\left(f_n - \bar{f}\right)\left(r_n - \bar{r}\right)}{\sigma_f \sigma_r}$$

$$E'^2 = \frac{1}{N}\sum_{n=1}^{N}\left[\left(f_n - \bar{f}\right) - \left(r_n - \bar{r}\right)\right]^2$$

$$\sigma_f^2 = \frac{1}{N}\sum_{n=1}^{N}\left(f_n - \bar{f}\right)^2$$

$$\sigma_r^2 = \frac{1}{N}\sum_{n=1}^{N}\left(r_n - \bar{r}\right)^2$$

where the overall mean of a field is indicated by an overbar. In the case of a time-independent field, the sum is computed over all grid cells. For the typical spatial grid, the grid cell area is not uniform, so each grid cell must be weighted by the fraction of the total area represented by that grid cell. In the case of a time varying field, the sum is a double-sum computed over all grid cells and all time samples.

**References:**

Taylor, K.E.: Summarizing multiple aspects of model performance in a single diagram. *J. Geophys. Res.,* **106**, 7183-7192, 2001 (also see PCMDI Report 55, http://www-pcmdi.llnl.gov/publications/ab55.html)

IPCC, 2001: *Climate Change 2001: The Scientific Basis, Contribution of Working Group I to the Third Assessment Report of the Intergovernmental Panel on Climate Change* [Houghton, J.T., Y. Ding, D.J. Griggs, M. Noguer, P.J. van der Linden, X. Dai, K. Maskell, and C.A. Johnson (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, 881 pp. (see http://www.grida.no/climate/ipcc_tar/wg1/317.htm#fig84)