Lecture 2: Turbulence

Announcements

- reading: Stull ch. 2 (sections 2.4 and 2.8 optional)
- video: Turbulence w/Robert W. Stewart

Today's Lecture

1. Turbulence characteristics
2. Reynolds (exp, #, decomp, avg, stress)
3. Law of the Wall
4. TKE (brief)

25 January 2023

Lecture 2: Turbulence

25 January 2023

Lecture 2: Turbulence

Announcements

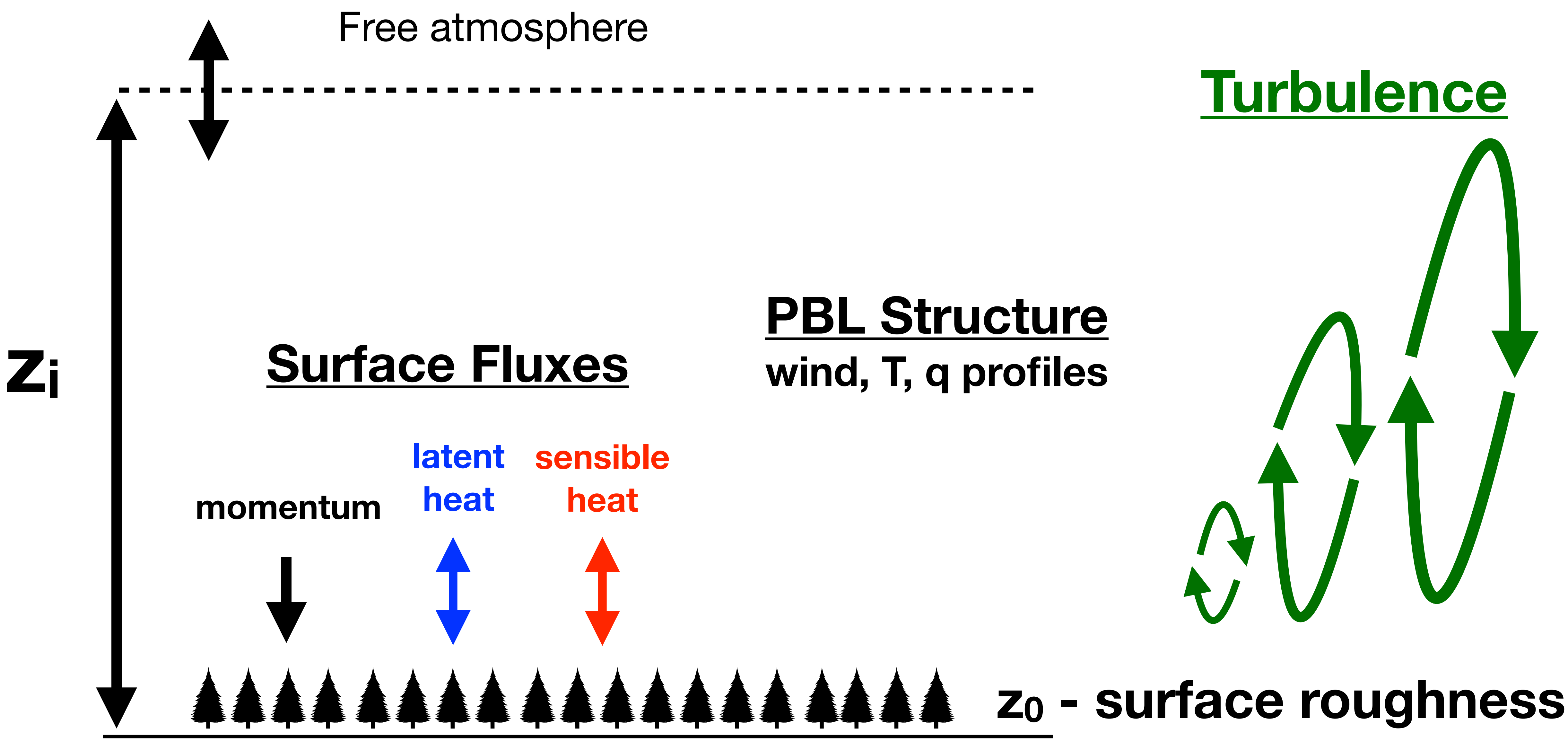
- reading: Stull ch. 2 (sections 2.4 and 2.8 optional)
- video: Turbulence w/Robert W. Stewart

Today's Lecture

1. Turbulence characteristics
2. Reynolds (exp, #, decomp, avg, stress)
3. Law of the Wall
4. TKE (brief)

25 January 2023

Conceptual Model of the ABL



horizontally homogeneous

Vertical Layers

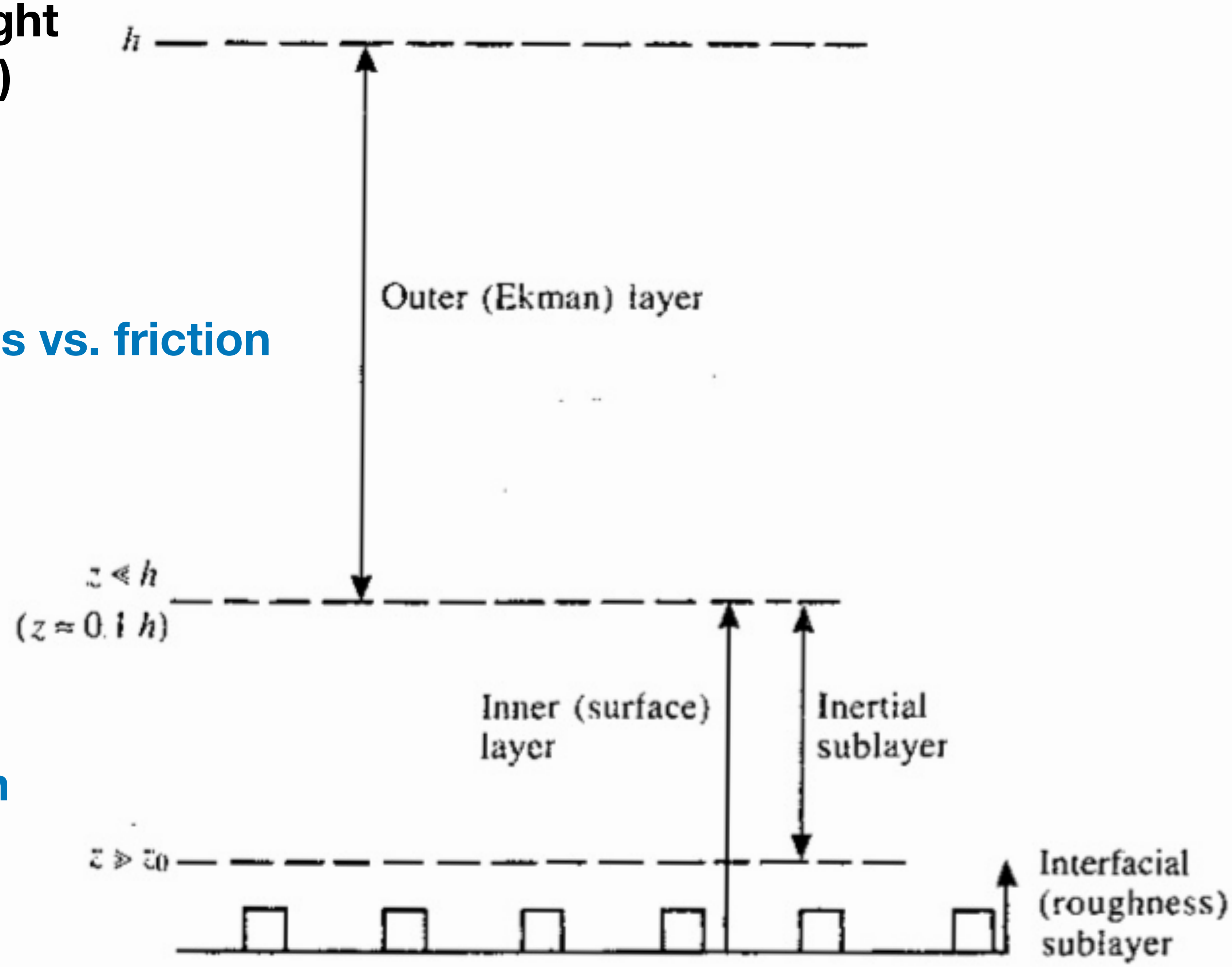
Free atmosphere: pressure gradient vs. Coriolis

PBL height
(~1km)

Outer BL:
pressure grad vs. Coriolis vs. friction

(~100 m)

Inner BL (surface layer):
pressure grad vs. friction



Garrat (1994)

Spectrum of Wind Speed

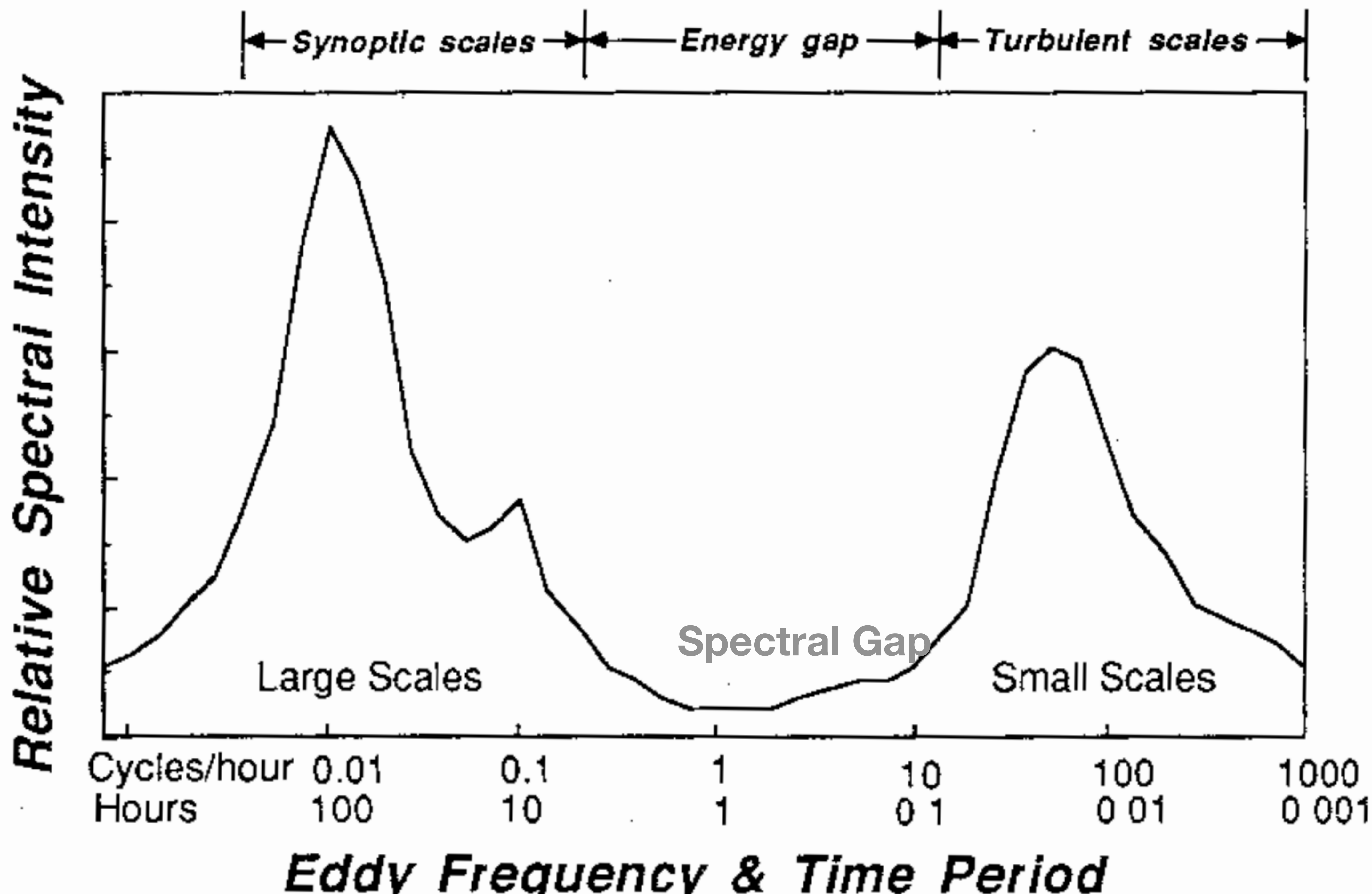


Fig. 2.2 Schematic spectrum of wind speed near the ground estimated from a study of Van der Hoven (1957)

Spectrum of Wind Speed

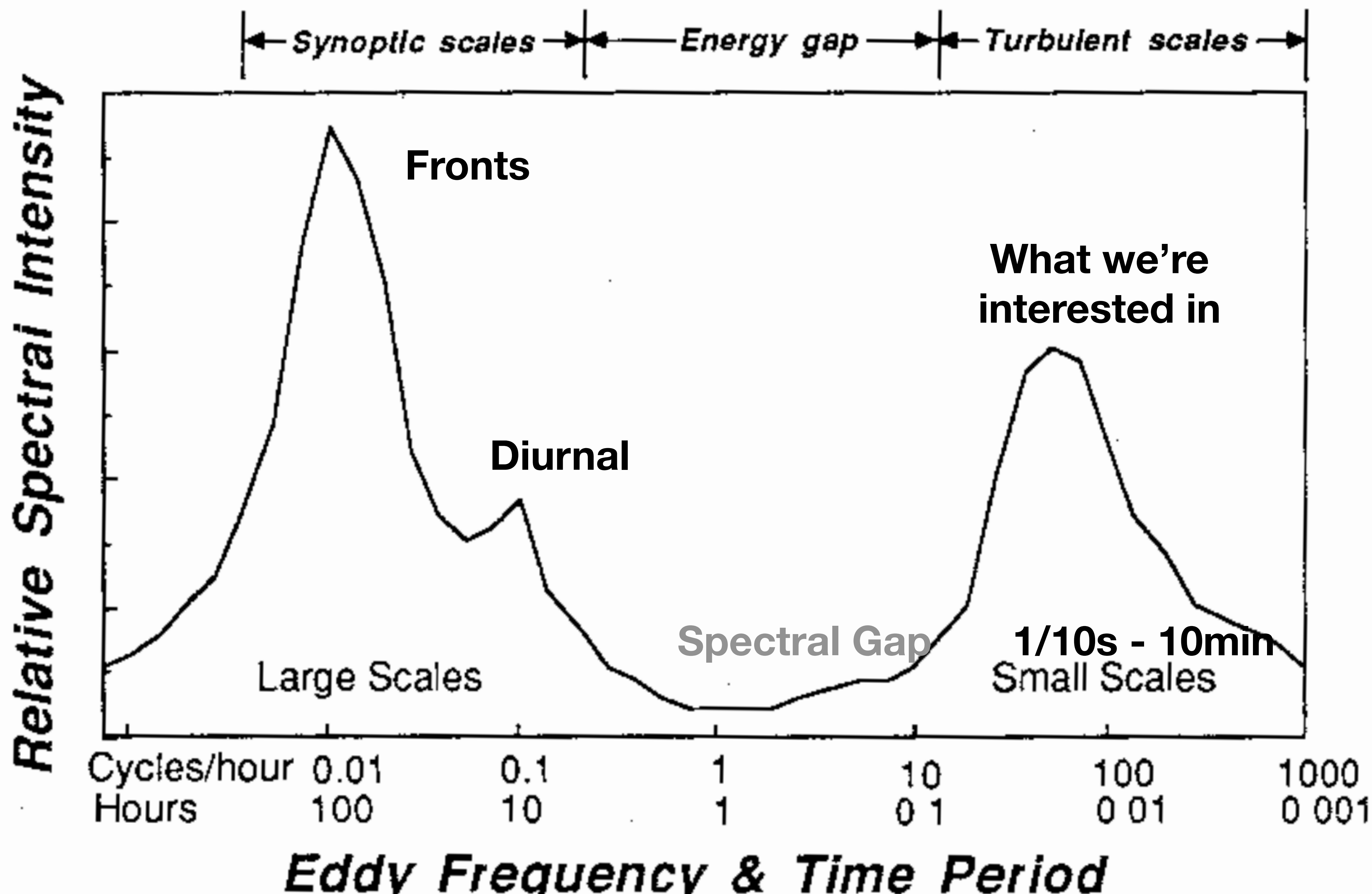


Fig. 2.2 Schematic spectrum of wind speed near the ground estimated from a study of Van der Hoven (1957)

Temporal Scales & Spectral Gap

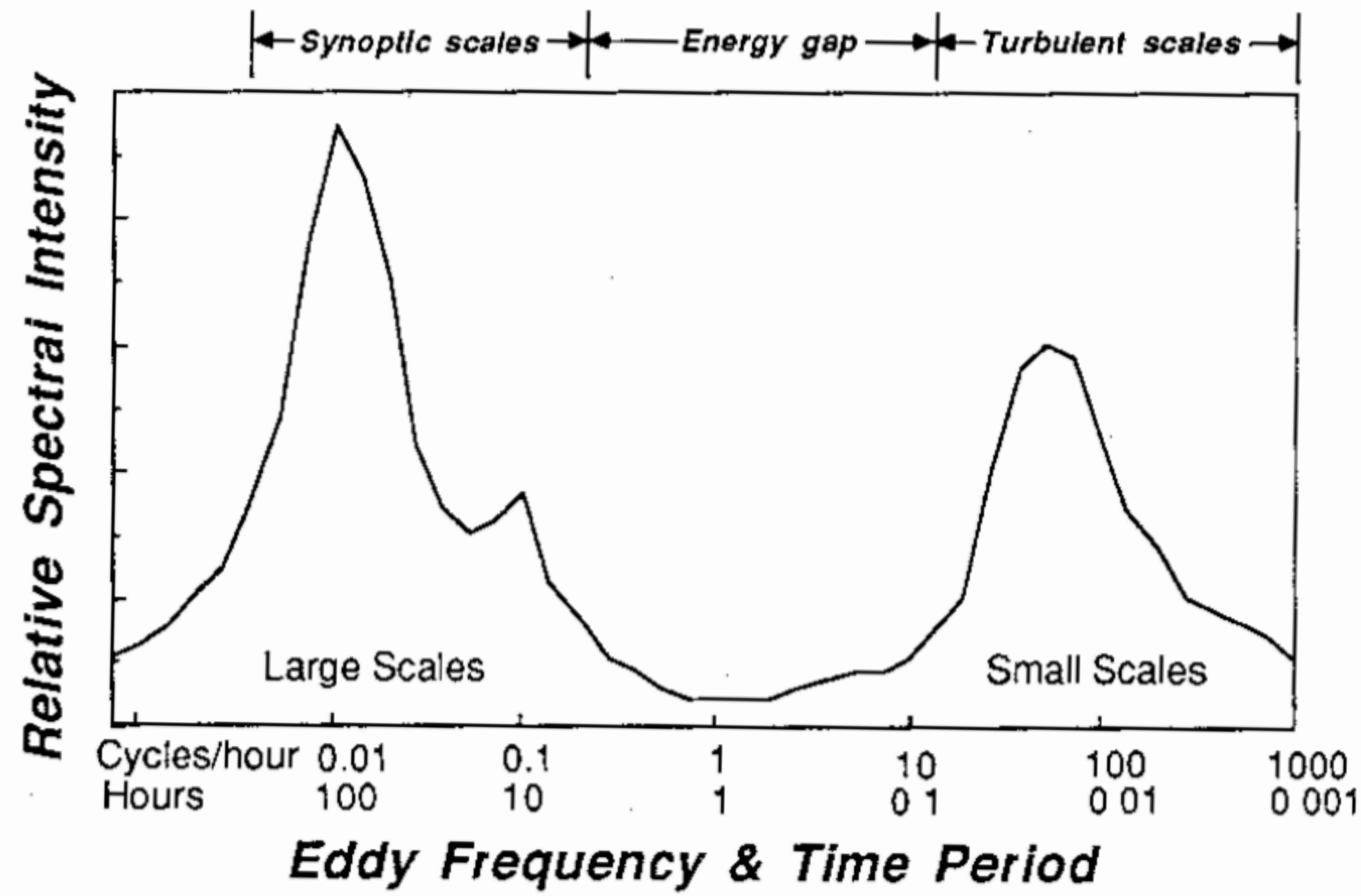
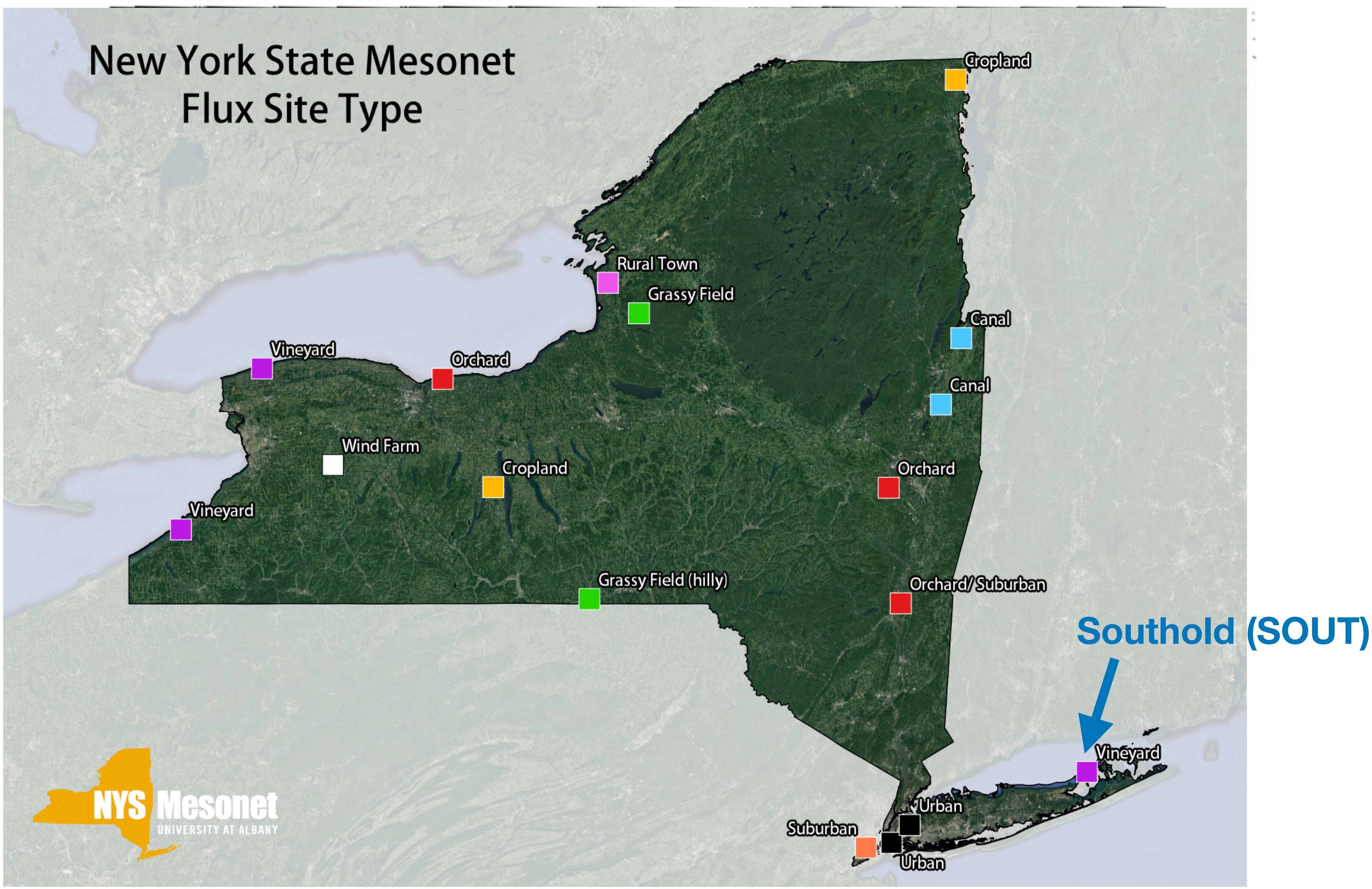


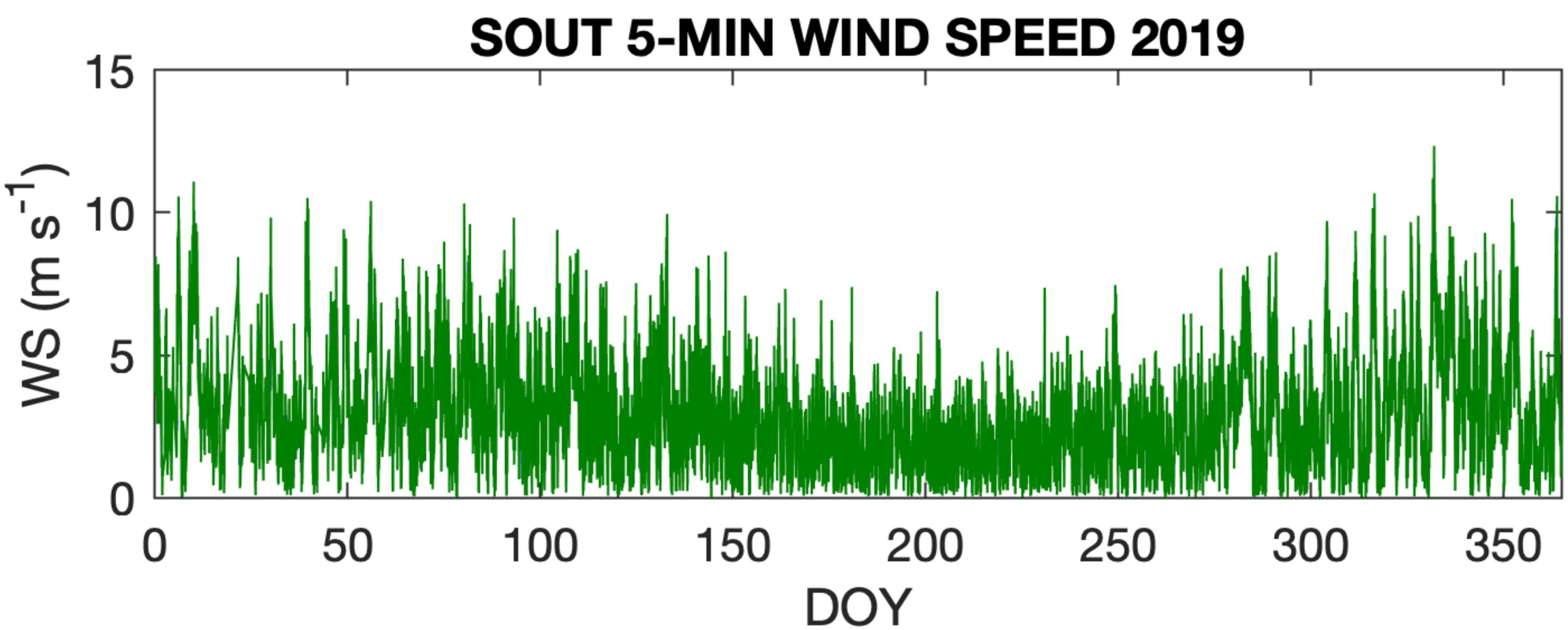
Fig. 2.2 Schematic spectrum of wind speed near the ground estimated from a study of Van der Hoven (1957)

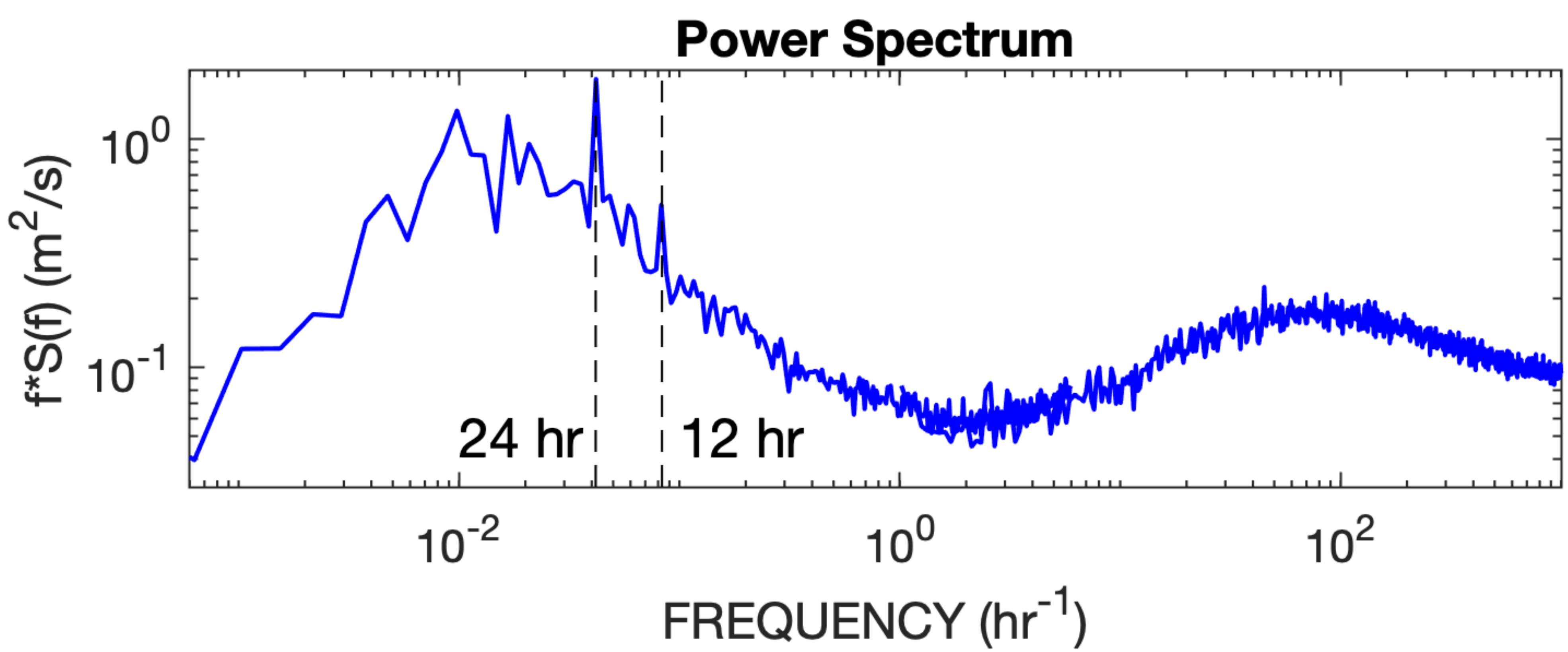
Stull (1988)

Temporal Scales & Spectral Gap



Temporal Scales & Spectral Gap





Taylors Frozen Flow Hypothesis

$$time \Leftrightarrow space$$

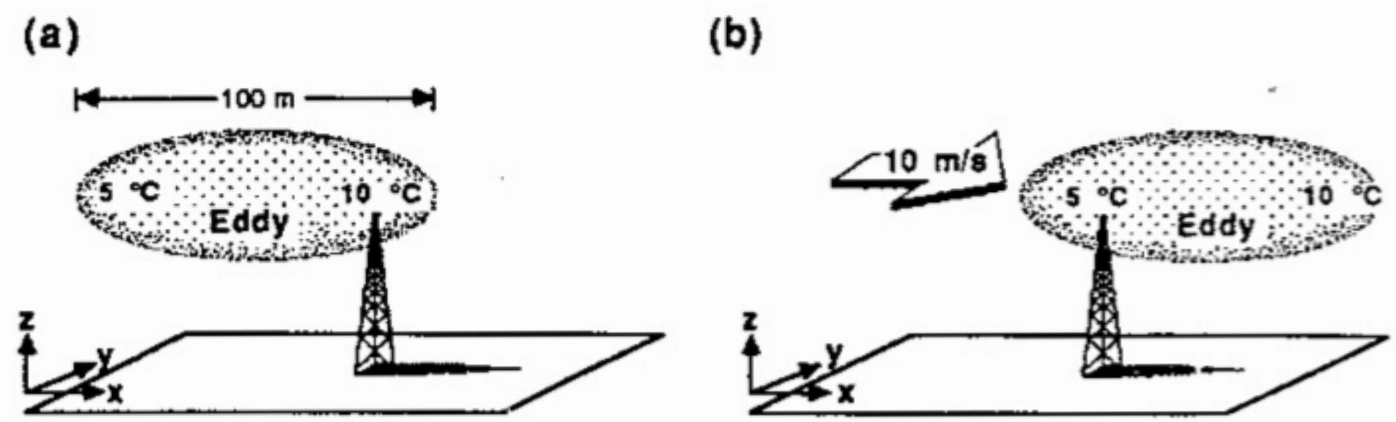


Fig. 1.4 Illustration of Taylor's hypothesis. (a) An eddy that is 100 m in diameter has a 5 ° C temperature difference across it. (b) The same eddy 10 seconds later is blown downwind at a wind speed of 10 m/s.

Stull (1988)

Taylors Frozen Flow Hypothesis

$$time \Leftrightarrow space$$

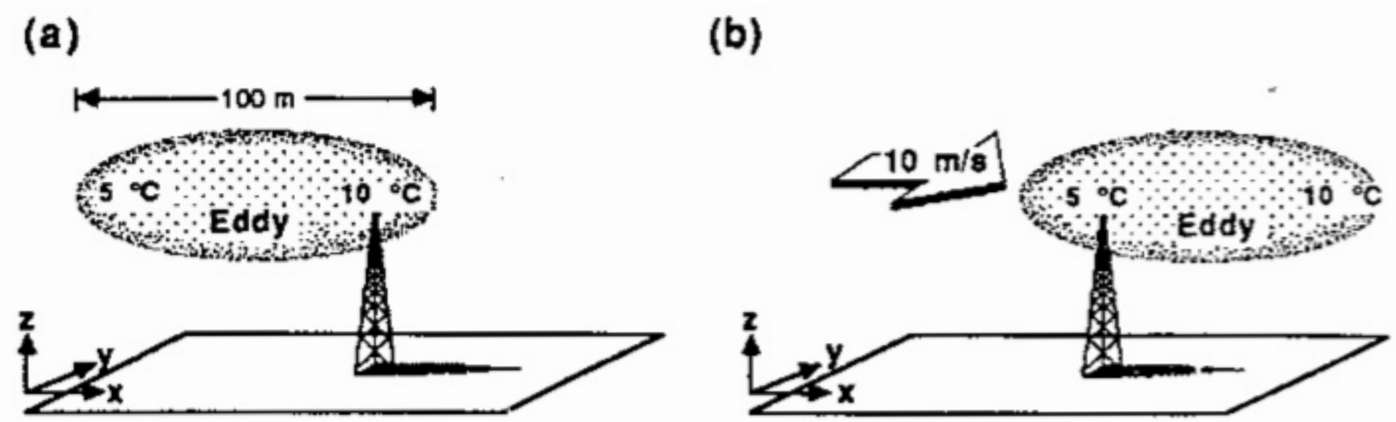


Fig. 1.4 Illustration of Taylor's hypothesis. (a) An eddy that is 100 m in diameter has a 5 ° C temperature difference across it. (b) The same eddy 10 seconds later is blown downwind at a wind speed of 10 m/s.

$$\frac{d}{dt} = 0$$

Stull (1988)

Taylors Frozen Flow Hypothesis

$$time \Leftrightarrow space$$

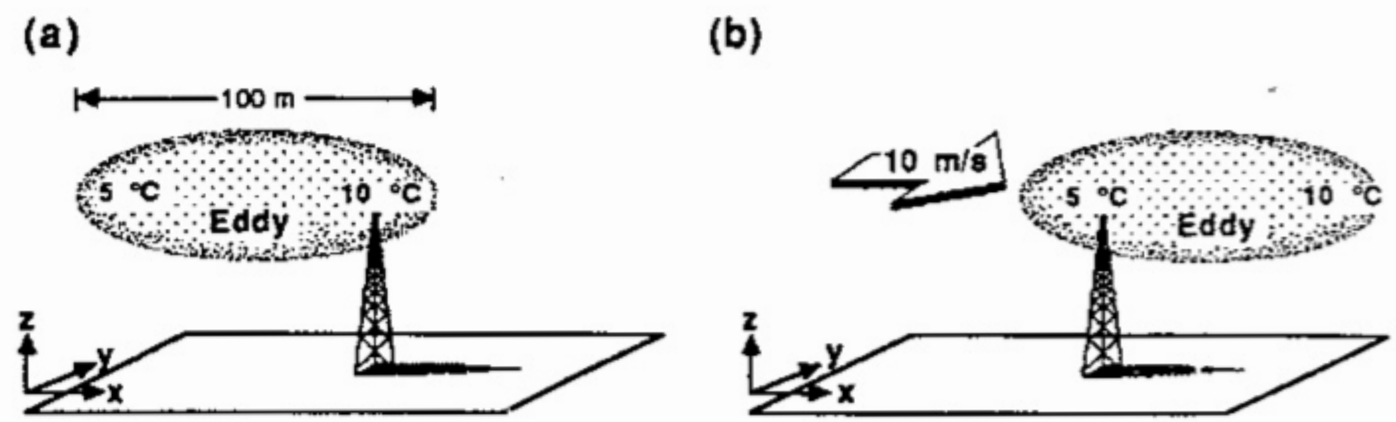


Fig. 1.4 Illustration of Taylor's hypothesis. (a) An eddy that is 100 m in diameter has a 5 ° C temperature difference across it. (b) The same eddy 10 seconds later is blown downwind at a wind speed of 10 m/s.

$$\frac{d}{dt} = 0 \longrightarrow \frac{\partial}{\partial t} + U\frac{\partial}{\partial x} = 0$$

Stull (1988)

Taylors Frozen Flow Hypothesis

$$time \Leftrightarrow space$$

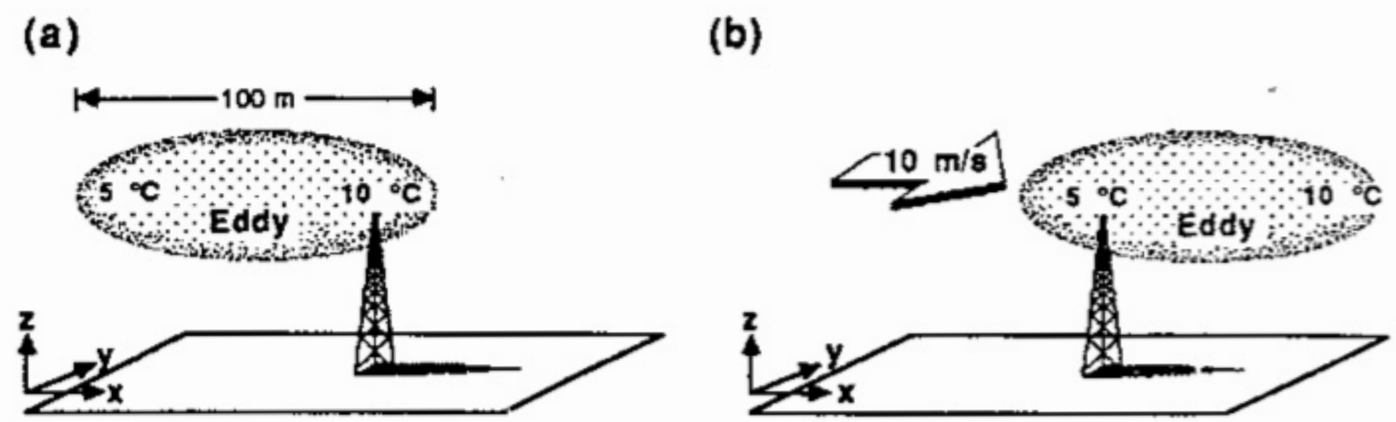


Fig. 1.4 Illustration of Taylor's hypothesis. (a) An eddy that is 100 m in diameter has a 5 ° C temperature difference across it. (b) The same eddy 10 seconds later is blown downwind at a wind speed of 10 m/s.

$$\frac{d}{dt} = 0 \longrightarrow \frac{\partial}{\partial t} + U\frac{\partial}{\partial x} = 0 \longrightarrow \frac{\partial}{\partial x} = -\frac{1}{U}\frac{\partial}{\partial t}$$

Stull (1988)

Taylors Frozen Flow Hypothesis

$$time \Leftrightarrow space$$

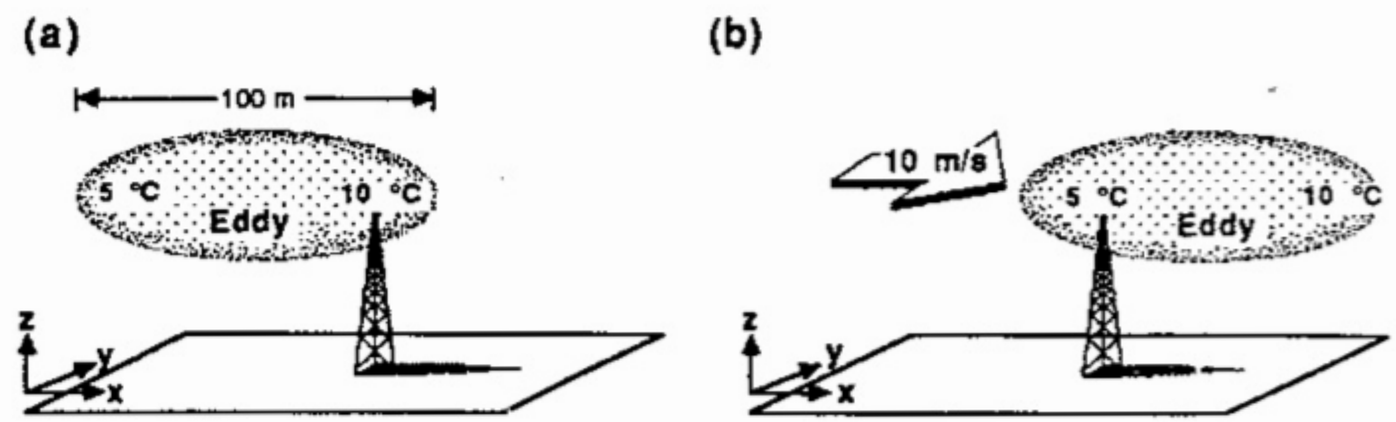


Fig. 1.4 Illustration of Taylor's hypothesis. (a) An eddy that is 100 m in diameter has a 5 ° C temperature difference across it. (b) The same eddy 10 seconds later is blown downwind at a wind speed of 10 m/s.

$$\frac{d}{dt} = 0 \longrightarrow \frac{\partial}{\partial t} + U\frac{\partial}{\partial x} = 0 \longrightarrow \frac{\partial}{\partial x} = -\frac{1}{U}\frac{\partial}{\partial t}$$

$$\frac{\Delta T}{\Delta x} = -\frac{1}{U}\frac{\Delta T}{\Delta t} = -\frac{1}{10ms^{-1}} \times \frac{-5C}{10s} = 0.05^{o}Cm^{-1}$$

Stull (1988)

scalar flux

$$\frac{J}{m^2 \cdot s} = \frac{W}{m^2}$$

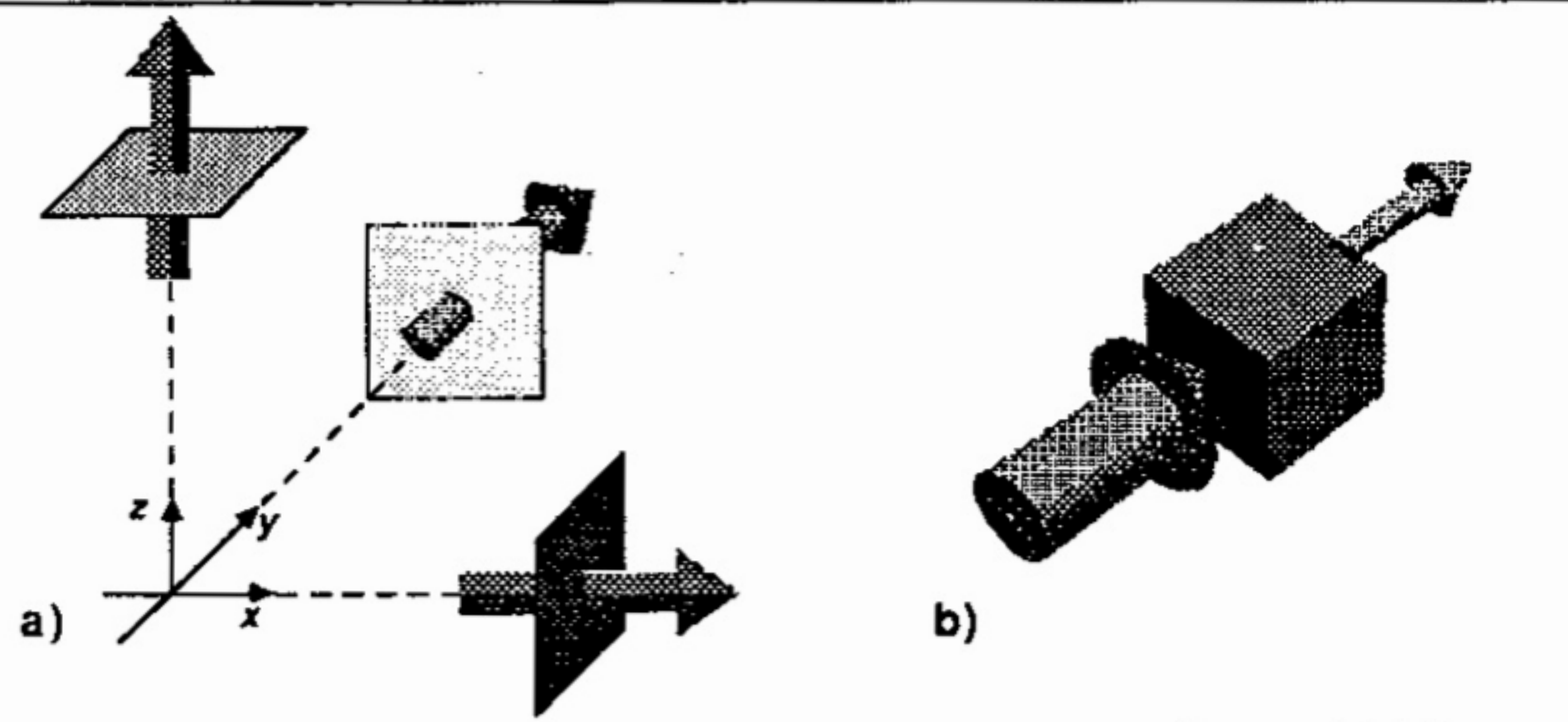
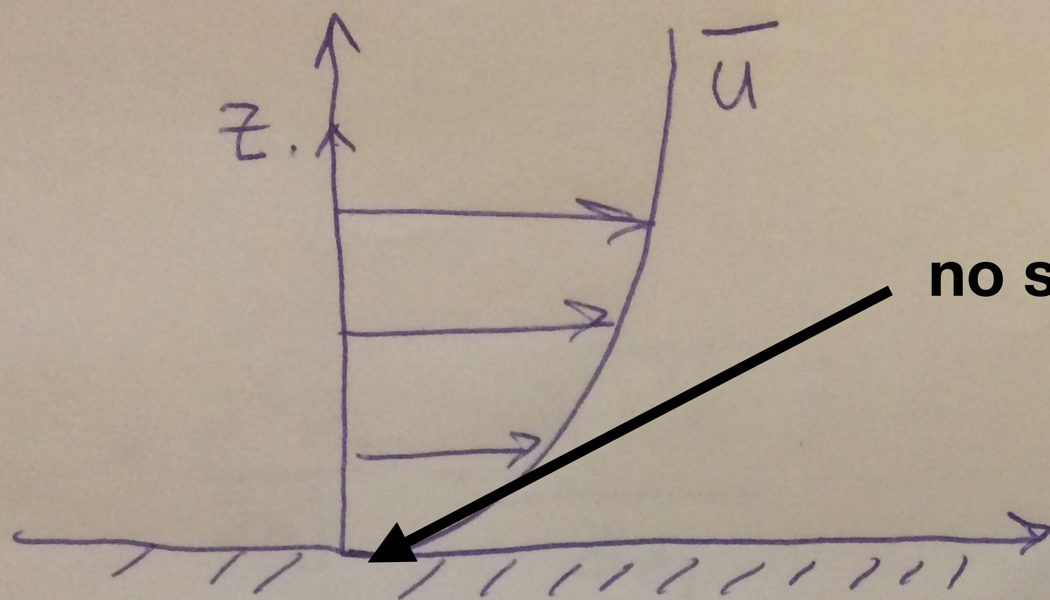


Fig. 2.10 Flux is the rate of transfer of a quantity across a unit area. (a) The net flux can be split into components in the x,y, and z directions, as shown here. (b) If a greater flux enters a volume than leaves, then there must be a net increase of that quantity within the volume.

Stull (1988)

Viscous (molecular) momentum flux



no slip boundary condition

property of the fluid

$$\tau = \eta \frac{d\bar{u}}{dz} \qquad \eta = \text{molecular viscosity} \quad (kg/m/s)$$

$$= \left(\frac{kg}{m\,s}\,\frac{m}{s}\right)\frac{1}{m} = \frac{N}{m^2}$$

Horizontal force on surface due to vertical transport of horizontal momentum

Reynolds Dye Experiment

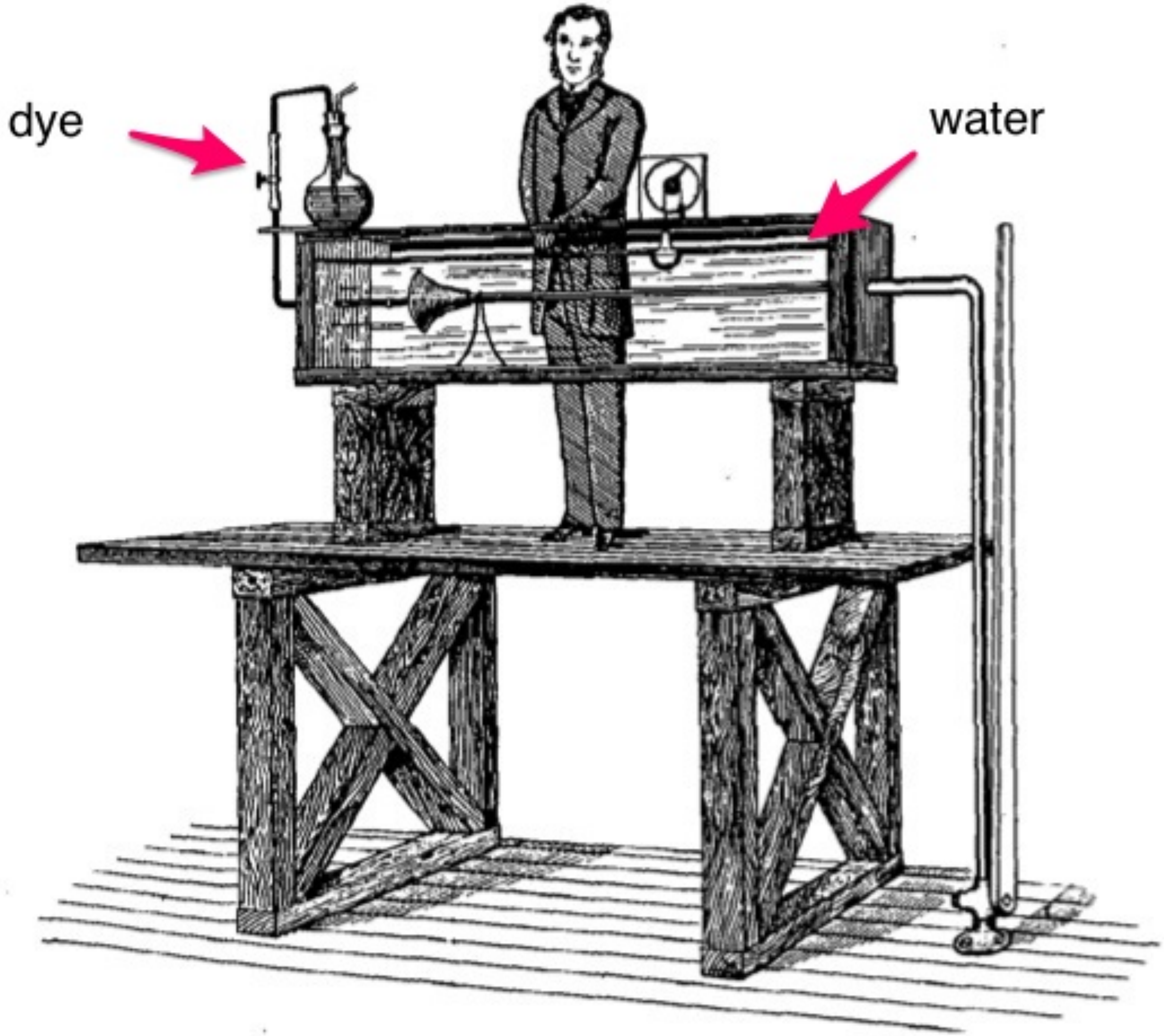


Figure 1 Artist's concept of Reynolds' flow-visualization experiment.

Rott, N. "Note on the history of the Reynolds Number", Ann. Rev. Fluid Mech (1990)

Laminar to Turbulent Flow Transition

increasing U

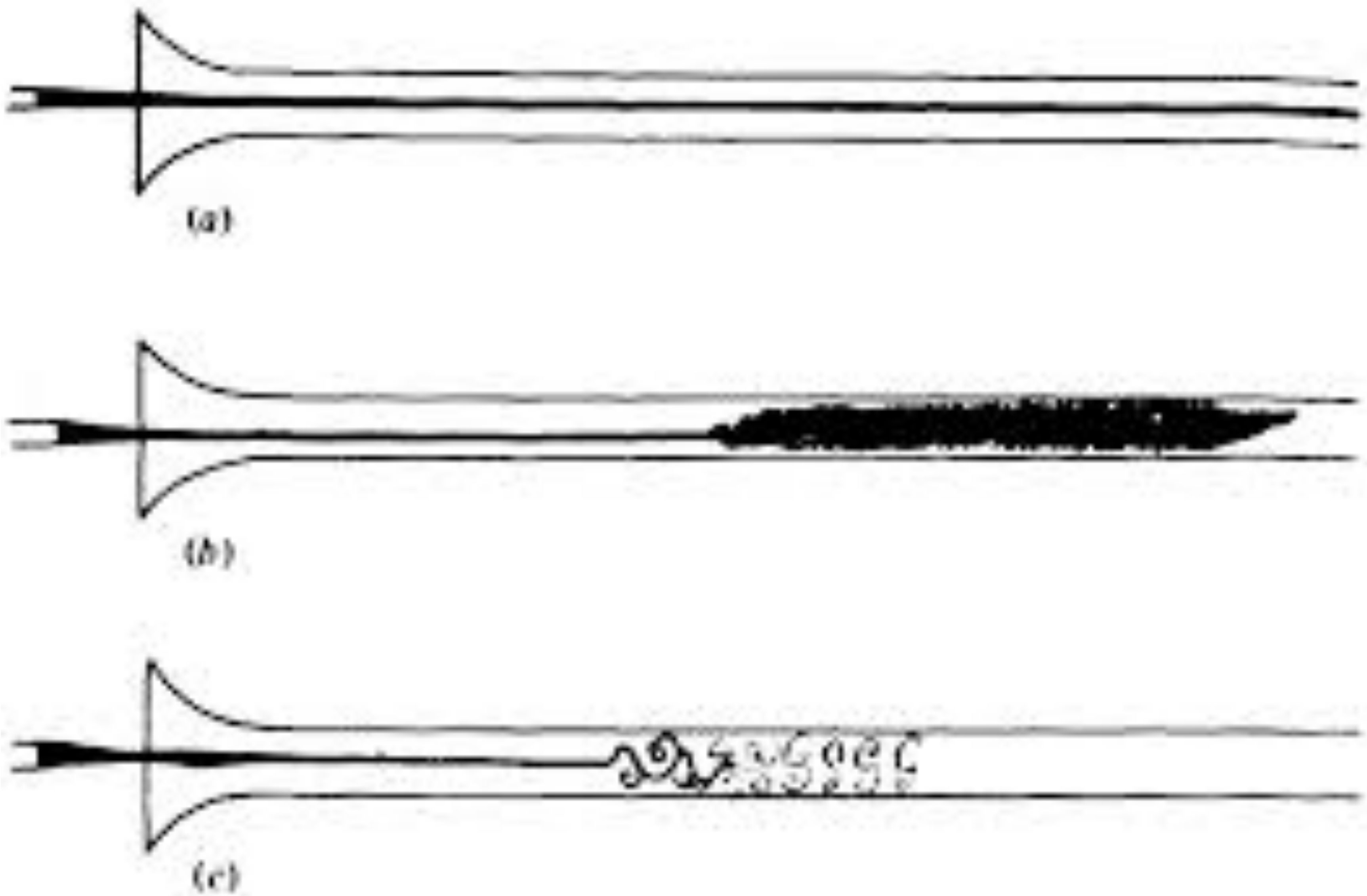
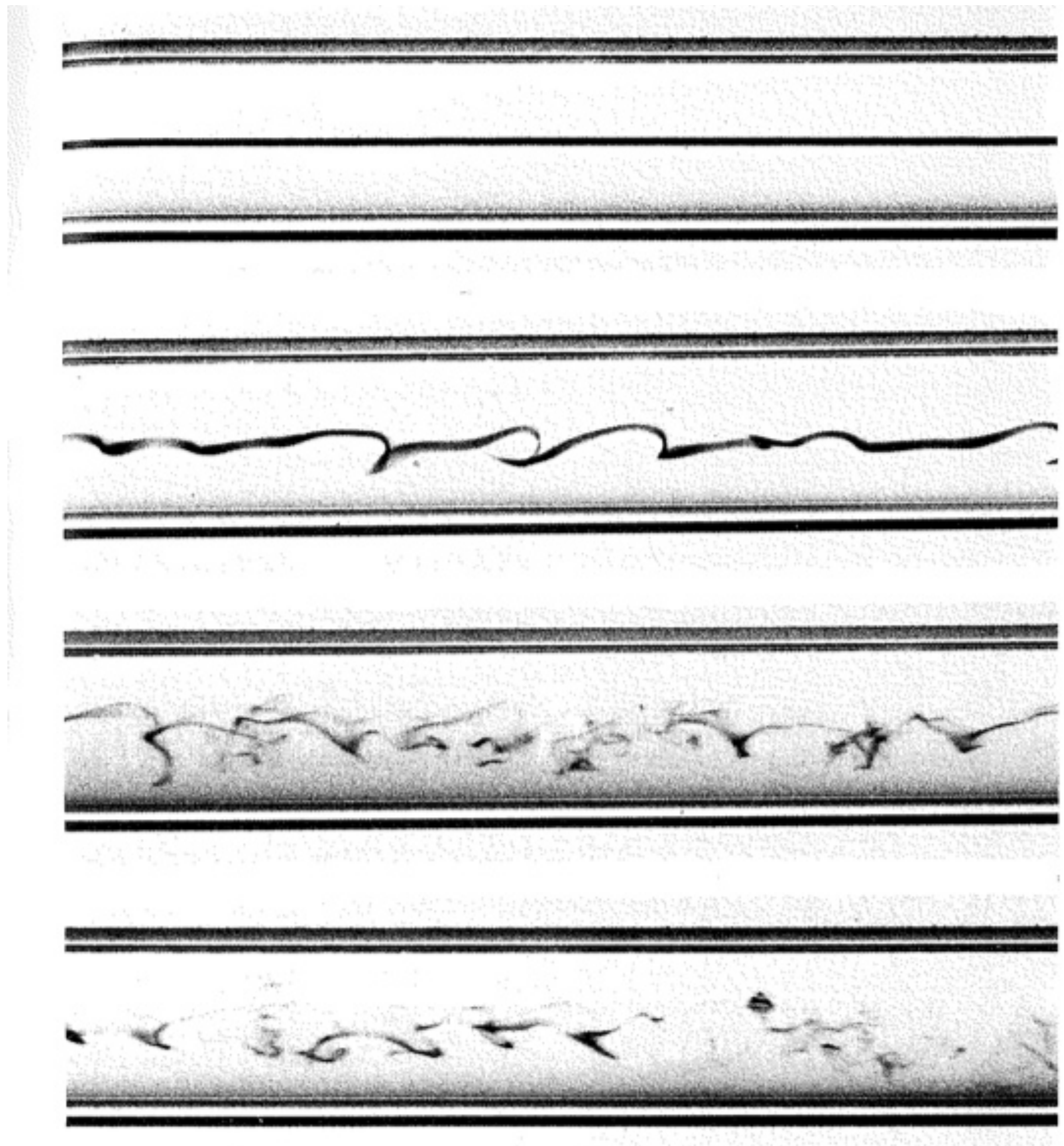


Fig. 9.2. Reynolds's drawings of the flow in his dye experiment.

Laminar to Turbulent Flow Transition

increasing U



Van Dyke (1982) *<u>An Album of Fluid Motion</u>. The Parabolic Press, Stanford, CA, p. 61.*

Reynolds number - key dimensionless parameter in turbulence

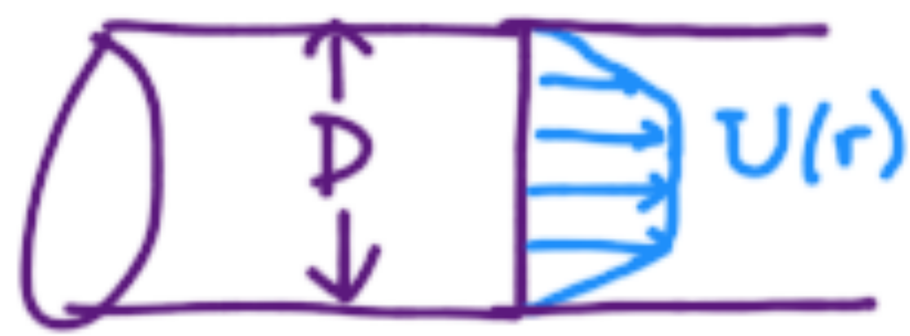
Reynolds #

$$Re = \frac{U D}{\nu}$$

$$Re > 2300$$

turbulent

PIPE FLOW



$$U = \frac{2}{D}\int_0^r U(r)\,dr$$

Reynolds number - key dimensionless parameter in turbulence

In the ABL…

$U = 10\ ms^{-1}$

$\eta = 10^{-5}\ m^2 s^{-1}$

$D = 10^2$ to 10^3 m (surface layer/boundary layer height)

$Re = 10 * 10^3/10^{-5} = 10^9$

ABL is high Reynolds number turbulence - compare to Princeton SuperPipe

Reynolds Decomposition

$$U = \overline{U} + u'$$

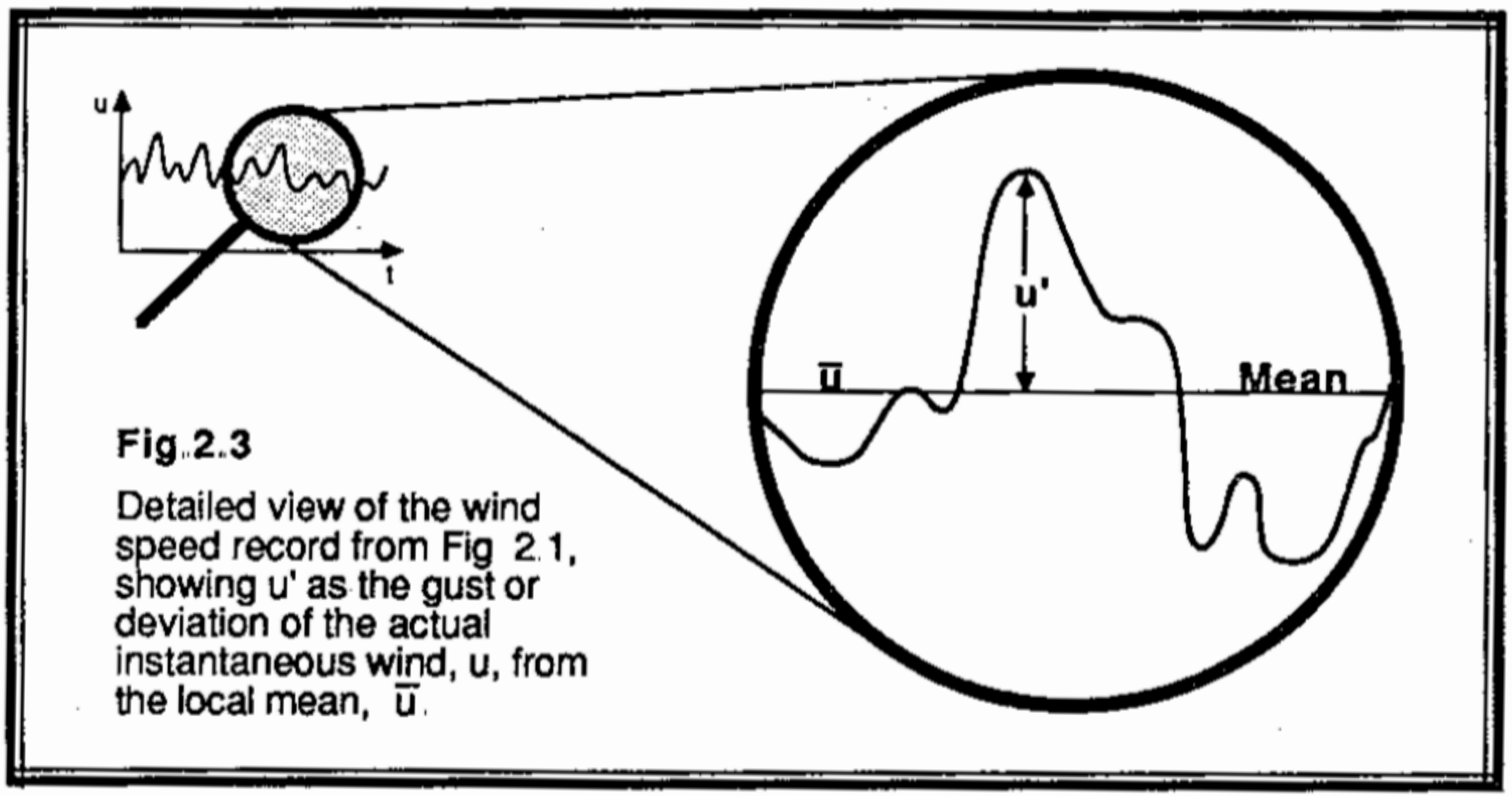


Fig 2.3

Detailed view of the wind speed record from Fig 2.1, showing u' as the gust or deviation of the actual instantaneous wind, u, from the local mean, $\overline{u}$.

Stull (1988)

Governing Equations

x-momentum (neglect Coriolis)

$$\frac{\partial U}{\partial t} + U_j \frac{\partial U}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 U}{\partial z^2}$$

I **II** **V** **VI**

Term I represents storage of momentum (inertia).
Term II describes advection.

Term V describes pressure-gradient forces.
Term VI represents the influence of viscous stress.

Governing Equations

$U = \overline{U} + u'$

$V = \overline{V} + v'$

$W = \overline{W} + w'$

$\theta_v = \overline{\theta_v} + \theta_v'$

$q = \overline{q} + q'$

$c = \overline{c} + c'$

x-momentum (neglect Coriolis)

$$\frac{\partial U}{\partial t} + U_j \frac{\partial U}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 U}{\partial z^2}$$

I **II** **V** **VI**

Term I represents storage of momentum (inertia).
Term II describes advection.

Term V describes pressure-gradient forces.
Term VI represents the influence of viscous stress.

Governing Equations

$U = \overline{U} + u'$

$V = \overline{V} + v'$

$W = \overline{W} + w'$

$\theta_v = \overline{\theta_v} + \theta_v'$

$q = \overline{q} + q'$

$c = \overline{c} + c'$

x-momentum (neglect Coriolis)

$$\frac{\partial U}{\partial t} + U_j \frac{\partial U}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 U}{\partial z^2}$$

I **II** **V** **VI**

Term I represents storage of momentum (inertia).
Term II describes advection.

Term V describes pressure-gradient forces.
Term VI represents the influence of viscous stress.

- plug in, Reynolds average, manipulate, simplify

Governing Equations

$U = \overline{U} + u'$

$V = \overline{V} + v'$

$W = \overline{W} + w'$

$\theta_v = \overline{\theta_v} + \theta_v'$

$q = \overline{q} + q'$

$c = \overline{c} + c'$

x-momentum (neglect Coriolis)

$$\frac{\partial U}{\partial t} + U_j \frac{\partial U}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 U}{\partial z^2}$$

I **II** **V** **VI**

Term I represents storage of momentum (inertia).
Term II describes advection.

Term V describes pressure-gradient forces.
Term VI represents the influence of viscous stress.

- plug in, Reynolds average, manipulate, simplify
- end up with a new **advection** term on left side:

$$\frac{\partial\, \overline{u'w'}}{\partial z}$$

Reynolds Stress

$$\frac{\partial \overline{u_i' u_j'}}{\partial x_j}$$ new term on left

$$\nu \frac{\partial^2 \bar{u}_i}{\partial x_j^2} - \frac{\partial \left(\overline{u_i' u_j'}\right)}{\partial x_j}$$

$$\frac{\partial}{\partial x_j}\left(\nu \frac{\partial \bar{u}_i}{\partial x_j} - \overline{u_i' u_j'}\right)$$

↑ Visious Stress

↑ Reynolds Stress

Turbulent Momentum Flux (Reynolds Stress)

$$\tau = -\rho\overline{u'w'}$$

$\overline{u'} = \overline{w'} = 0$ but $\overline{u'w'} \neq 0$

covariance $\overline{u'w'}$

- **sample fast enough to capture small eddies (~10 Hz)**
- **averaged long enough to sample largest eddies (~30 min)**

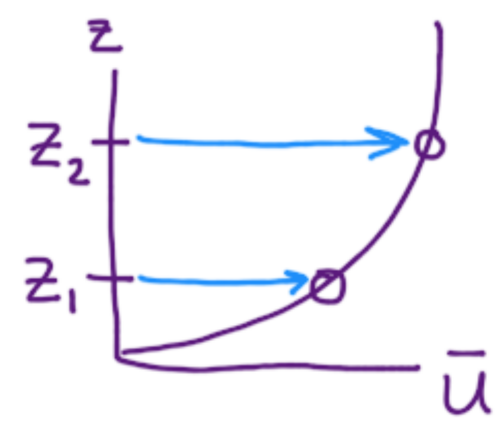
Vertical velocity fluctuations and turbulent transport

Wind Profile

$\rho \bar{U}$ = mean momentum

$w' > 0 \quad u' < 0 \rightarrow w'u' < 0$

$w' < 0 \quad u' > 0 \rightarrow w'u' < 0$



$z_2 > z_1$

$\rho \bar{U}(z_2) > \rho \bar{U}(z_1)$

- updrafts and downdrafts both have $w'u' < 0$
- avg over many updrafts / downdrafts (15-60 min)
- $\overline{u'w'} < 0$ momentum flux downward toward surface
- transfer is "down the gradient"
- similar case for θ, q if profile like right side
- left panel - surface warmer/moister, fluxes upward

Turbulent heat flux

$$H_s = \rho_a C_p \overline{w'T'}$$

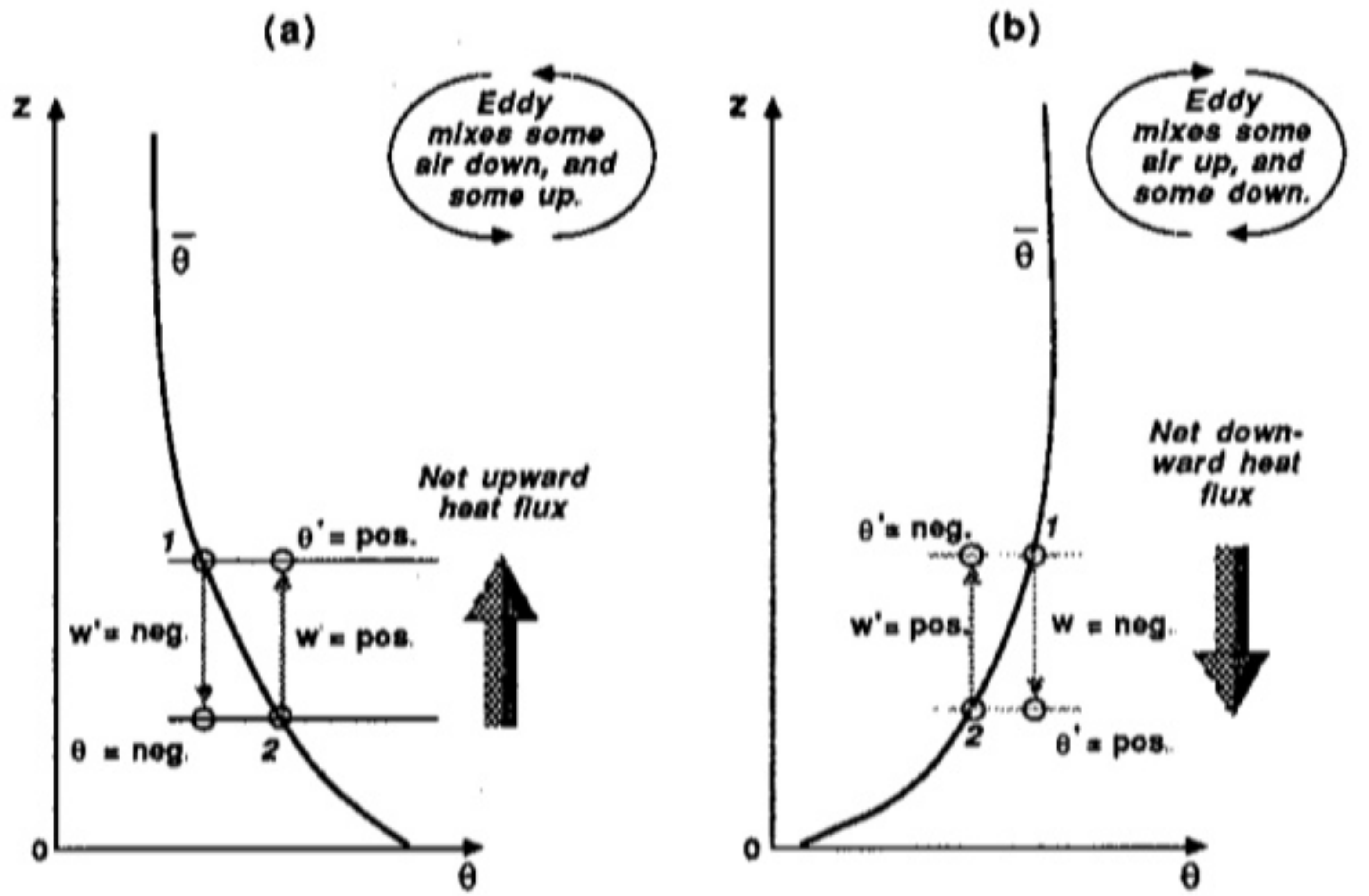
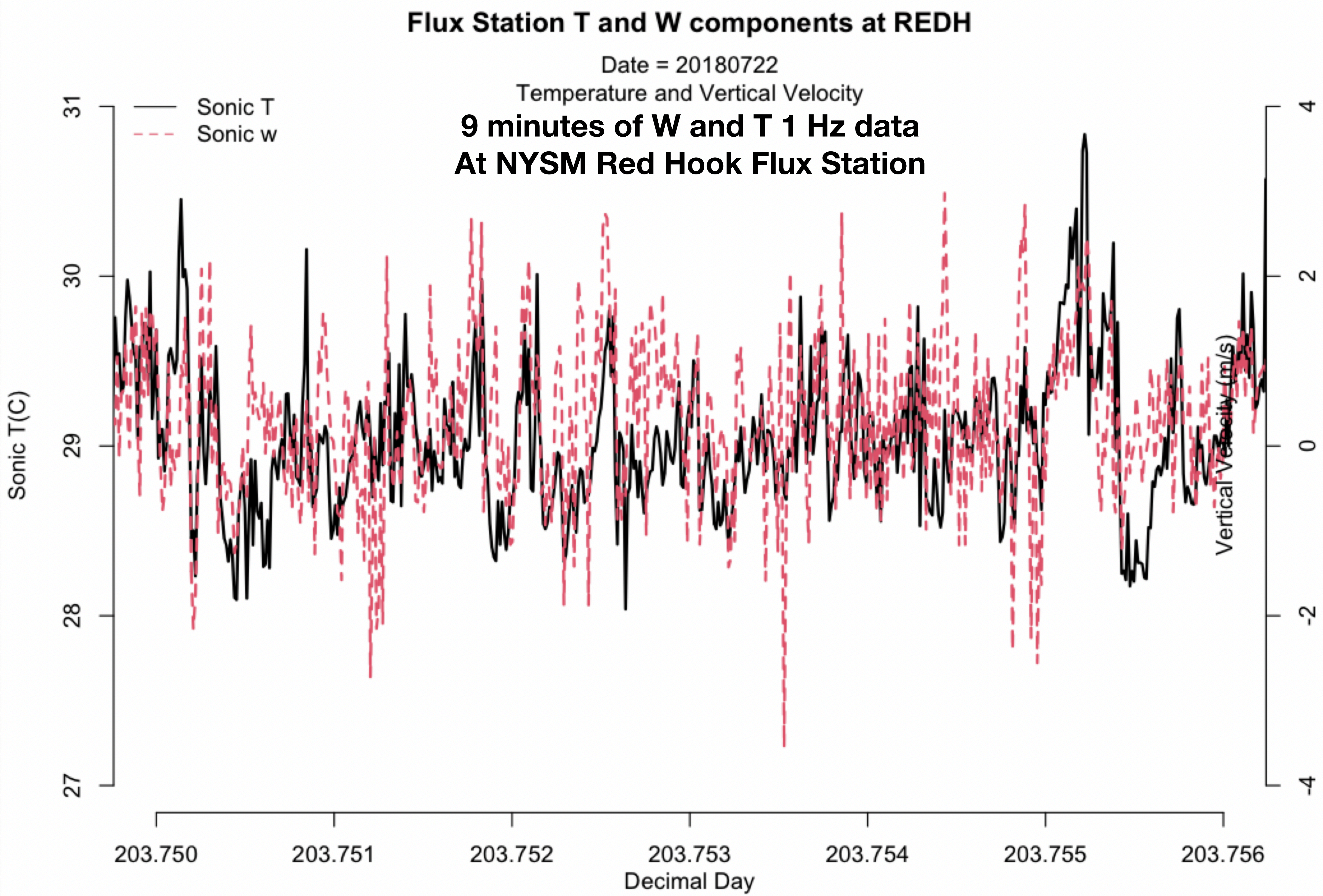


Fig. 2.12 Idealization of the small eddy mixing process, showing (a) net upward turbulent heat flux in a statically unstable environment, and (b) net downward turbulent heat flux in a stable environment.

Stull (1988)


Flux Station T and W components at REDH
Date = 20180722
Temperature and Vertical Velocity
9 minutes of W and T 1 Hz data
At NYSM Red Hook Flux Station
Sonic T
Sonic w
Sonic T(C)
Vertical Velocity (m/s)
Decimal Day
27
28
29
30
31
-4
-2
0
2
4
203.750
203.751
203.752
203.753
203.754
203.755
203.756

100 second timeseries of $w'T'$

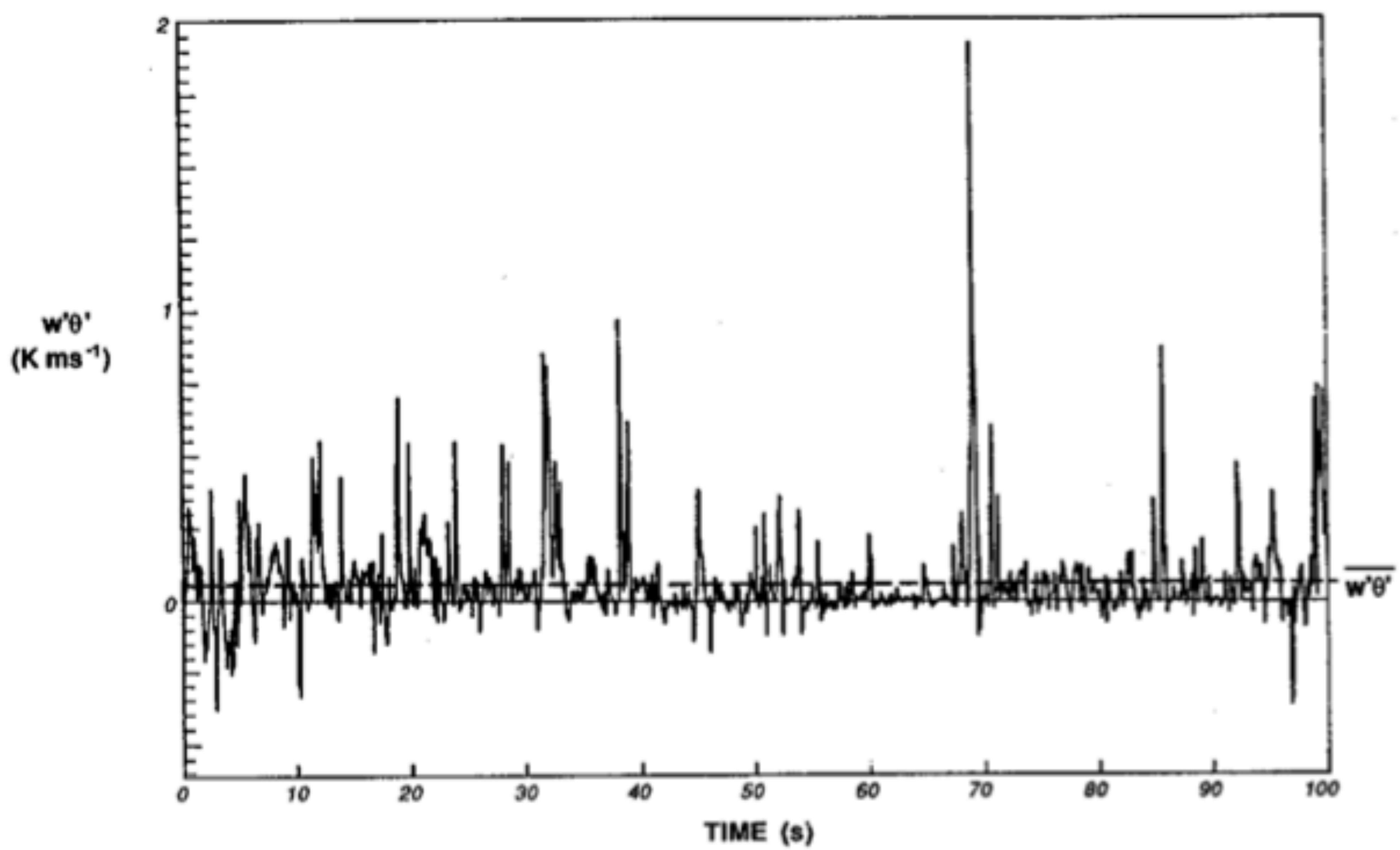


Fig. 2.13 Instantaneous kinematic surface heat flux. $w'\theta'$ trace measured by aircraft (BLX83 flight 3, leg 3) May 28, 1983 Oklahoma. The dashed line indicates the average heat flux.

Stull (1988)

Histogram of $w'T'$

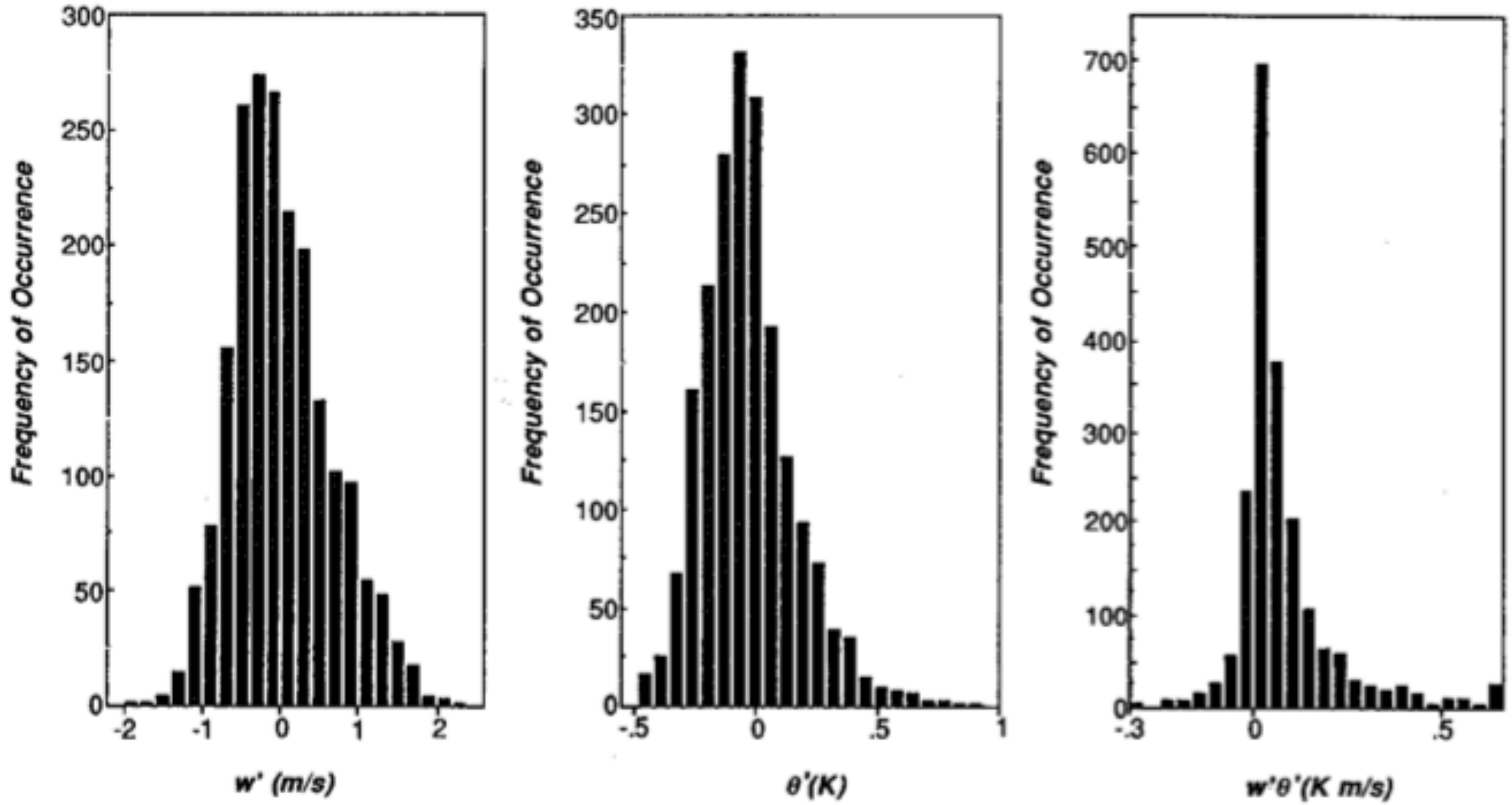
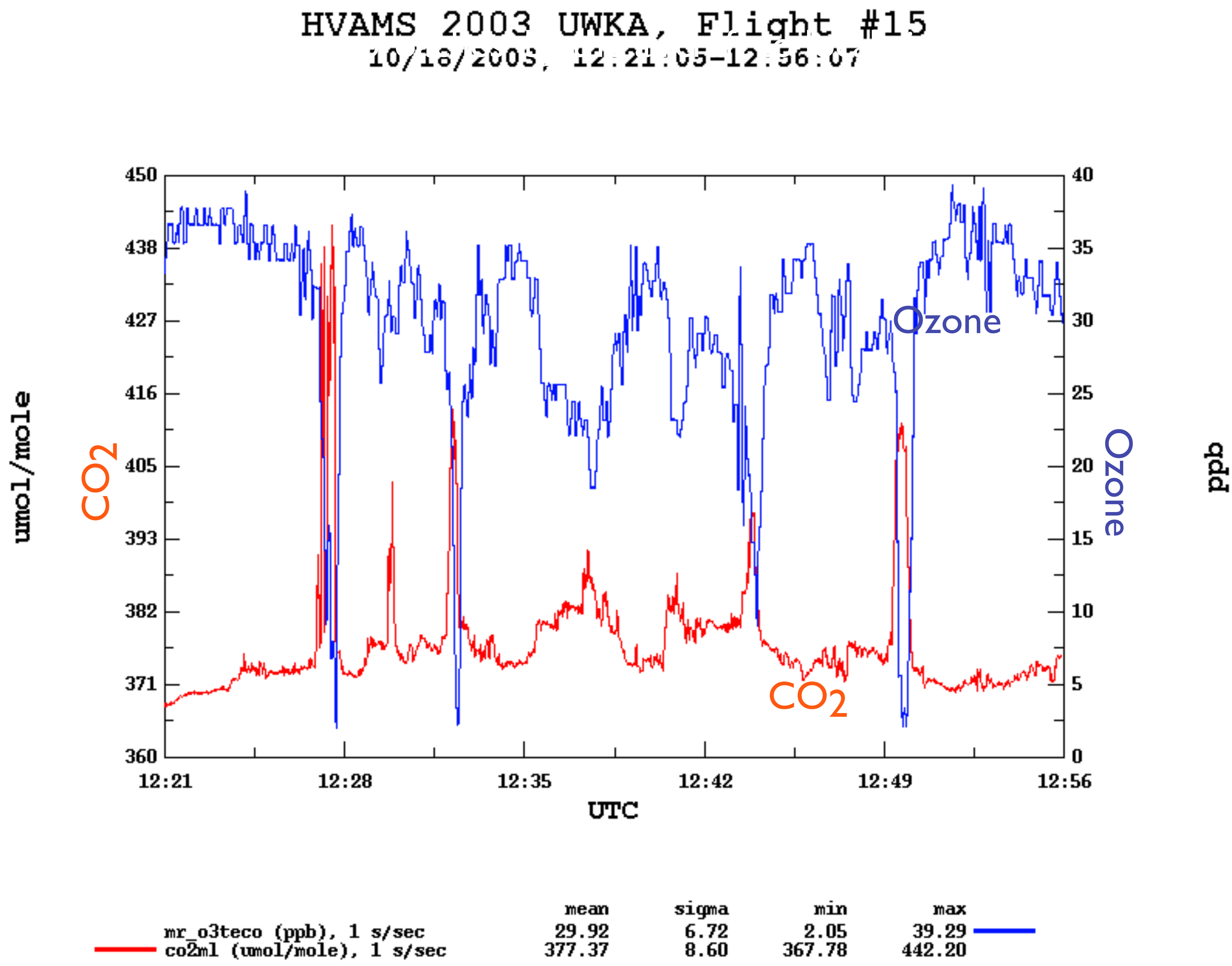


Fig. 2.14 Frequency of occurrence of w', θ', and w'θ'.

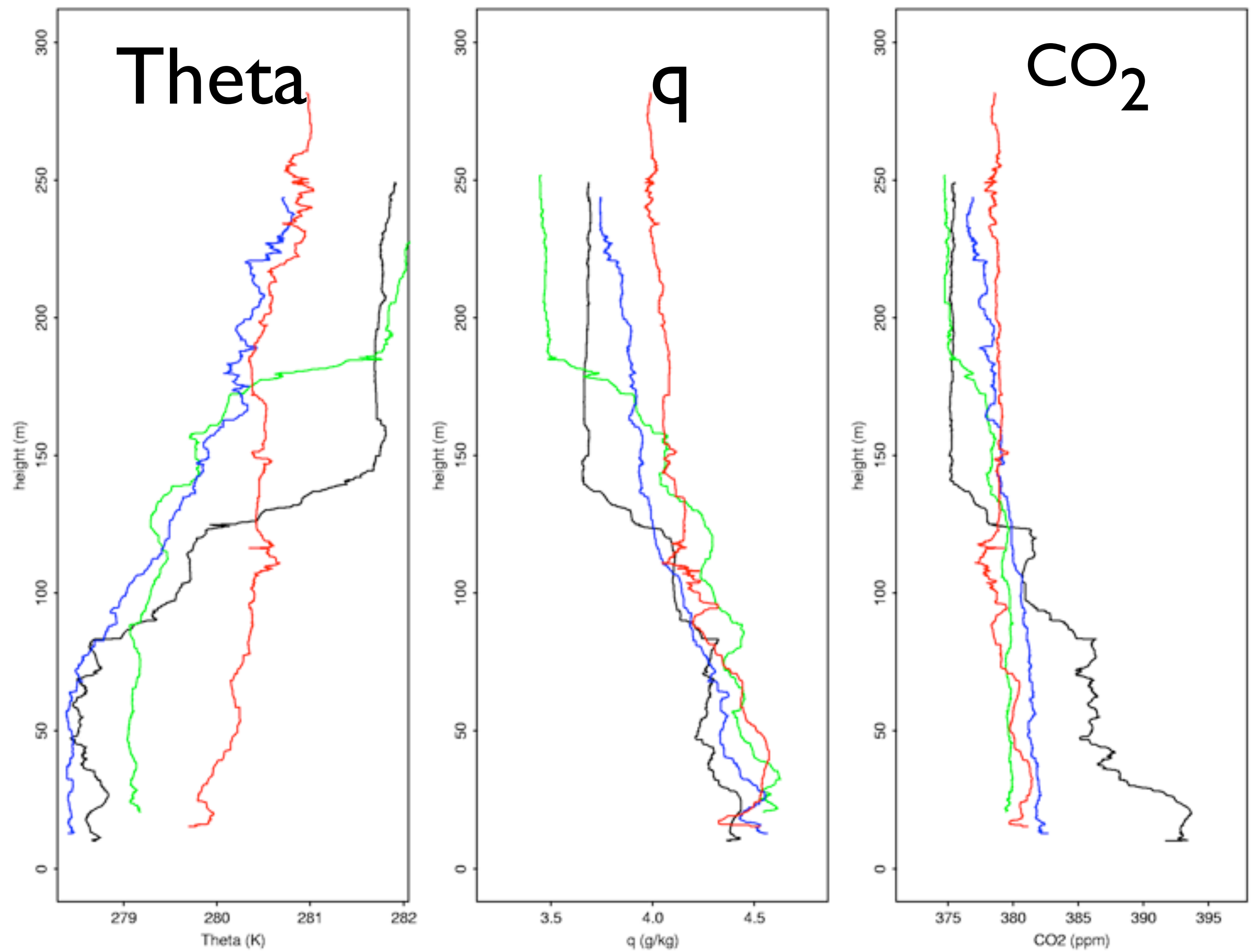
Stull (1988)



HVAMS 2003 UWKA, Flight #15
10/18/2003, 12:21:05-12:56:07
umol/mole
CO2
ppb
Ozone
Ozone
CO2
450
438
427
416
405
393
382
371
360
40
35
30
25
20
15
10
5
0
12:21
12:28
12:35
12:42
12:49
12:56
UTC
mean
sigma
min
max
mr_o3teco (ppb), 1 s/sec
29.92
6.72
2.05
39.29
co2ml (umol/mole), 1 s/sec
377.37
8.60
367.78
442.20

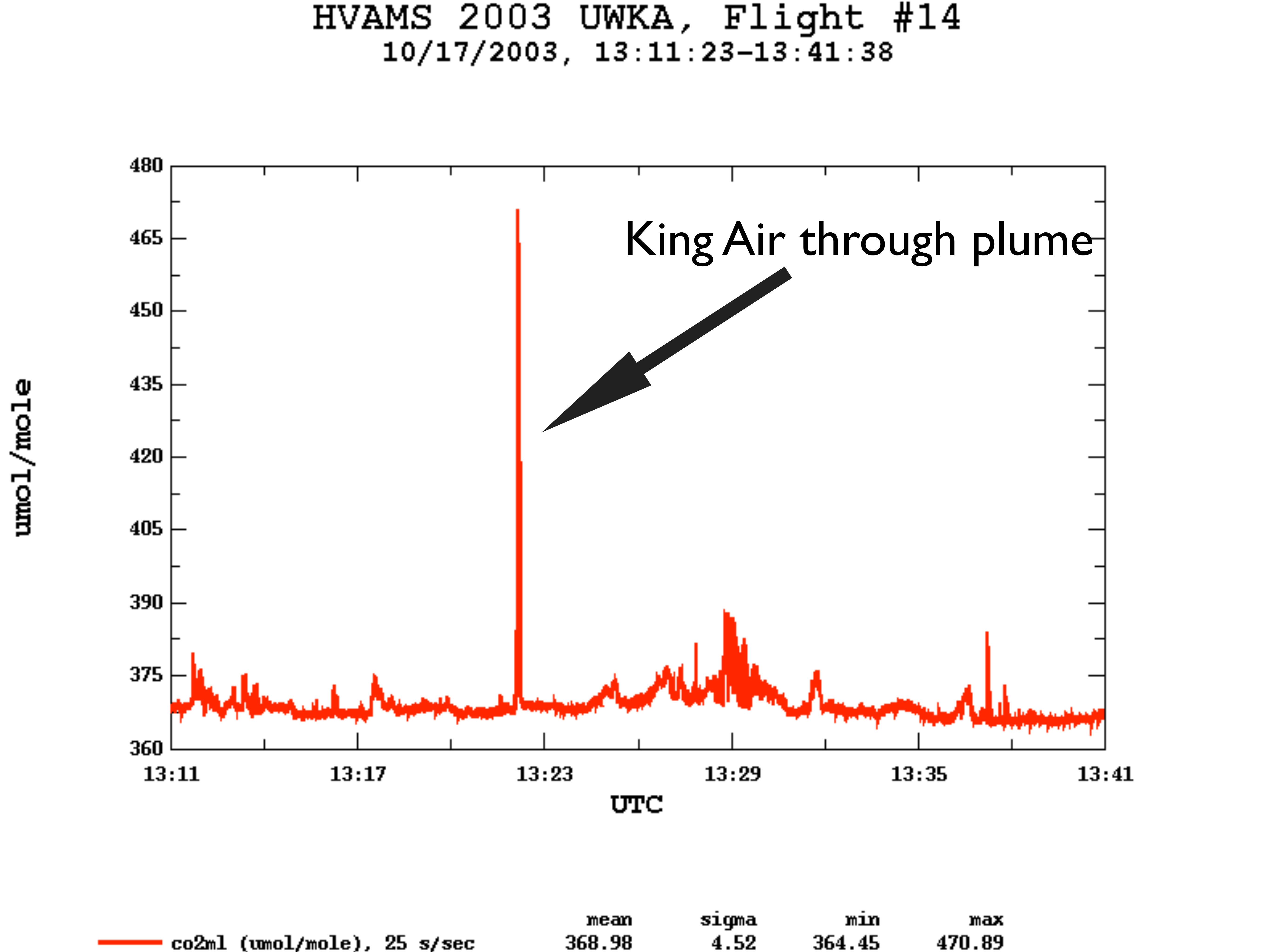
King Air missed approach profiles

Nocturnal accumulation of CO_2 (anthropogenic and respiratory origin) in the Hudson Valley

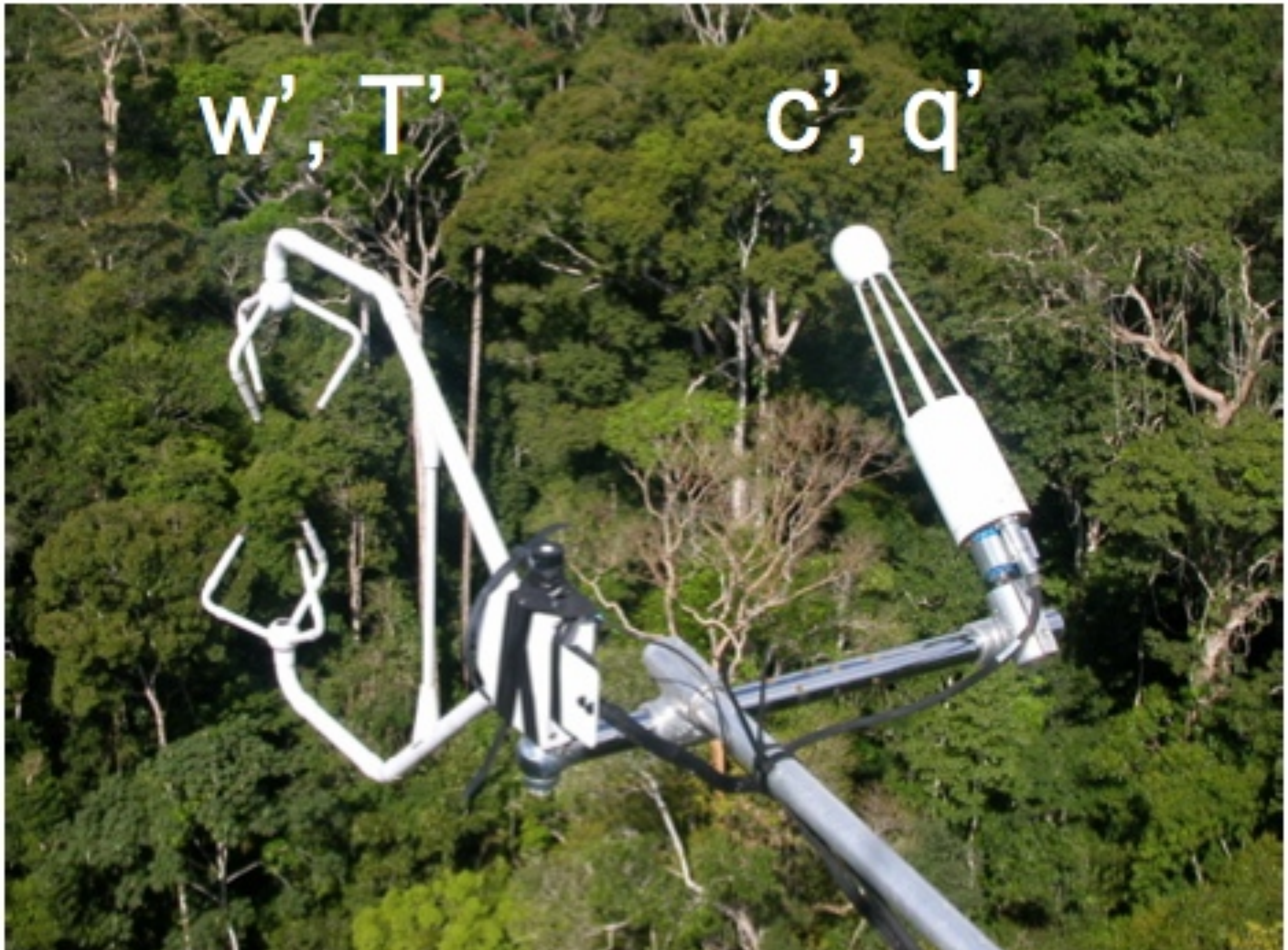


Kingston-Ulster: black
Green Acres: green
Columbia County: blue
Alexander Farm: red

Waves on a density interface made visible by St. Lawrence Cement Plant plume, October 17, 2003. Photo by King Air pilot Thomas Drew.

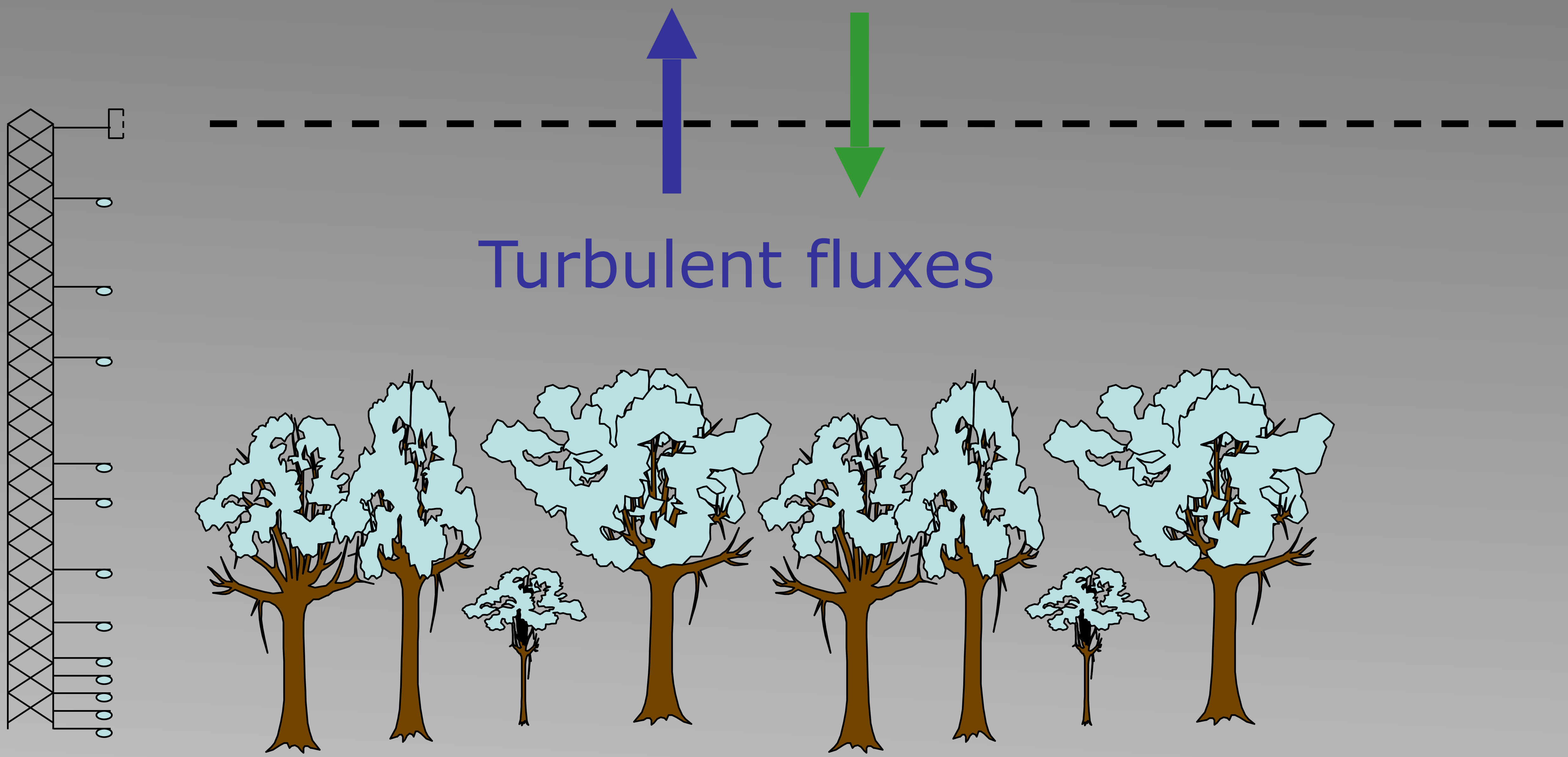

HVAMS 2003 UWKA, Flight #14
10/17/2003, 13:11:23-13:41:38
King Air through plume
480
465
450
435
420
405
390
375
360
umol/mole
13:11
13:17
13:23
13:29
13:35
13:41
UTC
mean
sigma
min
max
co2ml (umol/mole), 25 s/sec
368.98
4.52
364.45
470.89

Example of CO_2 exchange above a forest

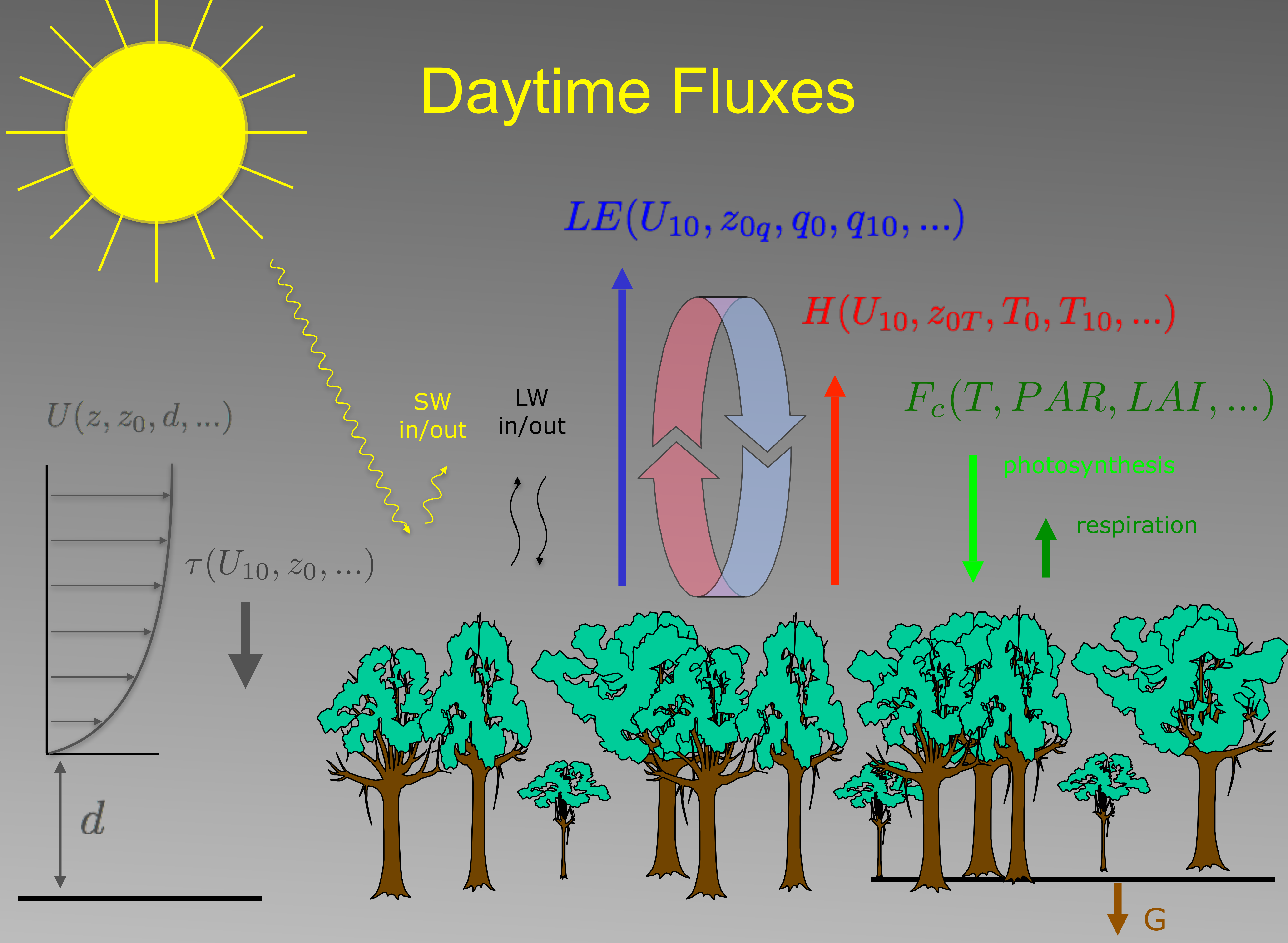


Measurement Approach

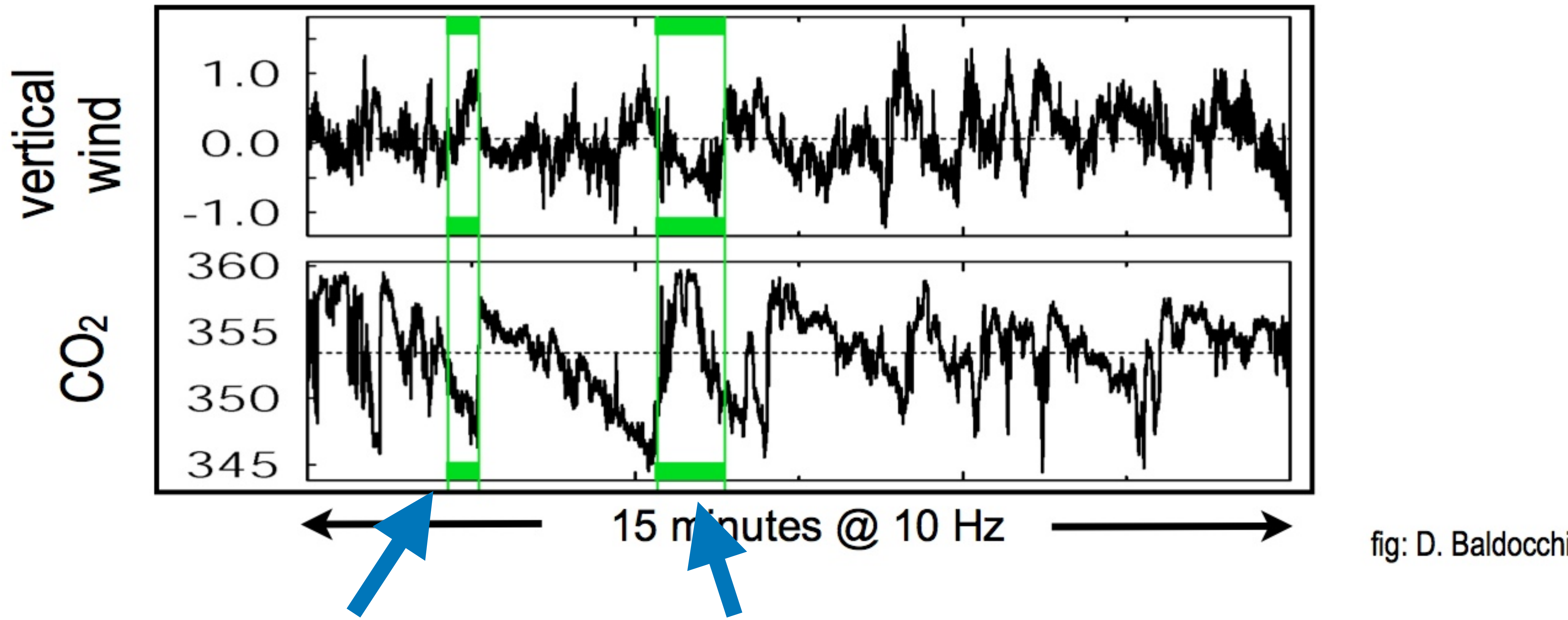
Control volume = atmospheric air below sensors



Daytime Fluxes



Example of CO_2 exchange above a forest



updraft

$w' > 0$

$c' < 0$

$w'c' < 0$

downdraft

$w' < 0$

$c' > 0$

$w'c' < 0$

$\rightarrow \overline{w'c'} < 0$

forest absorbing carbon

Law of the Wall

Log Wind Profile

$\tau = -\rho \overline{u'w'}$ Reynolds Stress, turbulent momentum flux, wind stress

$\tau = \mu \frac{d\bar{u}}{dz} = \rho \nu \frac{d\bar{u}}{dz}$

μ = molecular viscosity ($kg\, m^{-1} s^{-1}$)

ν = kinematic viscosity ($m^2 s^{-1}$)

$\tau = \rho K_m \frac{d\bar{u}}{dz}$ turbulent flow (molecular analogy)

K_m = eddy diffusivity, eddy exchange coefficient, eddy viscosity ($m^2 s^{-1}$)

ν = property of fluid

K_m = property of flow

Law of the Wall

$u_* = |\overline{u'w'}|^{1/2}$ ms^{-1} - friction velocity. Turbulence scaling parameter

$\tau = -\rho\overline{u'w'} = \rho u_*^2$

Hypothesis:

$K_m \sim u_* z$

1) related to turb. intensity. Increase u_*, increase K_m, increase momentum flux

2) scales with distance from wall as eddies are larger. Eddy size diminishes close to wall. Limited by z.

$K_m = \kappa u_* z$

κ = von Karman's "constant"

Law of the Wall

$$\tau = \rho K_m \frac{d\bar{u}}{dz}$$

$$\rho u_*^2 = \rho \kappa u_* z \frac{d\bar{u}}{dz}$$

$$\frac{d\bar{u}}{dz} = \frac{u_*}{\kappa z}$$

$$\frac{d\bar{u}}{d\ln z} = \frac{u_*}{\kappa}$$

$$\bar{u}(z) = \frac{u_*}{\kappa} \ln z + C$$ ← const

Define C so that $\bar{u} = 0$ when $z = z_0$ (roughness length). Recall no-slip boundary condition

$$\boxed{\bar{u}(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right)}$$

law of the wall

Law of the Wall

Adjust winds at height z_1 to height z_2

$$\bar{U}(z_1) = \frac{u_*}{k} \ln\left(\frac{z_1}{z_0}\right) = \frac{u_*}{k} \ln z_1 - \frac{u_*}{k} \ln z_0$$

$$\bar{U}(z_2) = \frac{u_*}{k} \ln z_2 - \frac{u_*}{k} \ln z_0$$

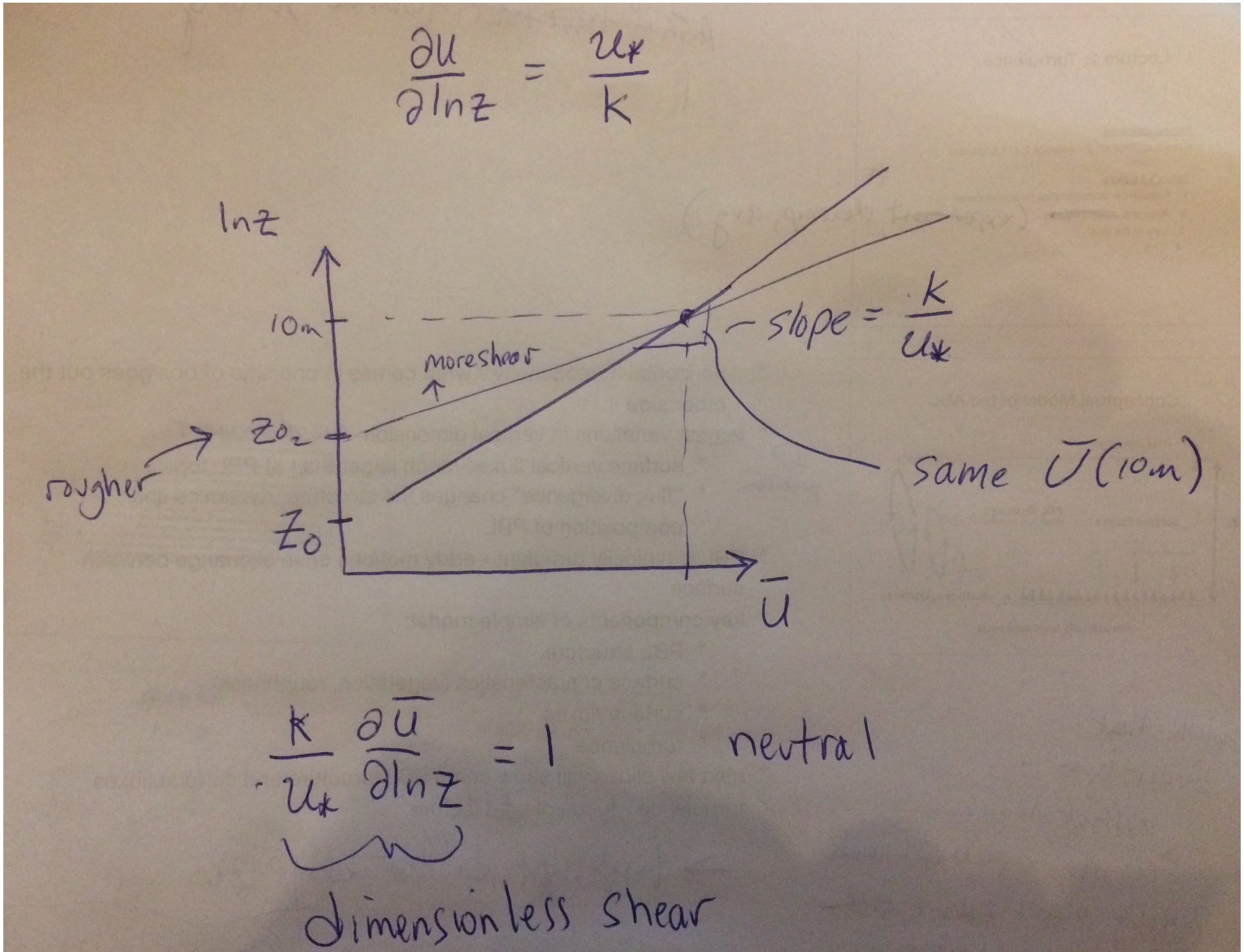
$$\frac{\bar{U}(z_2)}{\bar{U}(z_1)} = \frac{u_*/k}{u_*/k} \frac{\ln(z_2/z_0)}{\ln(z_1/z_0)}$$

$$\bar{U}(z_2) = \bar{U}(z_1) \ln \frac{z_2}{z_1}$$

know	calculate	
$\bar{u}(z)$ →	u_*, z_0	mom. flux, roughness
u_*, z_0 →	$\bar{u}(z)$	
$\bar{u}(z_1), z_0$ →	shear stress, ρu_*^2	
$\bar{U}(z_1), z_0$ →	$\bar{U}(z_2)$	

Note: for neutral stability.

Dimensionless Shear



Roughness length

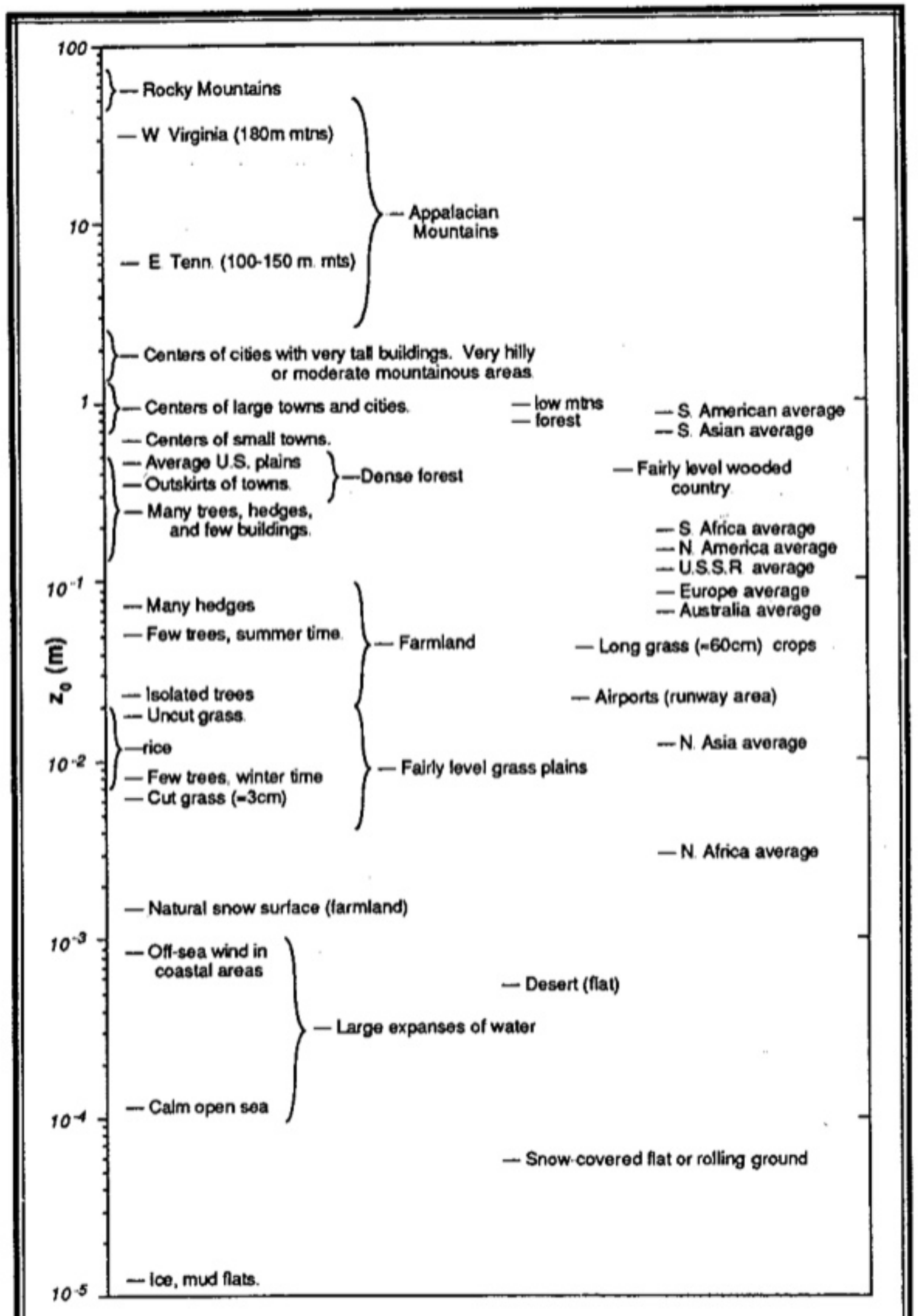


Fig. 9.6 Aerodynamic roughness lengths for typical terrain types. (After Garratt 1977, Smedman-Högström & Högström 1978, Kondo & Yamazawa 1986, Thompson 1978, Napo 1977, and Hicks 1975).

Stull (1988)

Mean and fluctuating kinetic energy

$$\text{MKE}/m = \frac{1}{2}\left(\overline{U}^2 + \overline{V}^2 + \overline{W}^2\right) \qquad (2.5a)$$

$$e = \frac{1}{2}\left(u'^2 + v'^2 + w'^2\right) \qquad (2.5b)$$

Turbulent Kinetic Energy

$$\frac{\text{TKE}}{m} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) = \overline{e} \qquad (2.5c)$$

Note KE = 0.5 m M^2 where m is mass
We want to talk about kinetic energy per unit mass
Which is just 0.5 M^2

Diurnal evolution of TKE

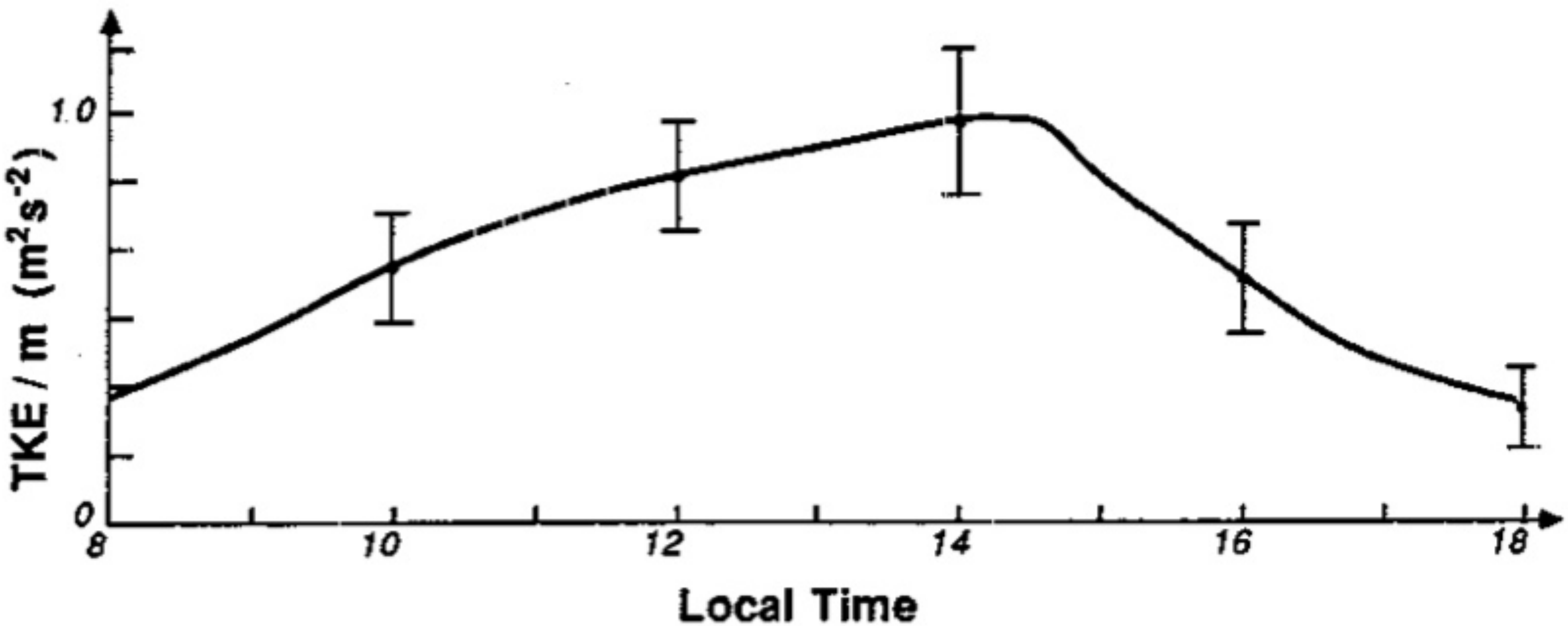


Fig. 2.8 Diurnal cycle of TKE/m = 1/2 ($\overline{u'^2} + \overline{v'^2} + \overline{w'^2}$) measured by aircraft below 300 m agl over Tennessee, August 1978. Adapted from Bean and Repoff, 1983.

Stull (1988)

Next class

Monday, 30 January: Similarity Scaling (really all about stability)

(Hopefully in person!)

Watch Turbulence Movie (in BOUNDARY_LAYERS folder).