Lecture 3: Stability

Announcements

- reading: Stull ch. 5, sections 5.1, 5.2, 5.5, 5.6, 5.7,5.8
- Reference: Stull ch. 9 (Similarity)
- Homework (on website)

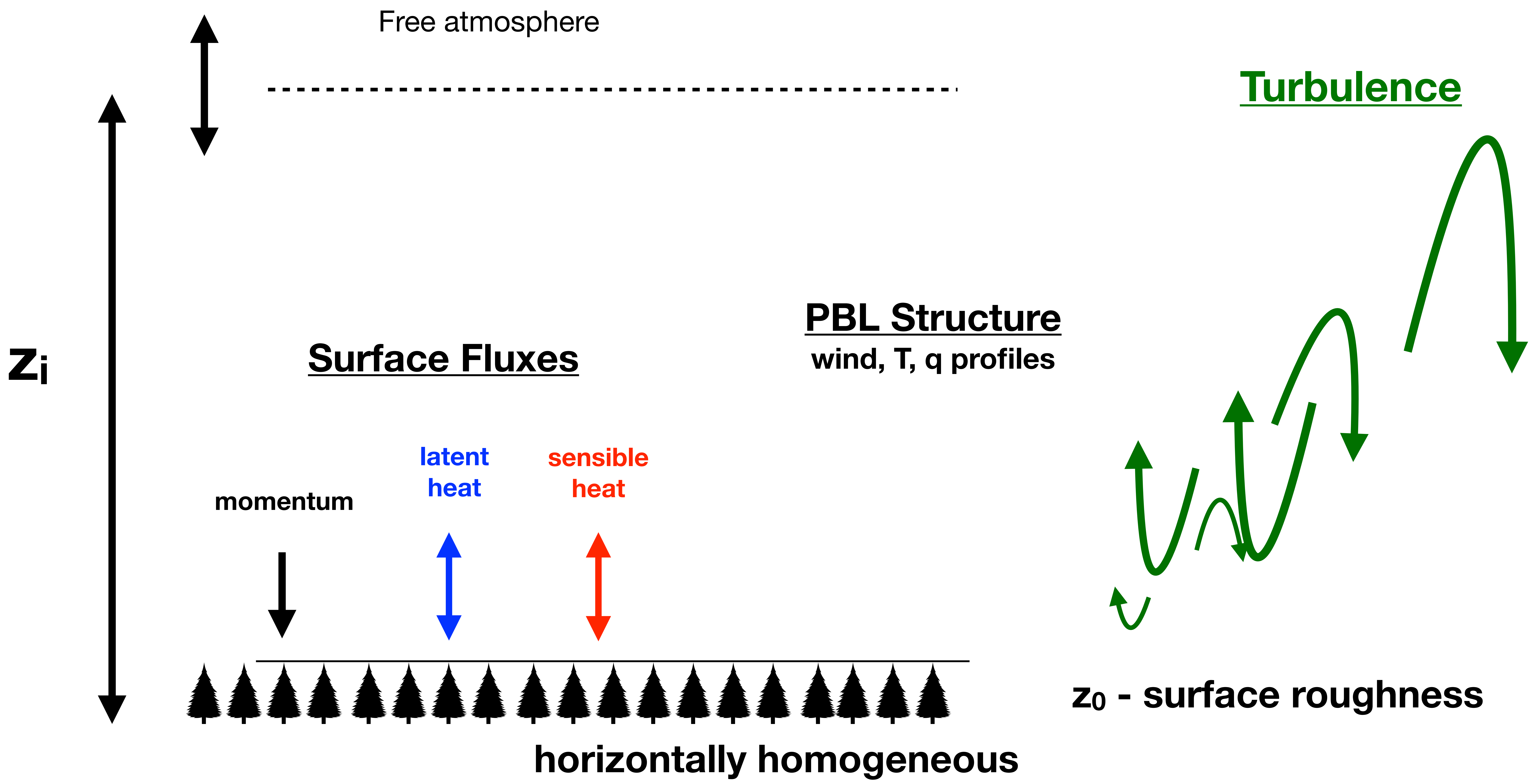
Today's Lecture

1. Brief review
2. Stability
3. Richardson number
4. Obukhov length
5. Diabatic wind profile

Thacher Park 6 December 2015

72518 ALB Albany
100
200
300
400
500
600
700
800
900
1000
16230 m
13710 m
11930 m
10530 m
9340 m
7380 m
5770 m
3181 m
1604 m
906 m
277 m
0.4
1
2
10
16
24
32
100 g/kg
-40 -30 -20 -10 0 10 20 30 40
12Z 06 Dec 2015
University of Wyoming
SLAT 42.69
SLON -73.83
SELV 95.00
SHOW 10.74
LIFT 21.61
LFTV 21.72
SWET 48.99
KINX -23.3
CTOT -0.90
VTOT 28.10
TOTL 27.20
CAPE 0.00
CAPV 0.00
CINS 0.00
CINV 0.00
EQLV -9999
EQTV -9999
LFCT -9999
LFCV -9999
BRCH 0.00
BRCV 0.00
LCLT 270.7
LCLP 970.5
LCLE 282.2
MLTH 273.1
MLMR 3.33
THCK 5493.
PWAT 6.80
Thacher Park 6 December 2015
Overlook Parking Lot (366 m)
Jan 29 2020

Conceptual Model of the ABL—review



Vertical Layers—review

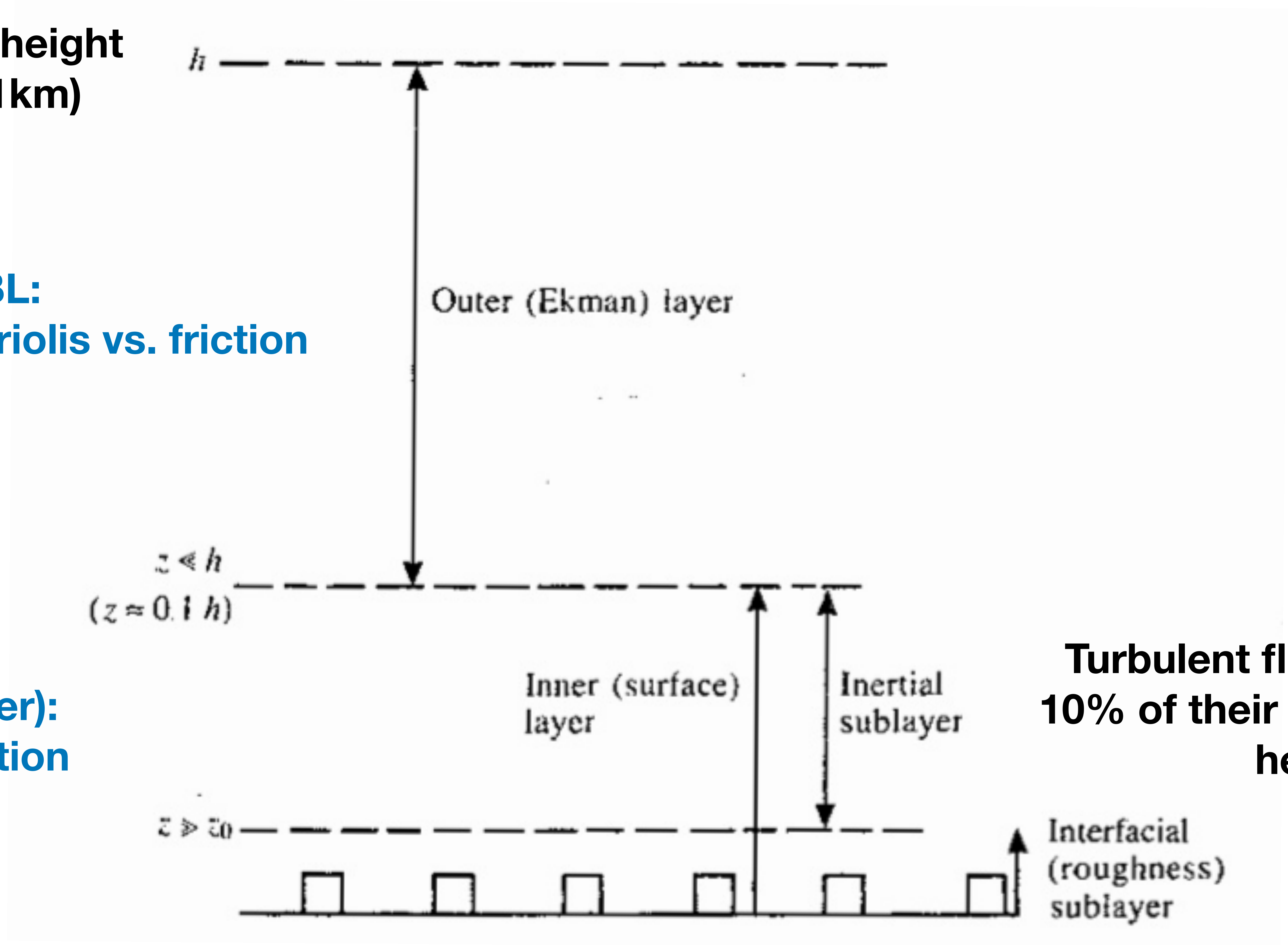
Free atmosphere: pressure gradient vs. Coriolis

PBL height
(~1km)

Outer BL:
pressure grad vs. Coriolis vs. friction

(~100 m)

Inner BL (surface layer):
pressure grad vs. friction



Turbulent fluxes vary by < 10% of their magnitude with height

Garratt (1994)

Vertical Layers—applying stability

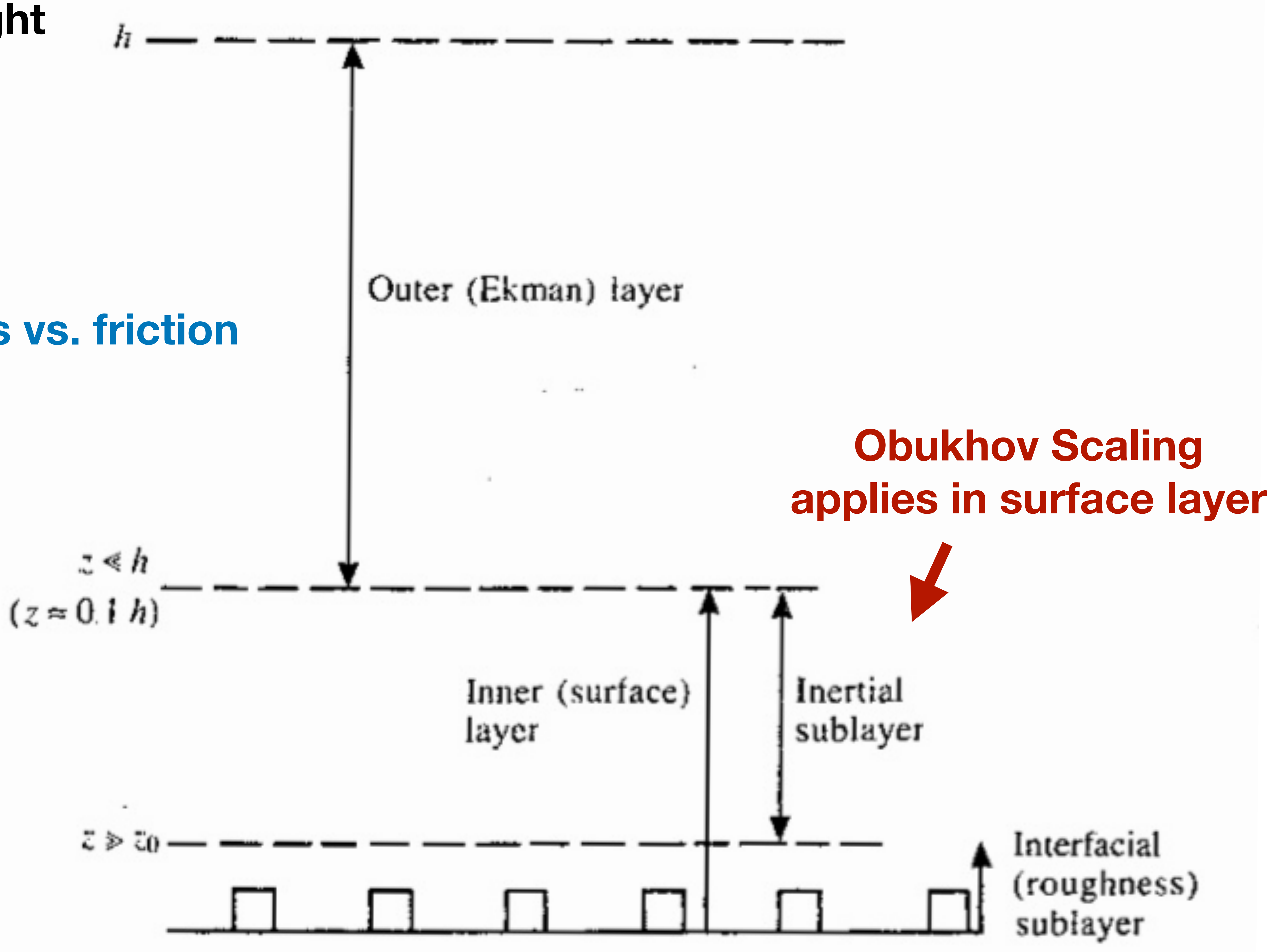
Free atmosphere: pressure gradient vs. Coriolis

PBL height
(~1km)

Outer BL:
pressure grad vs. Coriolis vs. friction

(~100 m)

Inner BL (surface layer):
pressure grad vs. friction



Obukhov Scaling
applies in surface layer

Garratt (1994)

(Review) Logarithmic Wind Profile (neutral)

$$\tau = \rho K_m \frac{d\bar{u}}{dz}$$

$$\rho u_*^2 = \rho \kappa u_* z \frac{d\bar{u}}{dz}$$

$$\frac{d\bar{u}}{dz} = \frac{u_*}{\kappa z}$$

$$\frac{d\bar{u}}{d\ln z} = \frac{u_*}{\kappa}$$

$$\bar{u}(z) = \frac{u_*}{\kappa} \ln z + C$$ ← const

Define C so that $\bar{u} = 0$ when $z = z_0$ (roughness length). Recall no-slip boundary condition

$$\bar{u}(z) = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right)$$

law of the wall

write as dimensionless shear

$$\frac{k}{u_*} \frac{d\overline{U}}{d\ln z} = 1$$

MO Similarity Hypothesis

$$\frac{k}{u_*} \frac{d\overline{U}}{d\ln z} = \phi_m\left(\frac{z}{L}\right)$$

Logarithmic wind profile (Law of the Wall)

Adjust winds at height z_1 to height z_2

$$\bar{U}(z_1) = \frac{u_*}{k} \ln\left(\frac{z_1}{z_0}\right) = \frac{u_*}{k} \ln z_1 - \frac{u_*}{k} \ln z_0$$

$$\bar{U}(z_2) = \frac{u_*}{k} \ln z_2 - \frac{U_*}{k} \ln z_0$$

$$\frac{\bar{U}(z_2)}{\bar{U}(z_1)} = \frac{U_*/k}{U_*/k} \, \frac{\ln(z_2/z_0)}{\ln(z_1/z_0)}$$

$$\boxed{\bar{U}(z_2) = \bar{U}(z_1) \ln \frac{z_2}{z_1}}$$

<u>know</u>		<u>calculate</u>	
$\bar{u}(z)$	$\rightarrow$	u_*, z_0	mom. flux, roughness
u_*, z_0	$\rightarrow$	$\bar{u}(z)$	
$\bar{u}(z_1), z_0$	$\rightarrow$	shear stress, ρu_*^2	
$\bar{U}(z_1), z_0$	$\rightarrow$	$\bar{U}(z_2)$	

Note: for neutral stability.

Stability in the Atmospheric Boundary Layer

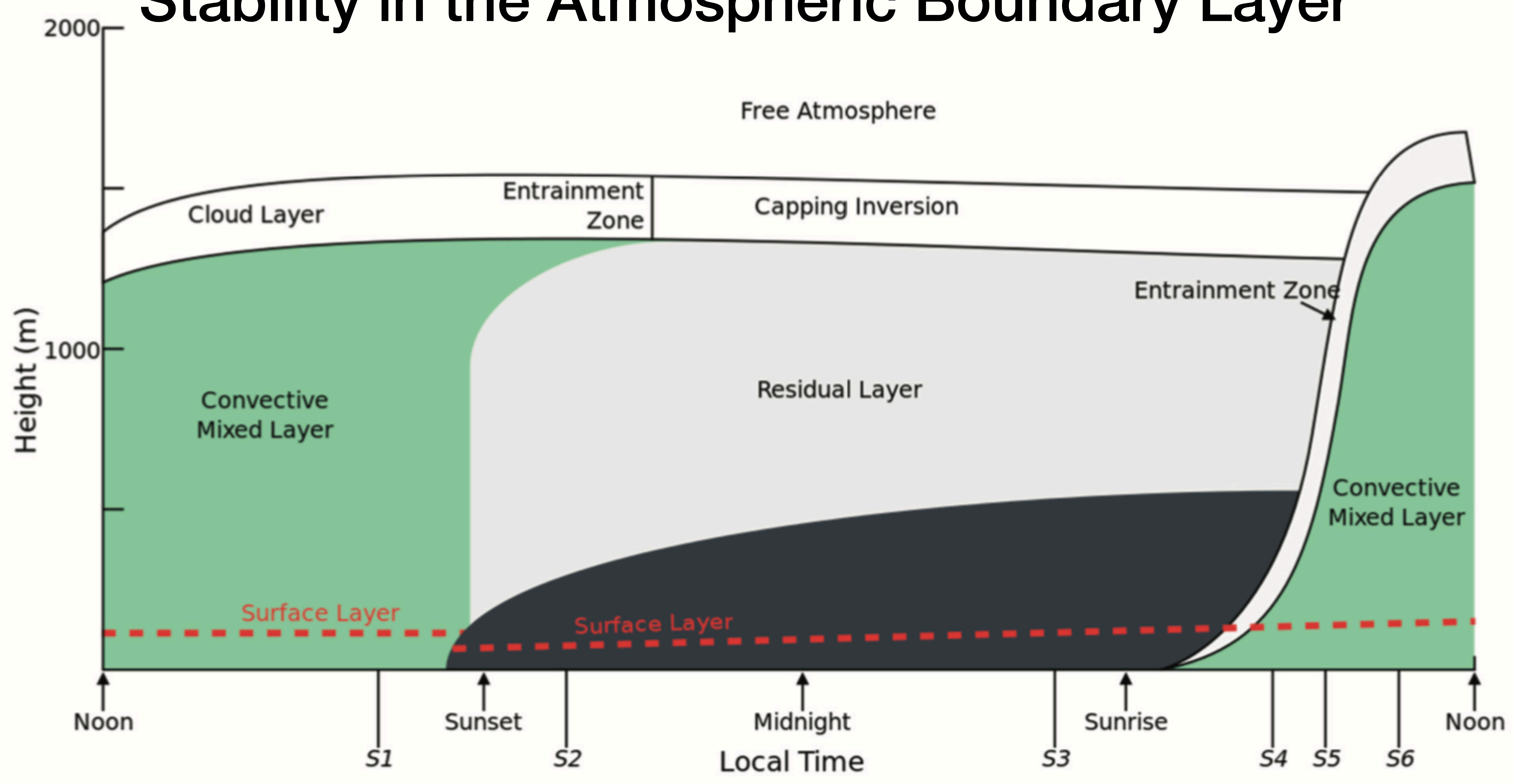
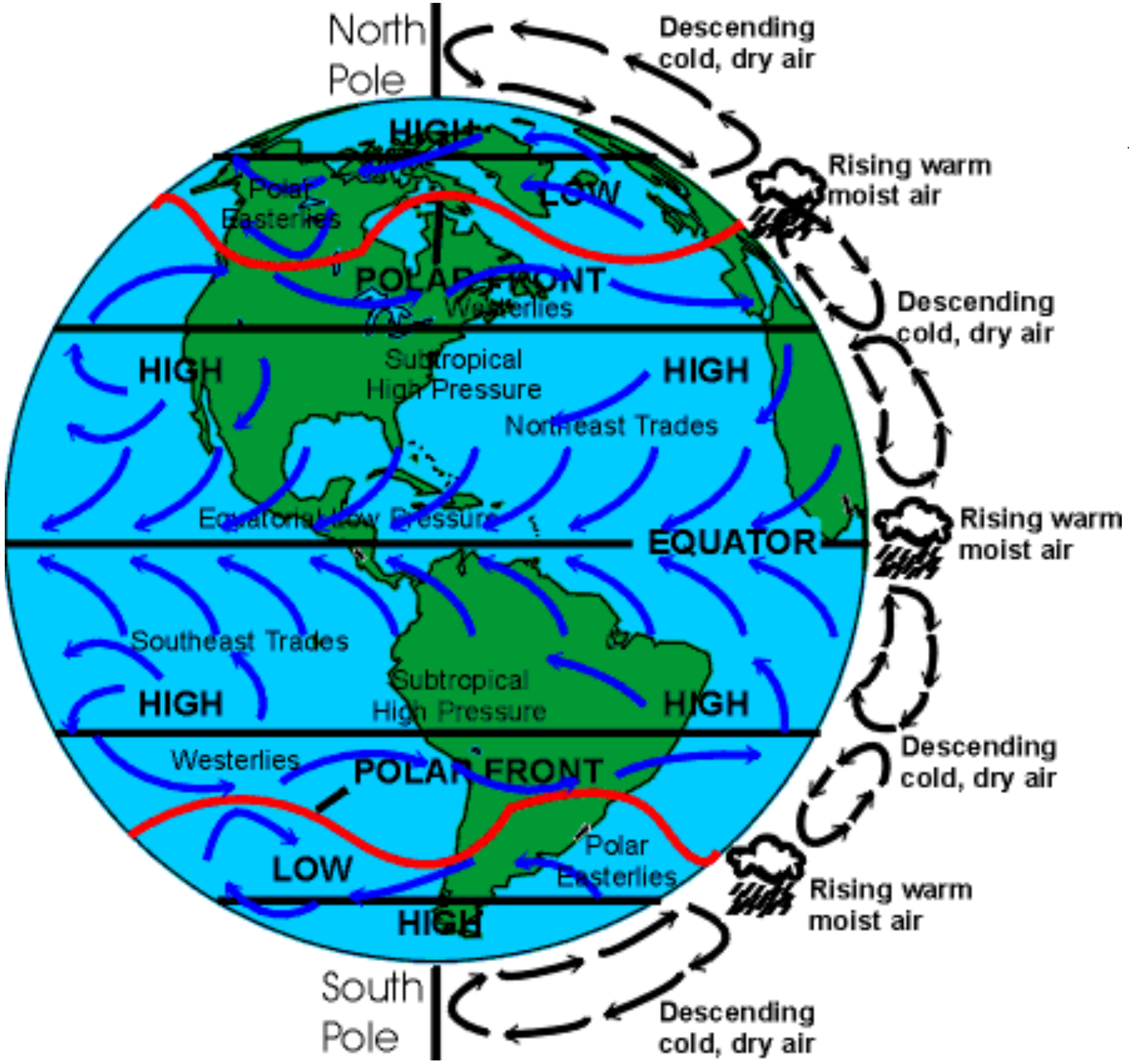


FIGURE 1. Schematic of the structure of the atmospheric boundary layer in high pressure regions over land, showing daily variations. SOURCE: Wikimedia Commons.

There are other types of inversions—Trade Winds



From https://propelsteps.wordpress.com/tag/geographic-illiteracy/

ON THE STRUCTURE OF THE TRADE-WIND MOIST LAYER 17

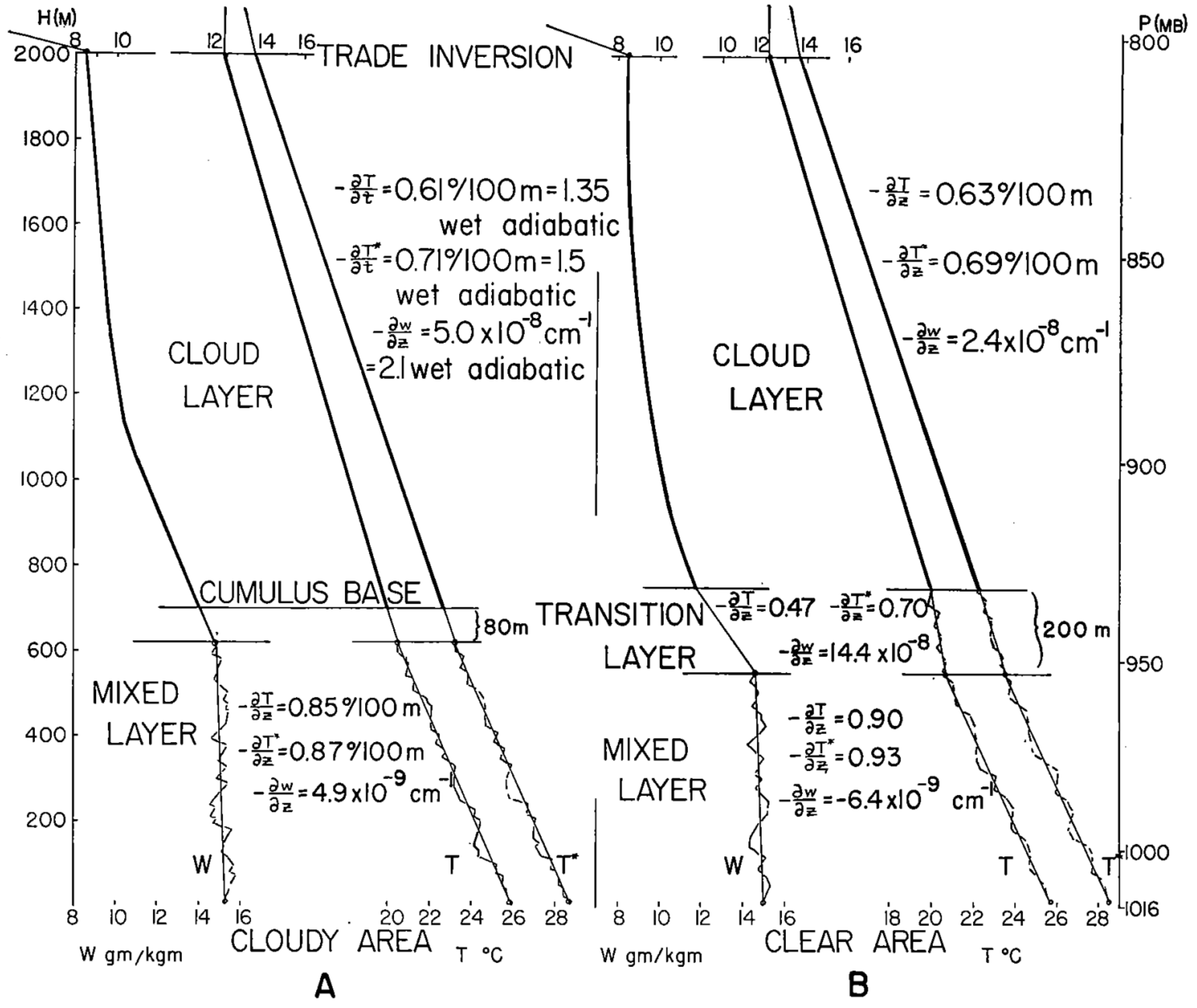
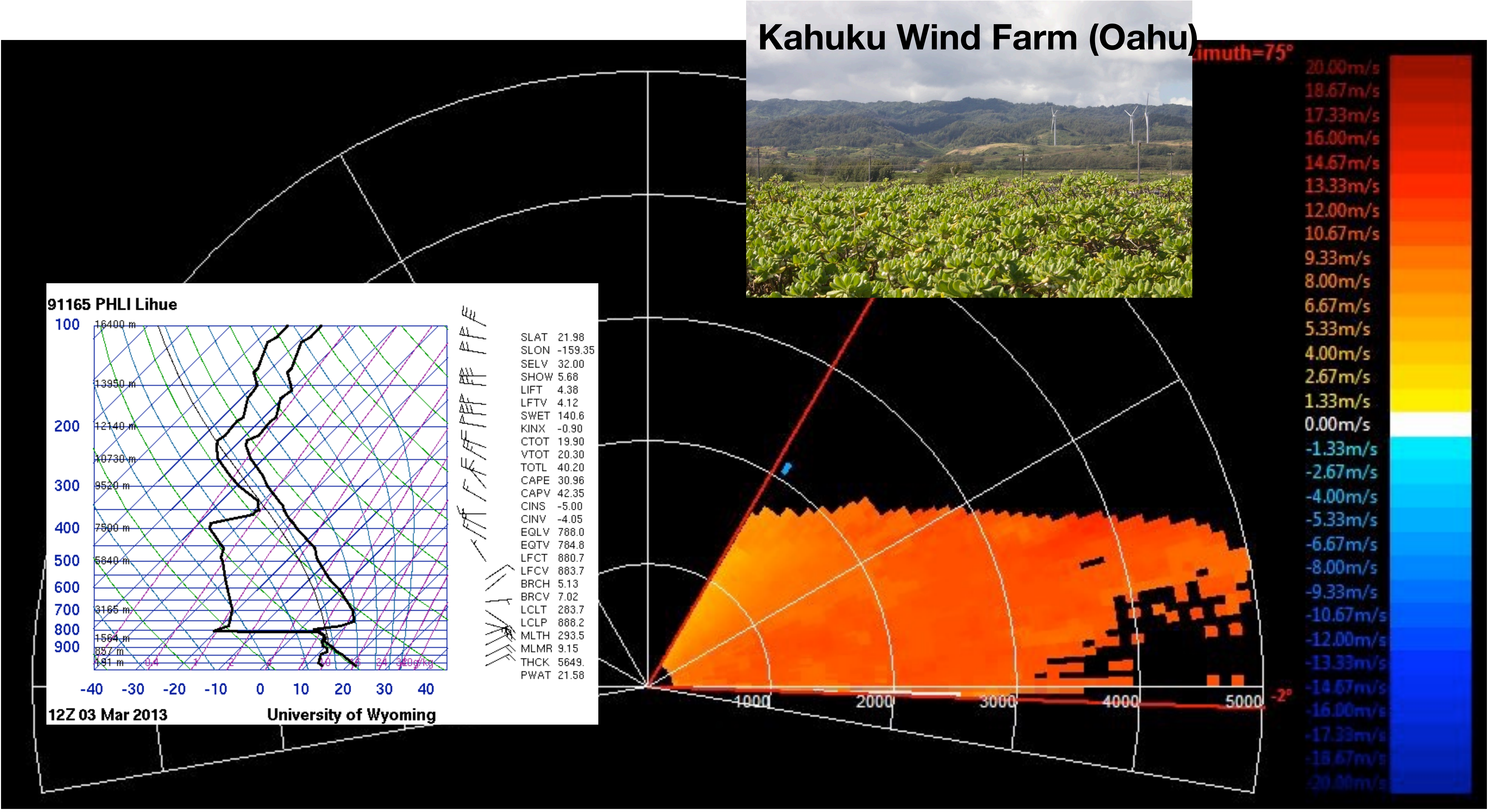


FIGURE 7. Average soundings for H period, compiled by taking the average temperature, T, virtual temperature, T*, and mixing ratio, w, at the base of each layer (see text), the average lapse rate within the layer of each property and the average vertical thickness of the layer. Height in meters is the ordinate. Figure 7A is the average cloudy area sounding (compiled from nine individual soundings) and Figure 7B is the average clear area sounding (compiled from sixteen individual soundings). Generally one or more soundings of each type was made on a given observing day.

Joanne Starr Malkus, 1958: ON THE STRUCTURE OF THE TRADE WIND MOIST LAYER

E.g.,Trade Inversion (Hawaii)

Range Height Indicator Scan 4 March 2013 0910:05 UTC
Azimuth = 75°



Kahuku Wind Farm (Oahu)

13796 ft (4205 m) top of Mauna Kea HI

Cloud tops above inversion

Driving through the inversion

Mean and Turbulent Kinetic Energy

$$\text{MKE}/m = \frac{1}{2}\left(\overline{U}^2 + \overline{V}^2 + \overline{W}^2\right)$$

Mean KE

$$e = \frac{1}{2}\left(u'^2 + v'^2 + w'^2\right)$$

Instantaneous KE

Turbulent Kinetic Energy

$$\frac{\text{TKE}}{m} = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) = \bar{e}$$

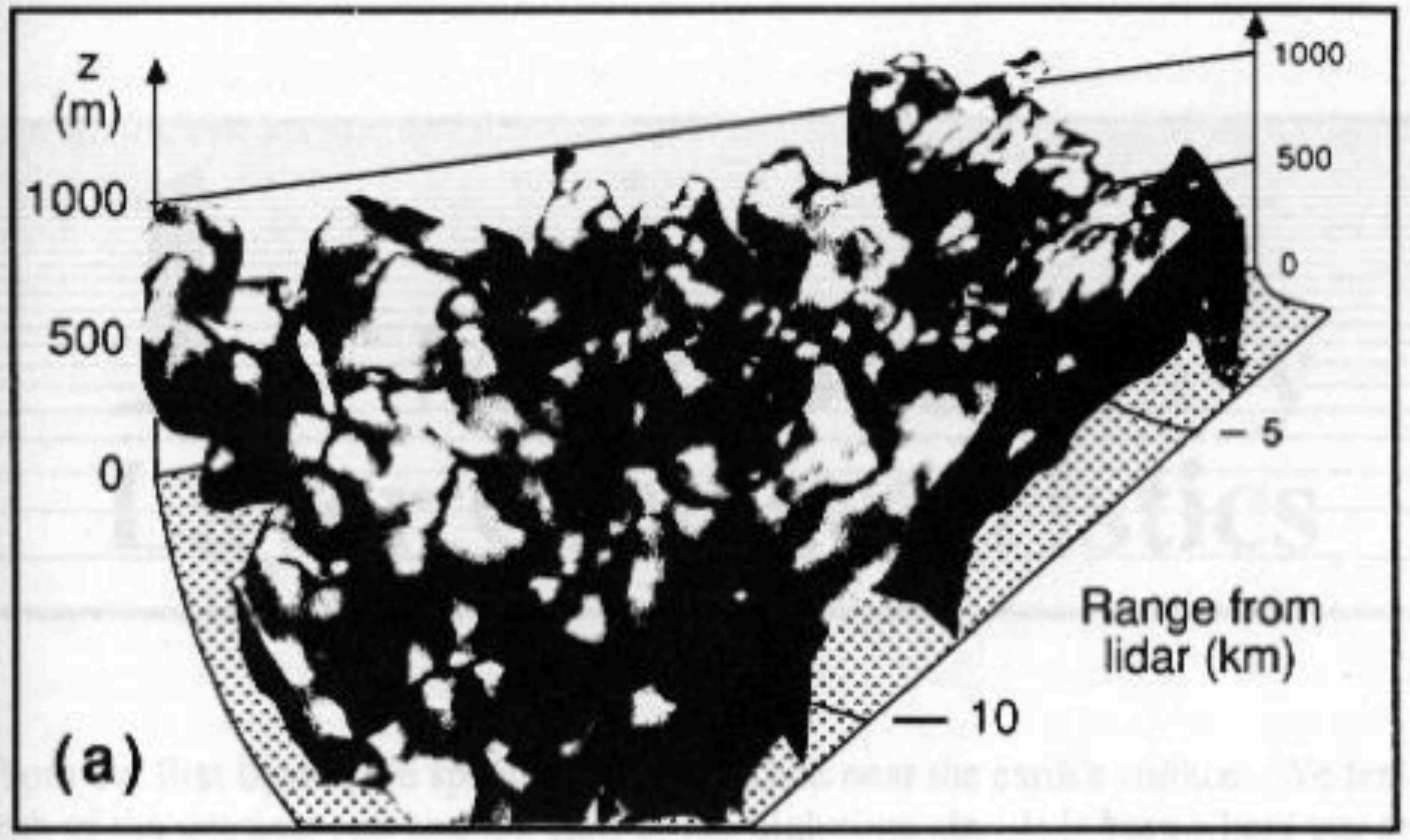
Turbulent Kinetic Energy Budget
(Stull Ch. 5)

$$\frac{\partial \bar{e}}{\partial t} + \overline{U}_j \frac{\partial \bar{e}}{\partial x_j} = + \delta_{i3} \frac{g}{\overline{\theta}_v}\left(\overline{u_i'\theta_v'}\right) - \overline{u_i'u_j'} \frac{\partial \overline{U}_i}{\partial x_j} - \frac{\partial\left(\overline{u_j'e}\right)}{\partial x_j} - \frac{1}{\overline{\rho}} \frac{\partial\left(\overline{u_i'p'}\right)}{\partial x_i} - \varepsilon$$

I	II	III	IV	V	VI	VII
Storage	**Advection**	**Buoyant Production Or consumption**	**Mechanical/Shear Production/Loss**	**Turbulent Transport**	**Pressure Perturbations**	**Dissipation**

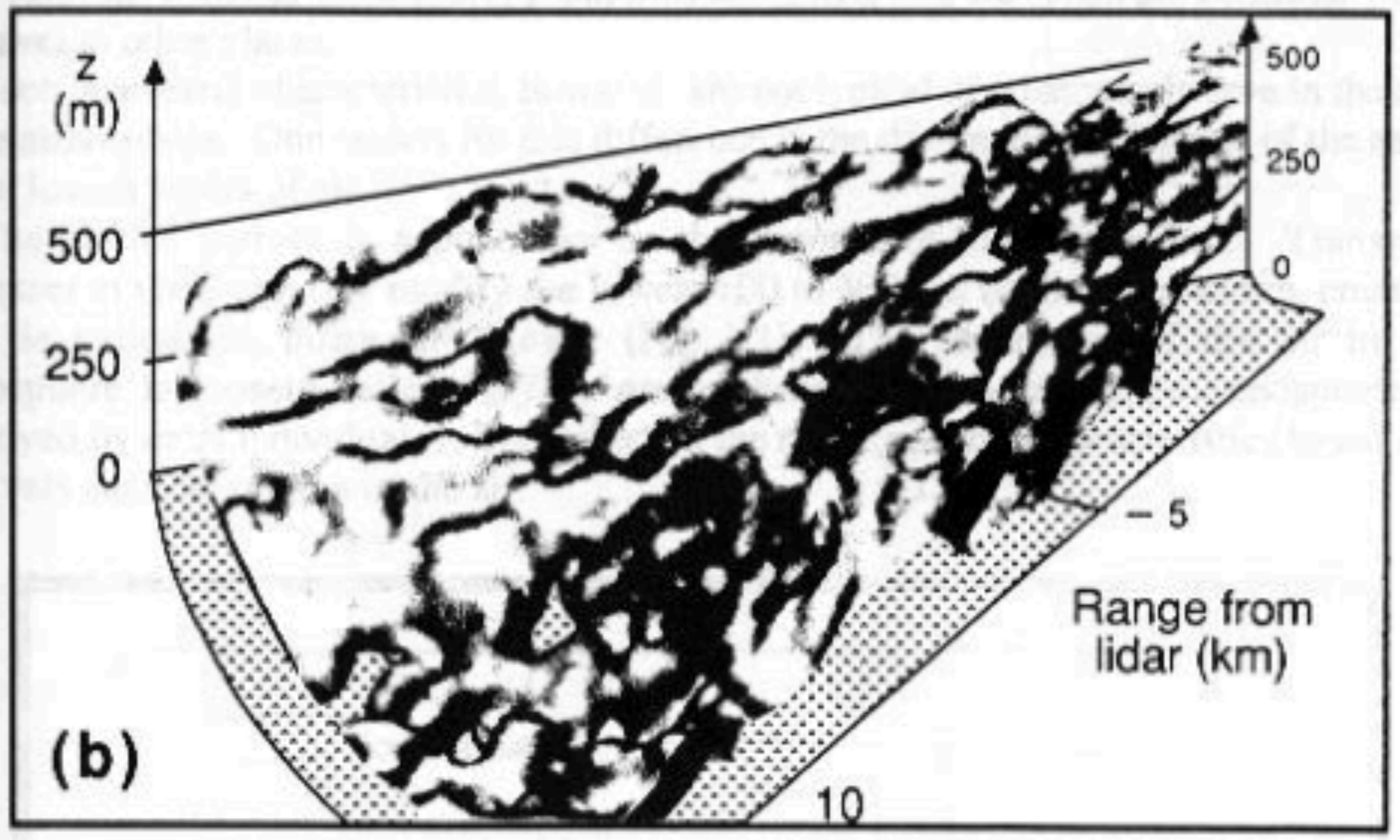
Stability enhances or suppresses turbulence (and fluxes)—determines the capability for buoyant convection

unstable



note higher vertical scale

slightly stable



Lidar images of the aerosol-laden boundary layer, obtained during the FIFE field experiment in Kansas. (a) Convective mixed layer observed at 1030 local time on 1 July 1987, when winds were generally less than 2 m/s. (b) Slightly-stable boundary layer with shear-generated turbulence, observed at 530 local time on 7 July 1987. Winds ranged from 5 m/s near the surface to 15 m/s near the top of the boundary layer. Photographs from the Univ. of Wisconsin lidar are courtesy of E. Eloranta, Boundary Layer Research Team.

Unstable flows → …become or remain turbulent

Stable flows → …become or remain laminar

Stull 1988

Static Stability

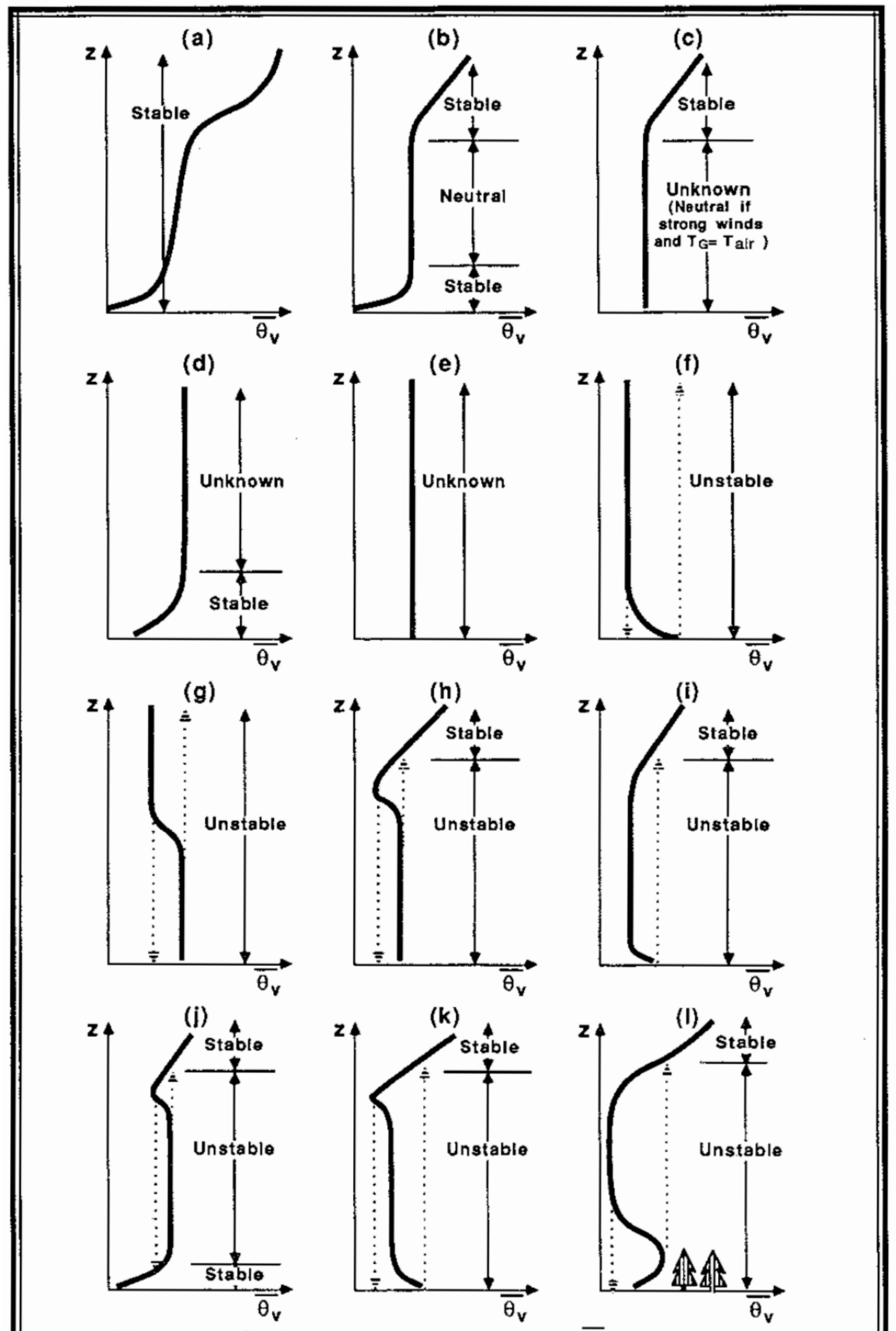
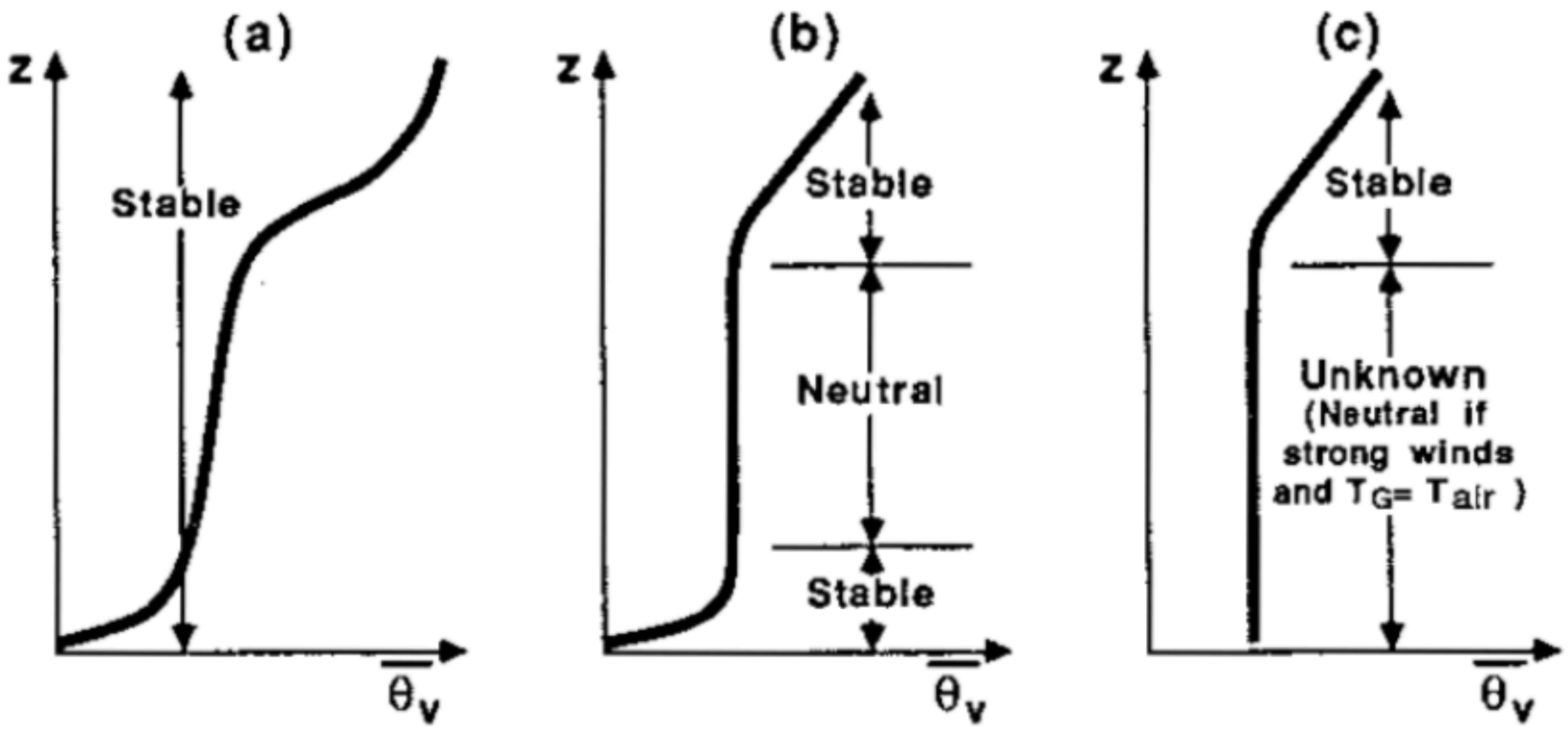


Fig. 5.17 Static stabilitiy as a function of the $\overline{\theta_v}$ profile. Dotted lines denote parcel movement.

- measure of capability for convection
- considers buoyancy only (does not consider wind/mechanical turbulence)
- local lapse rate (stability) insufficient - need the look at the whole profile *or* measure the buoyancy flux

Stull 1988

Static Stability



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Stull 1988

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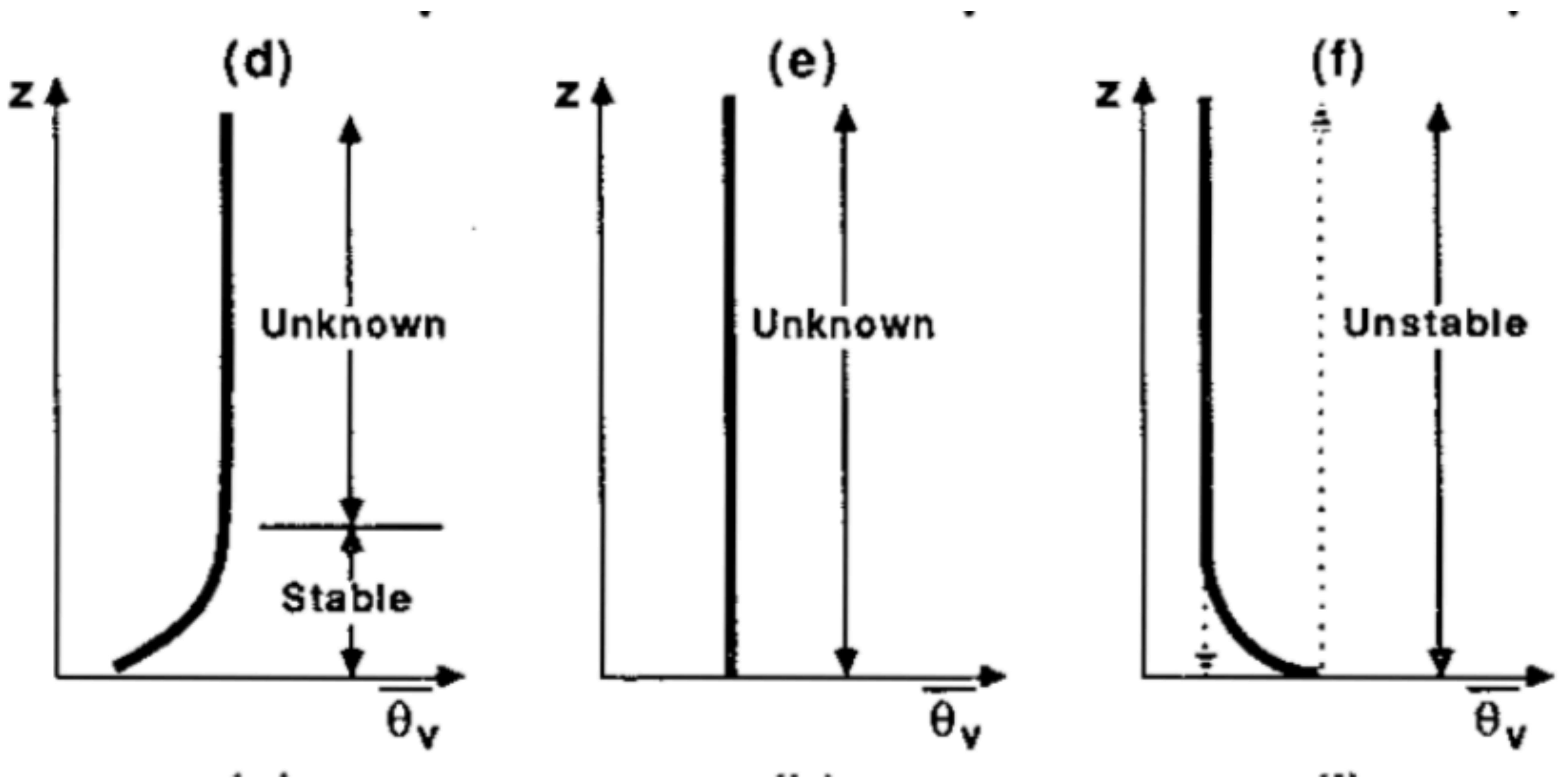

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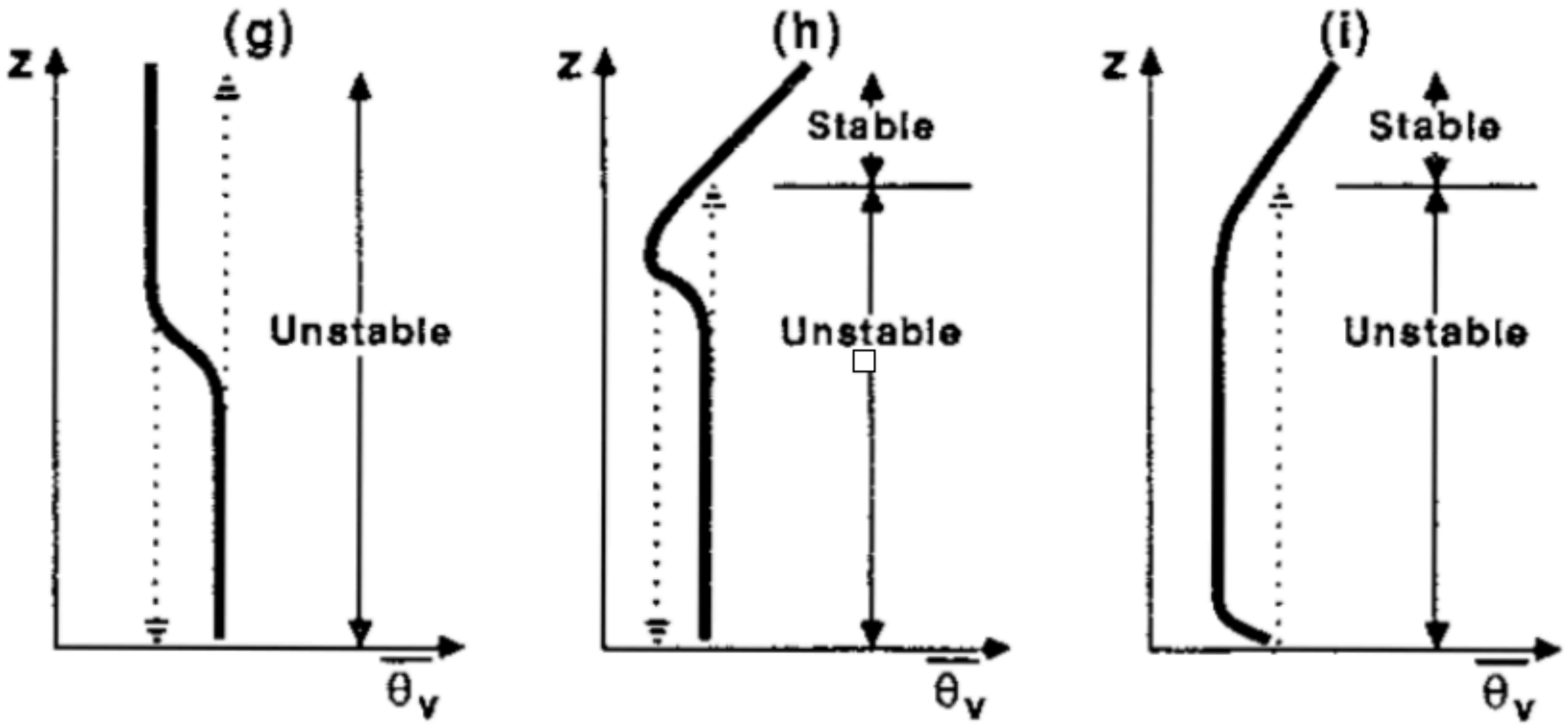
Stull 1988

Static Stability

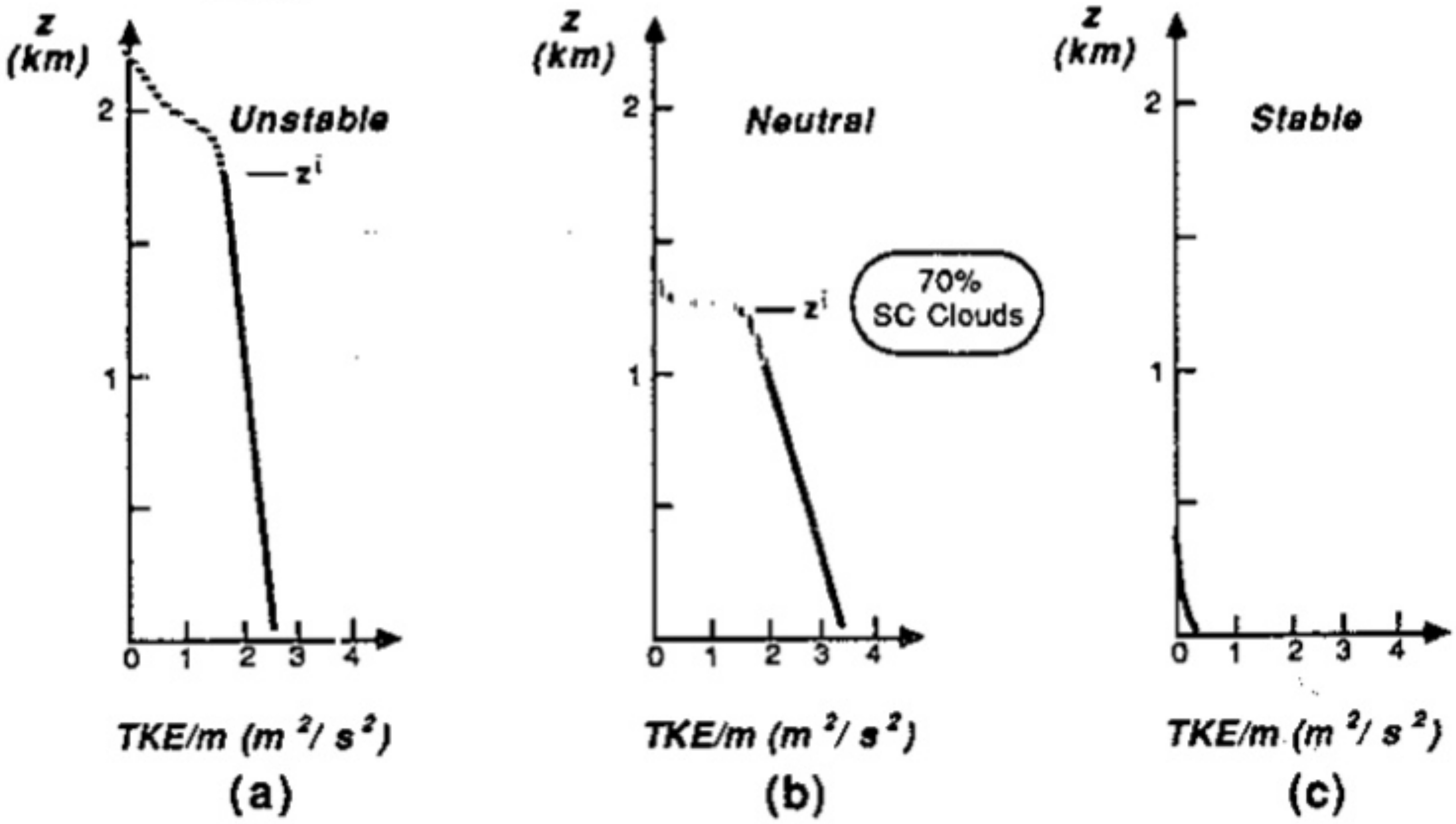


- measure of capability for convection
- considers buoyancy only (does not consider wind/mechanical turbulence)
- local lapse rate (stability) insufficient - need the look at the whole profile *or* measure the buoyancy flux

Stull 1988

Stability enhances or suppresses turbulence (and fluxes)

unstable **neutral** **stable**



- TKE is suppressed during stable conditions
- confined to shallower layer near surface

Stull 1988

Richardson Number, $R_{[n]}$

Dimensionless ratio of buoyant suppression of turbulence to shear generation of turbulence.

$$Ri \approx \frac{\text{buoyancy forcing}}{\text{shear forcing}}$$

Eddy diffusivity analogy to molecular diffusion:

momentum $$\tau = -\rho\, \overline{u'w'} \propto \frac{\partial \overline{U}}{\partial z}$$

heat $$H = \rho\, C_p \overline{w'T'} \propto \frac{\partial T}{\partial z}$$

Buoyancy $$\overline{w'\theta_v'} \propto \frac{\partial \overline{\theta_v}}{\partial z}$$

We can define different Richardson numbers….

Gradient Richardson, Ri

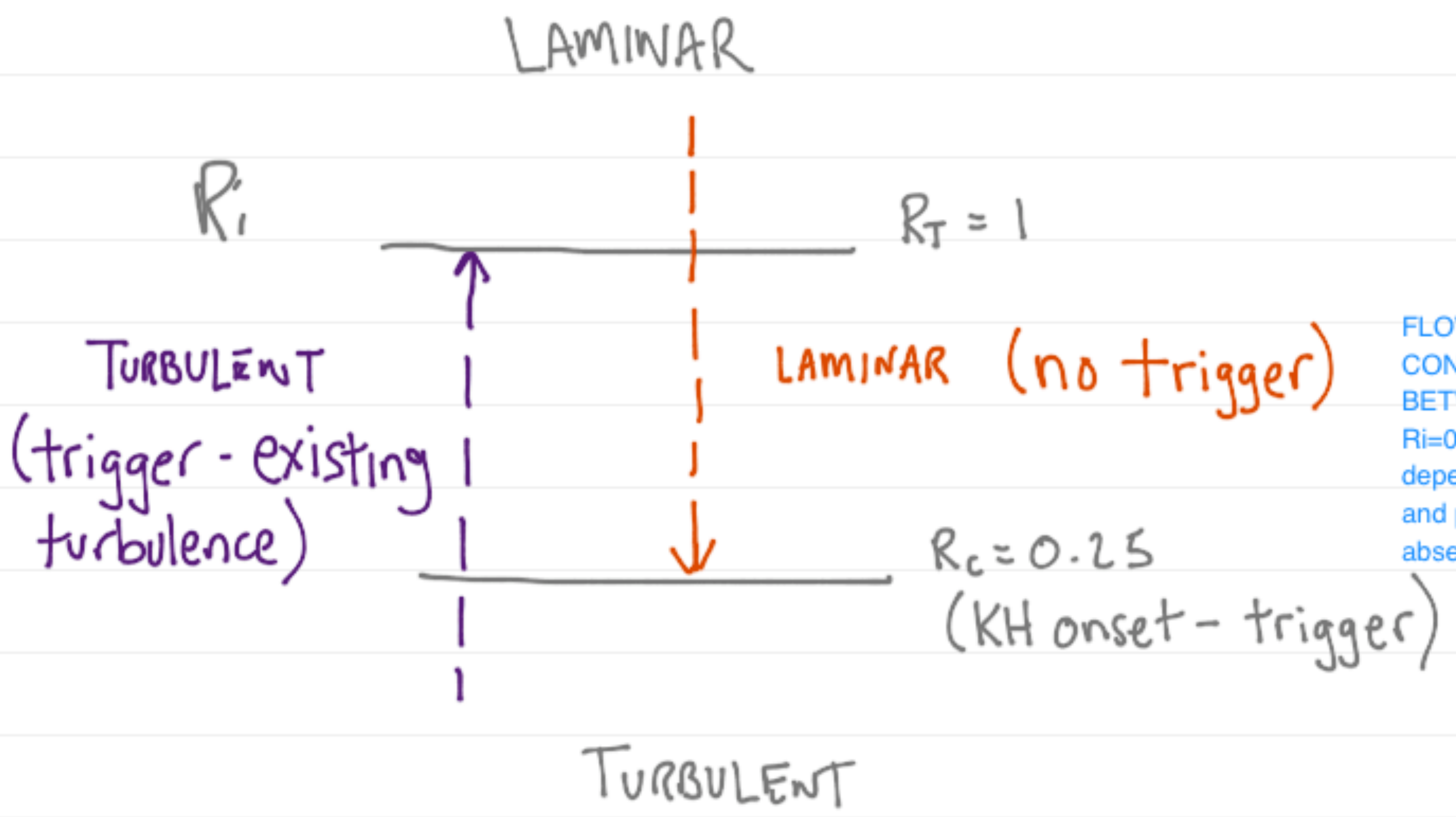
$$Ri = \frac{\frac{g}{\overline{\theta_v}} \frac{\partial \overline{\theta_v}}{\partial z}}{\left(\frac{\partial \overline{U}}{\partial z}\right)^2}$$

(Gradient) Richardson Number, R_i

$R_i < R_c$ → onset of turbulence

$R_i > R_T$ → turbulent to laminar

$R_c = 0.21 - 0.25$
$R_T = 1$
} hysteresis



FLOW CONDITION BETWEEN Ri=0.25 AND RI=1 depends on history and presences/ absence of trigger

Problem B: Given a fictitious SBL where $(g/\overline{\theta_v}) = 0.033$ m s^{-2} K^{-1}, $\partial\overline{U}/\partial z = [u_* / (0.4 \cdot z)]$ s^{-1}, $u_* = 0.4$ m/s, and where the lapse rate, c_1, is constant with height such that there is 6°C $\overline{\theta_v}$ increase with each 200 m of altitude gained. How deep is the turbulence?

Problem B p.180

Given: $g/\overline{\theta}_v = 0.033\ m\,s^{-2}K^{-1}$

$\partial\overline{U}/\partial z = U_*/0.4z\ \ s^{-1}$

$u_* = 0.4\ m\,s^{-1}$

lapse rate constant @ +6°C/200m (stable)

$$R_i = \frac{g}{\overline{\theta}_v}\frac{\partial\overline{\theta}_v}{\partial z} \Big/ \left(\frac{\partial\overline{U}}{\partial z}\right)^2$$

$$C_1 = \frac{\partial\overline{\theta}_v}{\partial z} = \frac{6C}{200\,m} = 0.03\ C\,m^{-1}$$

$$R_i = \frac{\frac{g}{\overline{\theta}_v} C_1}{\left(\frac{U_*}{0.4z}\right)^2} = \frac{(0.033\,m)}{s^2 K}\frac{(0.03\,K)}{m}\frac{s^2}{(0.4/0.4)^2} z^2$$

$$= 0.00099\ m^{-2}\ z^2$$

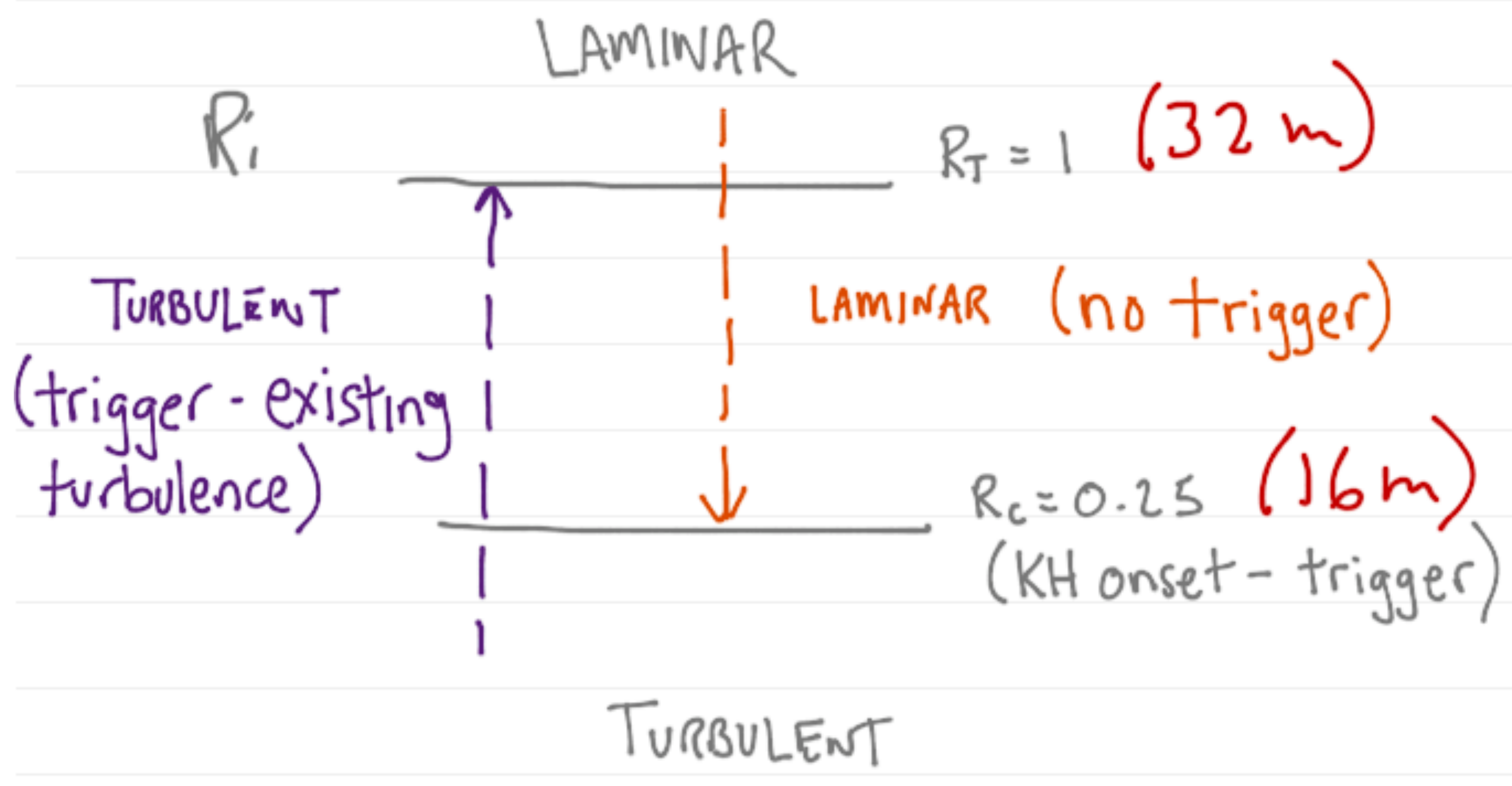
to determine height below which is turbulence:

set $R_i = R_c = 0.25$

$$z_c = \sqrt{(1010)(0.25)\,m^2} = \underline{16m}$$

$$z_T = \sqrt{(1010)(1)\,m^2} = \underline{32\ m}$$

- Below 16m, turbulent
- above 32 laminar
- in between - depends on history @ that ht



Introduce the Bulk Richardson Number R_b

Bulk Richardson

- gradients not always available
- approximate gradients from discrete measurements

$$\frac{\partial \overline{\theta}_v}{\partial z} \approx \frac{\Delta \overline{\theta}_v}{\Delta z} \quad , \quad \frac{\partial \overline{U}}{\partial z} \approx \frac{\Delta \overline{U}}{\Delta z}$$

$$R_B = \frac{g \Delta \overline{\theta}_v \Delta z}{\overline{\theta}_v \left(\Delta \overline{u}^2 + \Delta \overline{v}^2 \right)}$$

- for thinner layers $R_c = 0.25$
- thicker layers average out gradients

R_i time-height section

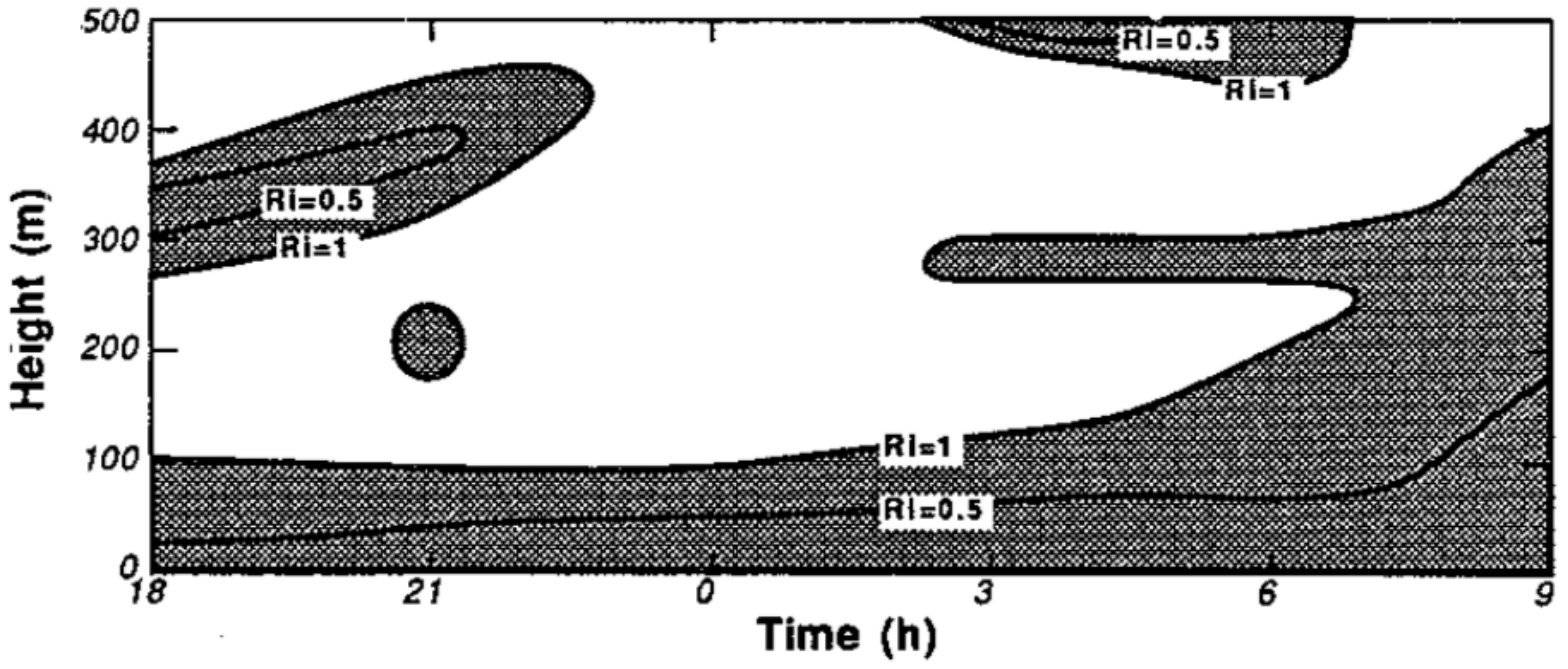


Fig. 5.20 Example of the evolution of local Richardson number with height and time during one night. Regions with Richardson number less than 1.0 are shaded, and are likely to be turbulent. After Mahrt (1981).

- grey areas turbulent
- Ri >1 so stable yet sufficient shear for turbulence
- near surface turbulent stable layer

Stull 1988

Stability—introducing the Obukhov Length (L)

$$L = -\frac{\bar{\theta}_v u_*^3}{gk\left(\overline{w'\theta_v'}\right)_s}$$ Obukhov Length

Don't forget that minus (-) sign!

$$L \approx -\frac{\text{shear forcing}}{\text{buoyancy forcing}}$$

$L < 0$ unstable

$L > 0$ stable

- $\overline{w'\theta_v'} > 0$, $L < 0$ unstable
- $\overline{w'\theta_v'} < 0$, $L > 0$ stable

$$Ri \approx \frac{\text{buoyancy forcing}}{\text{shear forcing}}$$

Obukhov Length

$$L = -\frac{\bar{\theta}_v u_*^3}{g k (\overline{w'\theta_v'})_s}$$ **meters**

- L negative during daytime (unstable) and positive at night (stable)
- larger L magnitude corresponds to more shear and/or less heat flux
- L blows up when surface heat flux transitions pos/neg or neg/pos

Physical interpretation: scale height where buoyancy dominates over shear

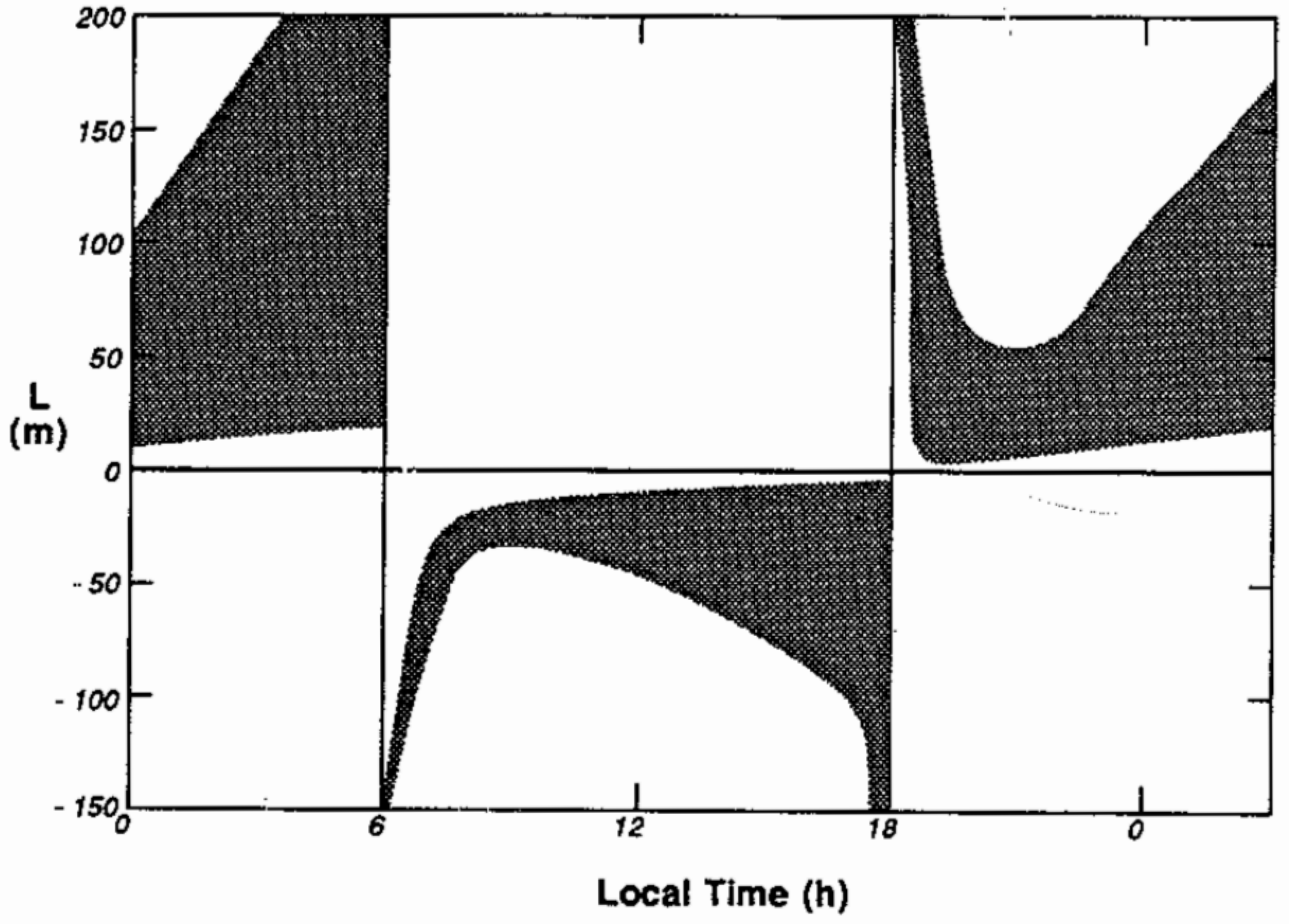


Fig. 5.21 Typical ranges of Obukhov length (L) evolution over a diurnal cycle.

Stull 1988

Similarity Example— dimensionless variables to collapse curves

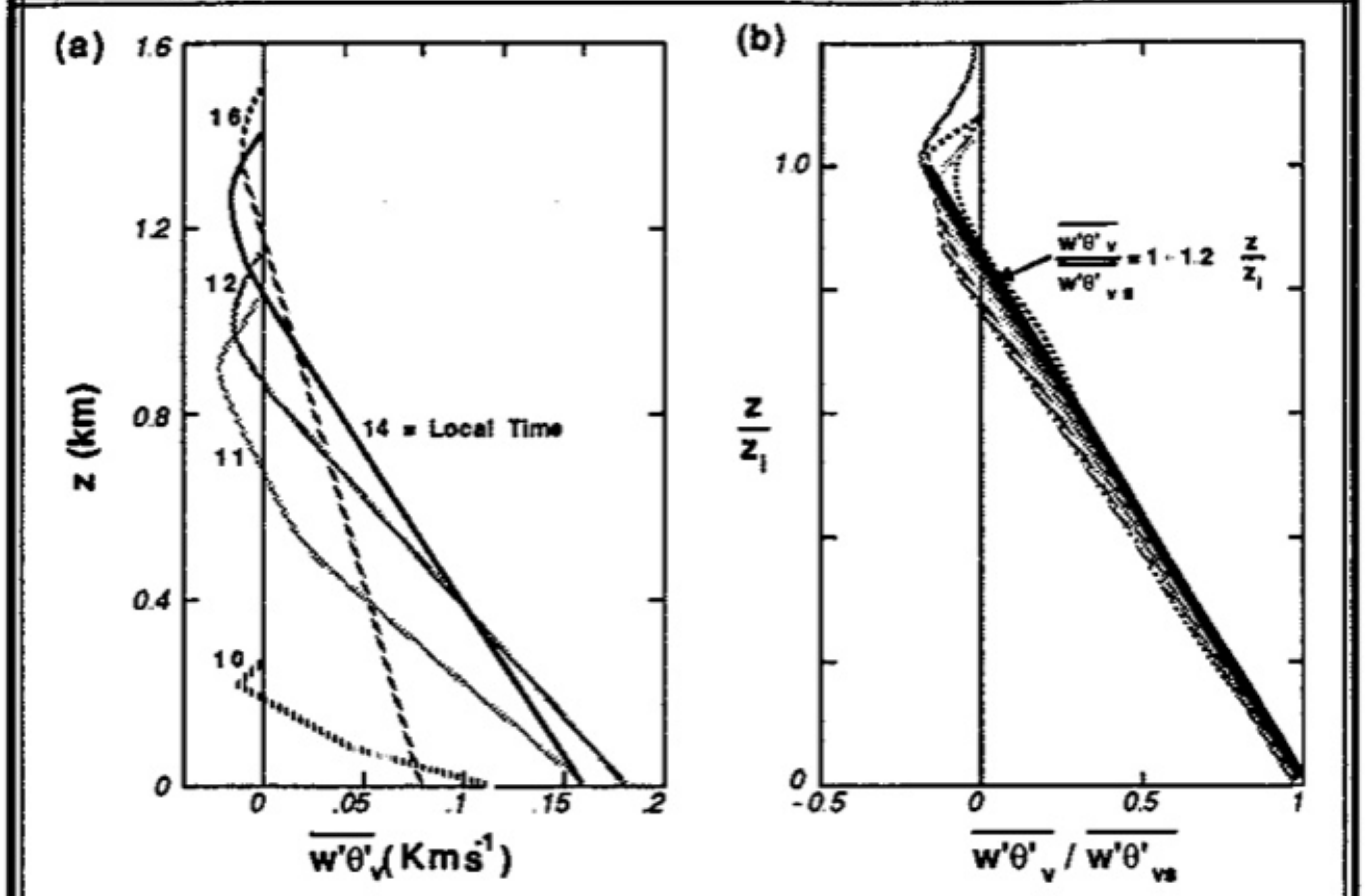


Fig. 9.1 Raw heat flux data from a simulation of Wangara Day 33 (a) replotted in a dimensionless framework (b). The empirical straight line estimate from similarity theory is also shown in (b).

- soundings of buoyancy flux at different times of day
- profile structure: positive at surface, decrease linearly with height, goes slightly negative, returns to zero
- normalizing height and flux collapses curves
- seeking "universal" relationships

Stull 1988

Obukhov Length—relate to TKE Equation

Obukhov Length

Normalize TKE by dividing by $-\frac{u_*^3}{kz}$

$\epsilon = u_*^3/kz$ (surface layer)

k = von Karmans constant = 0.4

$$S + A = -\underbrace{\frac{kzg(\overline{w'\theta_v'})_s}{\bar{\theta}_v u_*^3}}_{III} + \underbrace{\frac{kz(\overline{u'w'})}{u_*^3}\frac{\partial \bar{U}}{\partial z}}_{IV} + \underbrace{\frac{kz\,\epsilon_s}{u_*^3}}_{VII} + \dots$$

term III:

$$\zeta = \frac{z}{L} = \frac{-kzg\,\overline{w'\theta_v'}}{\bar{\theta}_v u_*^3}$$ dimensionless

- z is ht
- remaining terms units Length

$$\frac{\partial \bar{e}}{\partial t} + \bar{U}_j \frac{\partial \bar{e}}{\partial x_j} = + \delta_{i3}\frac{g}{\bar{\theta}_v}\left(\overline{u_i'\theta_v'}\right) - \overline{u_i'u_j'}\frac{\partial \bar{U}_i}{\partial x_j} - \frac{\partial\left(\overline{u_j'e}\right)}{\partial x_j} - \frac{1}{\bar{\rho}}\frac{\partial\left(\overline{u_i'p'}\right)}{\partial x_i} - \varepsilon \qquad (5.1a)$$

I II III IV V VI VII

Term I represents local ***storage*** or tendency of TKE.

Term II describes the ***advection*** of TKE by the mean wind.

Term III is the ***buoyant production or consumption term***. It is a production or loss term depending on whether the heat flux $\overline{u_i'\theta_v'}$ is positive (during daytime over land) or negative (at night over land).

Term IV is a ***mechanical or shear production/loss term***. The momentum flux $\overline{u_i'u_j'}$ is usually of opposite sign from the mean wind shear, because the momentum of the wind is usually lost downward to the ground. Thus, Term IV results in a positive contribution to TKE when multiplied by a negative sign.

Term V represents the ***turbulent transport*** of TKE. It describes how TKE is moved around by the turbulent eddies u_j'.

Term VI is a ***pressure correlation term*** that describes how TKE is redistributed by pressure perturbations. It is often associated with oscillations in the air (***buoyancy or gravity waves***).

Term VII represents the viscous ***dissipation*** of TKE; i.e., the conversion of TKE into heat.

Monin-Obukhov Similarity, z/L

- MO Similarity accounts for relative importance of shear (mechanical turbulence) and buoyancy in the generation of turbulence and affects on surface fluxes and surface layer profiles
- z= height above the surface
- ratio z/L is dimensionless
- can be described as a surface layer scaling parameter
- when z/L is small, buoyancy is less important
- as z increases, buoyancy increasingly important

Diabatic (non-neutral) wind profile

neutral $\phi_m = 1$

$$\bar{U}(z) = \frac{U_*}{k} \ln(z/z_0)$$ log wind profile "law of the wall"

- for non-neutral (diabatic) $\phi_m \neq 1$

$$\phi_m = \phi_m(z/L)$$ surface layer

- $\phi_m(z/L)$ determined empirically

$$\frac{\overline{M}}{u_*} = \left(\frac{1}{k}\right)\left[\ln\left(\frac{z}{z_o}\right) + \Psi_M\left(\frac{z}{L}\right)\right] \quad (9.7.5g)$$

where the function $\Psi(z/L)$ is given for stable conditions $(z/L > 0)$ by:

$$\Psi_M\left(\frac{z}{L}\right) = \frac{4.7\,z}{L} \quad (9.7.5h)$$

and for unstable $(z/L < 0)$ by:

$$\Psi_M\left(\frac{z}{L}\right) = -2\ln\left[\frac{(1+x)}{2}\right] - \ln\left[\frac{(1+x^2)}{2}\right] + 2\tan^{-1}(x) - \frac{\pi}{2} \quad (9.7.5i)$$

where $x = [1 - (15z/L)]^{1/4}$

How Φ relationships are used in stability

Integrate $\Phi_m(z/L)$ to get wind profile

Wind Profile

$$\phi_m(z/L) = \frac{kz}{U_*}\frac{d\bar{U}}{dz}$$

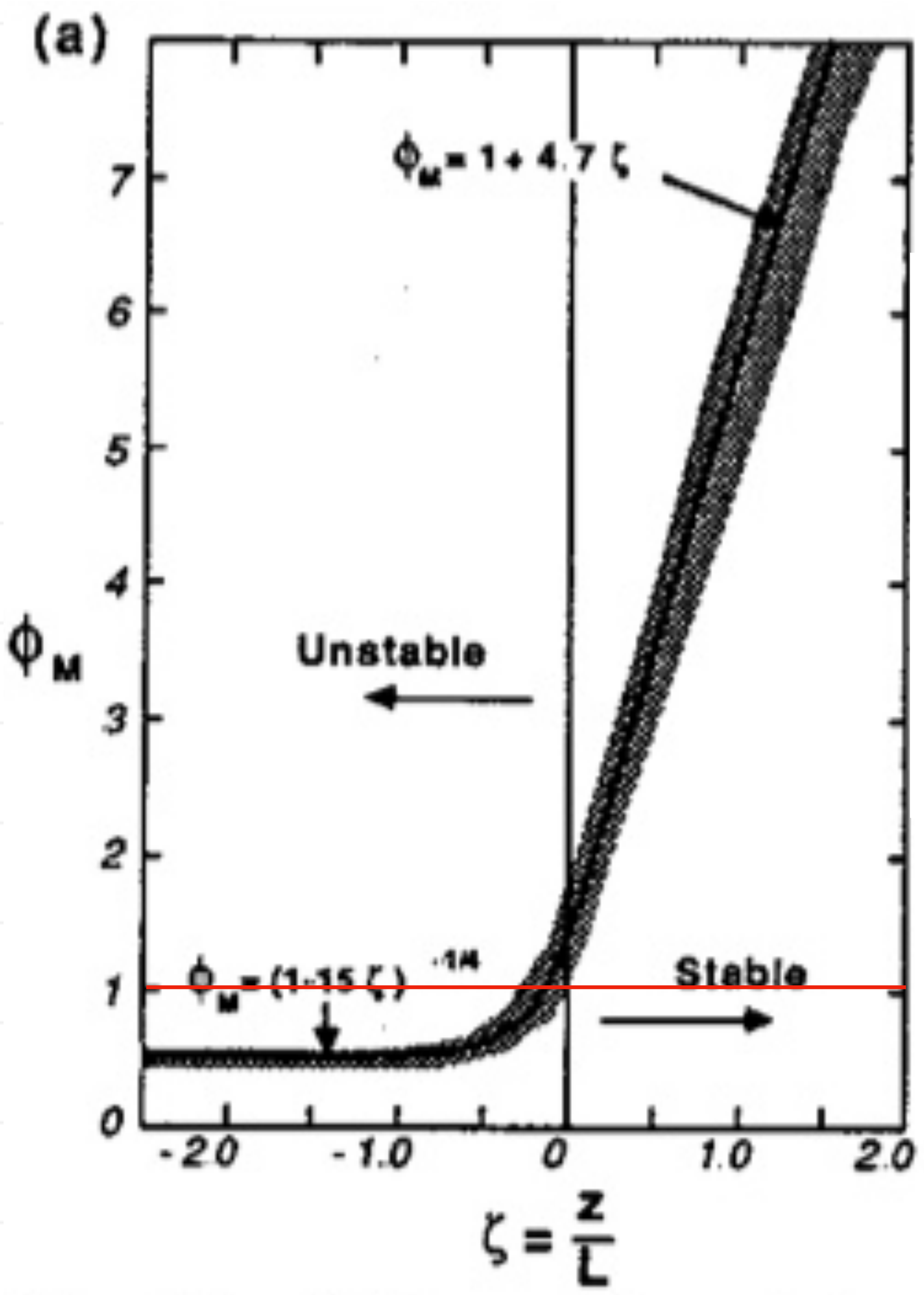
$$\frac{d\bar{U}}{dz} = \frac{U_*}{k}\frac{\phi_m(z/L)}{z}$$

$$\bar{U}(z) = \frac{U_*}{k}\int \frac{\phi_m(z/L)}{z}\, dz$$

- substitute $\phi_m(z/L)$ relationships
- integrate

$$\bar{U}(z) = \frac{U_*}{k}\left(\ln\frac{z}{z_0} + \psi_m(z/L)\right)$$

- $\psi_m = 0$ neutral
- $\psi_m(z/L)$ stable, unstable from Paulson (1972)



$$\phi_M = \begin{cases} 1 + \left(\frac{4.7\,z}{L}\right) & \text{for } \frac{z}{L} > 0 \text{ (stable)} \\ 1 & \text{for } \frac{z}{L} = 0 \text{ (neutral)} \\ \left[1 - \left(\frac{15z}{L}\right)\right]^{-1/4} & \text{for } \frac{z}{L} < 0 \text{ (unstable)} \end{cases}$$

Obukhov Length

$$L = \frac{-\bar{\theta}_v u_*^3}{gk(\overline{w'\theta_v'})_s}$$ Obukhov Length

- $\overline{w'\theta_v'} > 0$, $L < 0$ unstable
- $\overline{w'\theta_v'} < 0$, $L > 0$ stable
- L inverse to heat flux magnitude

- Figure 5.21

  - diurnal cycle L
  - at night, stable $L > 0$
  - $|L|$ smaller, closer to $y=0$, buoyancy larger wrt shear
  - $|L|$ large, shear dominates → neutral

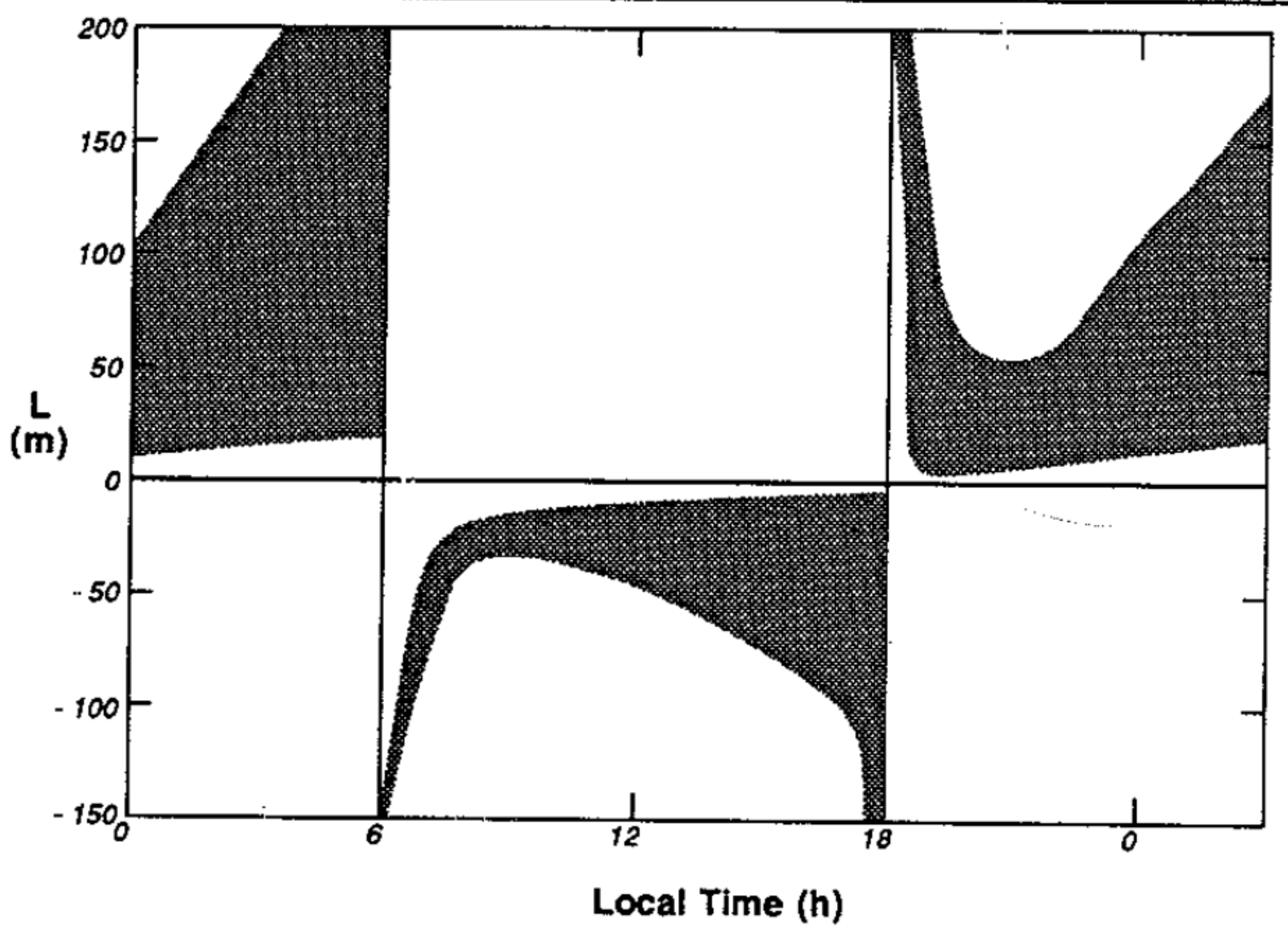


Fig. 5.21 Typical ranges of Obukhov length (L) evolution over a diurnal cycle.

Relationship between Ri and z/L

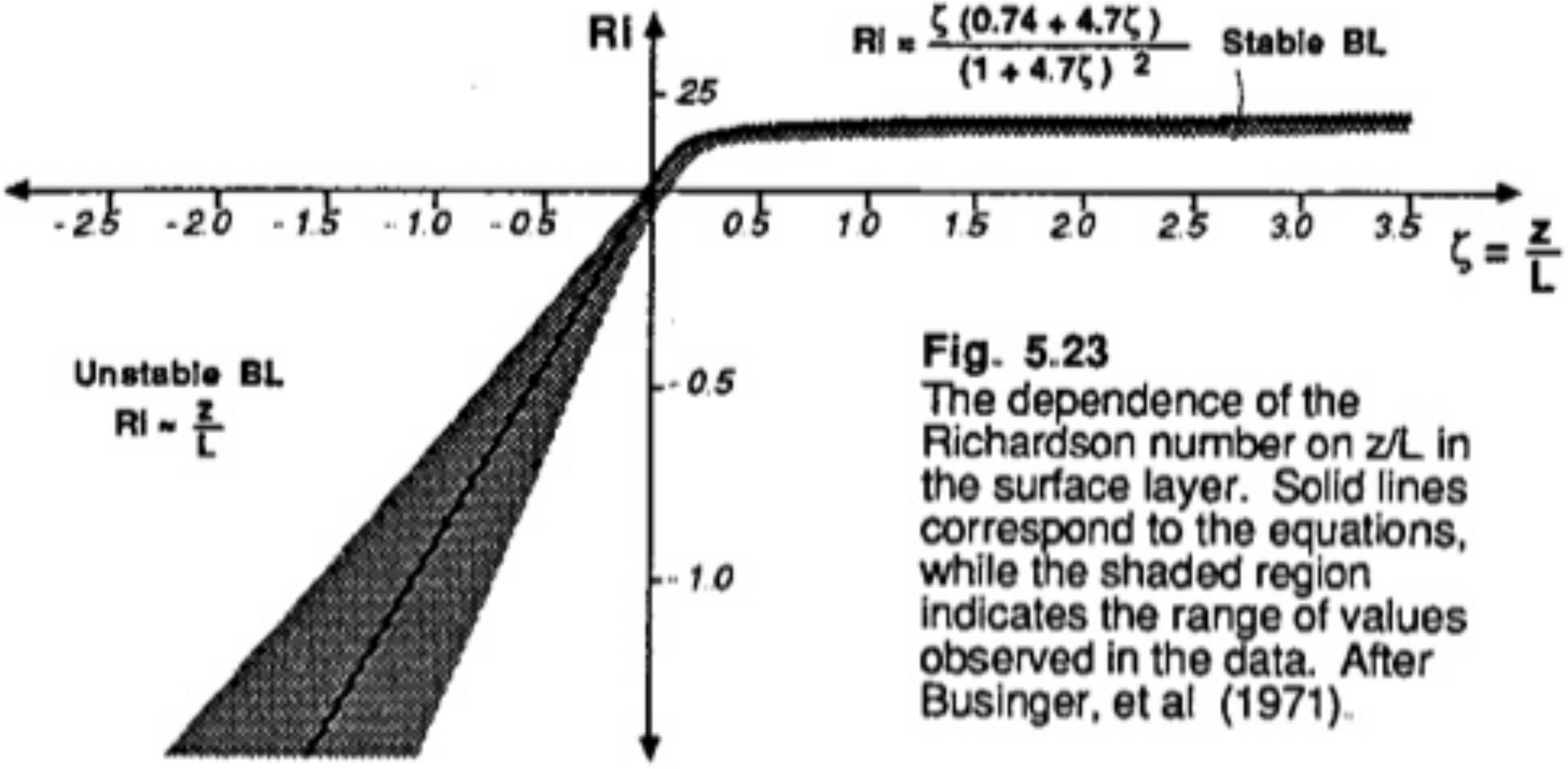


Fig. 5.23
The dependence of the Richardson number on z/L in the surface layer. Solid lines correspond to the equations, while the shaded region indicates the range of values observed in the data. After Businger, et al (1971).

Diabatic Wind Profiles

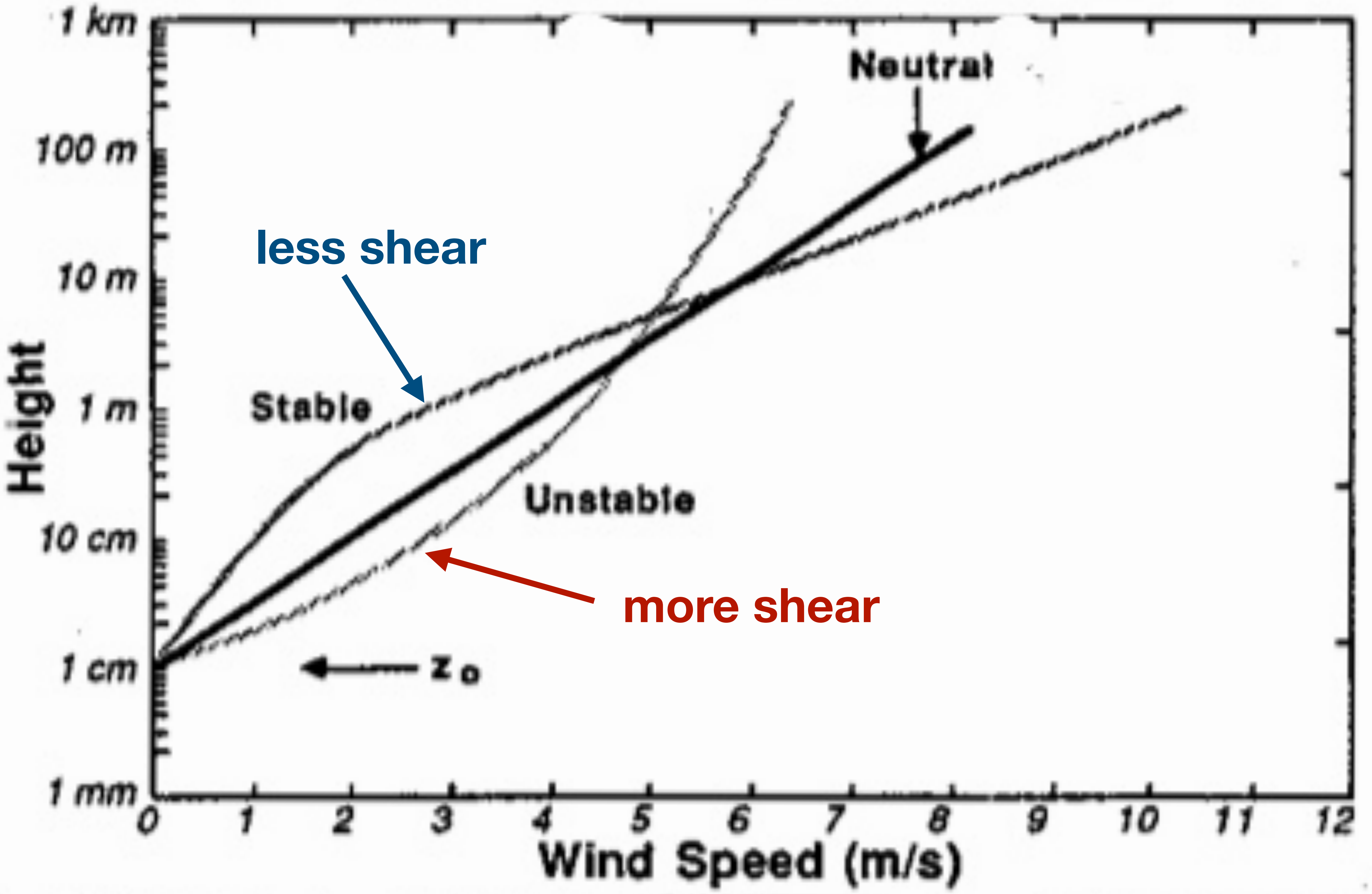
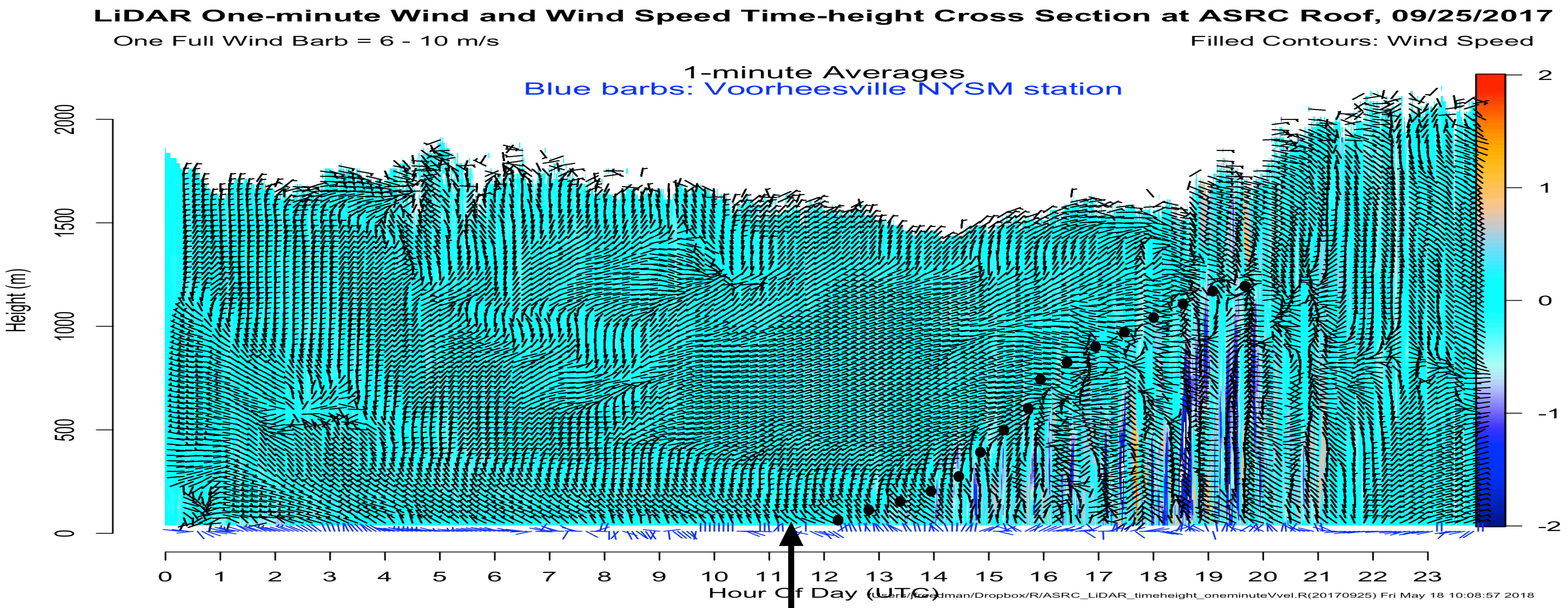


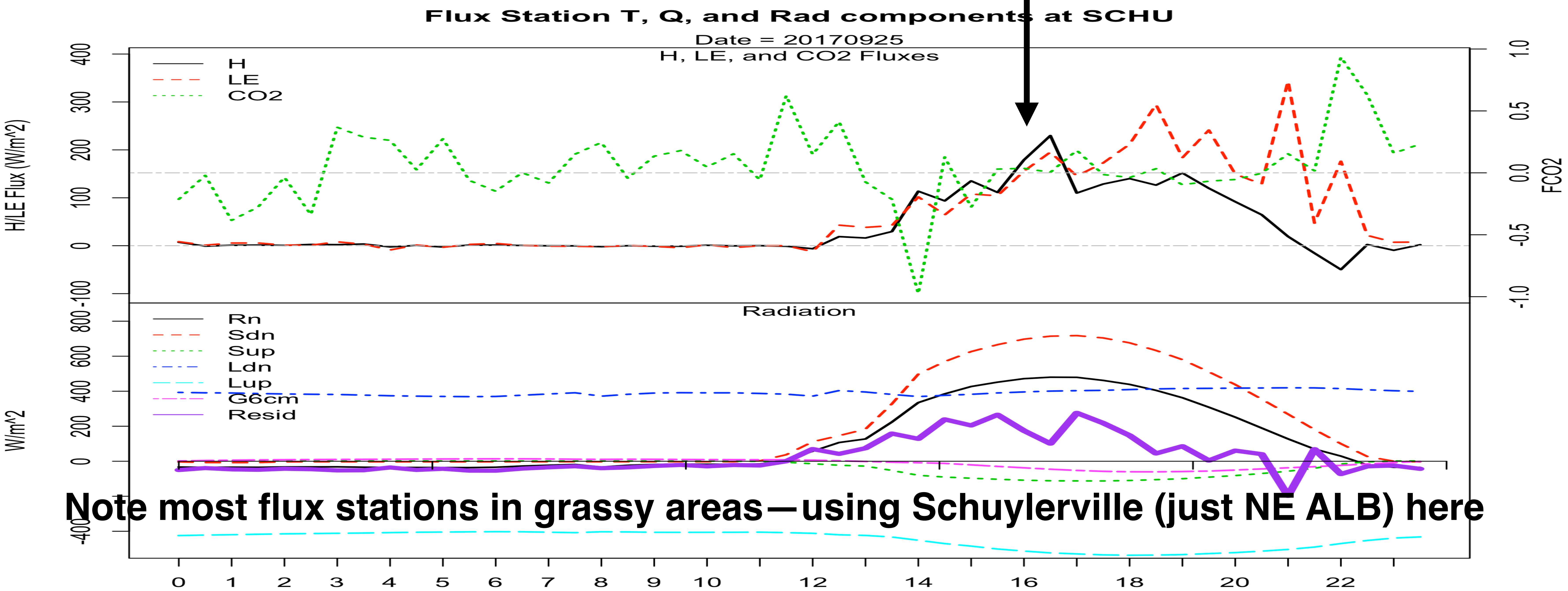
Fig. 9.5 Typical wind speed profiles vs. static stability in the surface layer

Stull 1988

Putting some things together….

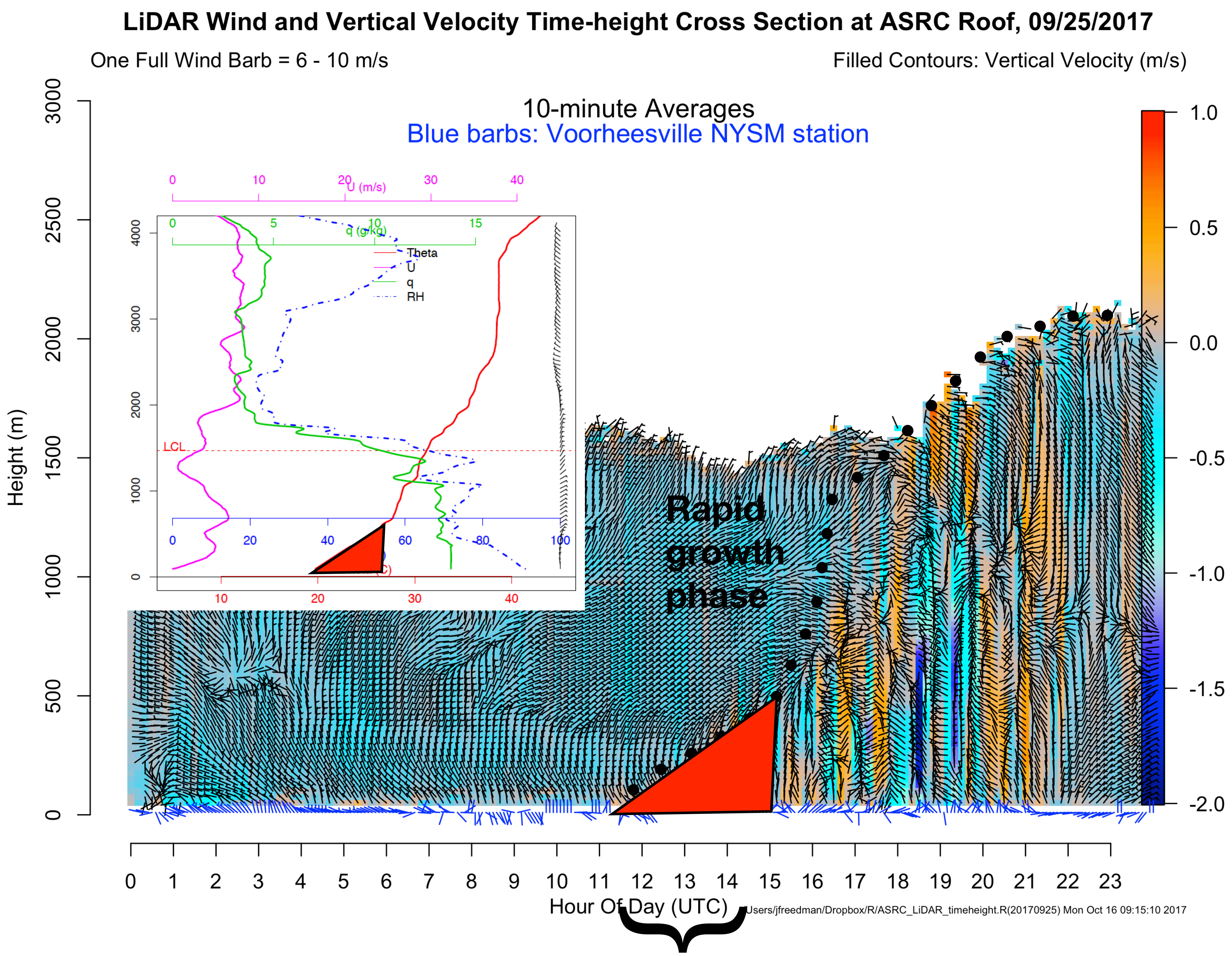


Sunrise **Noon**



Note most flux stations in grassy areas—using Schuylerville (just NE ALB) here

Can you predict when surface inversion will erode? A homework problem!

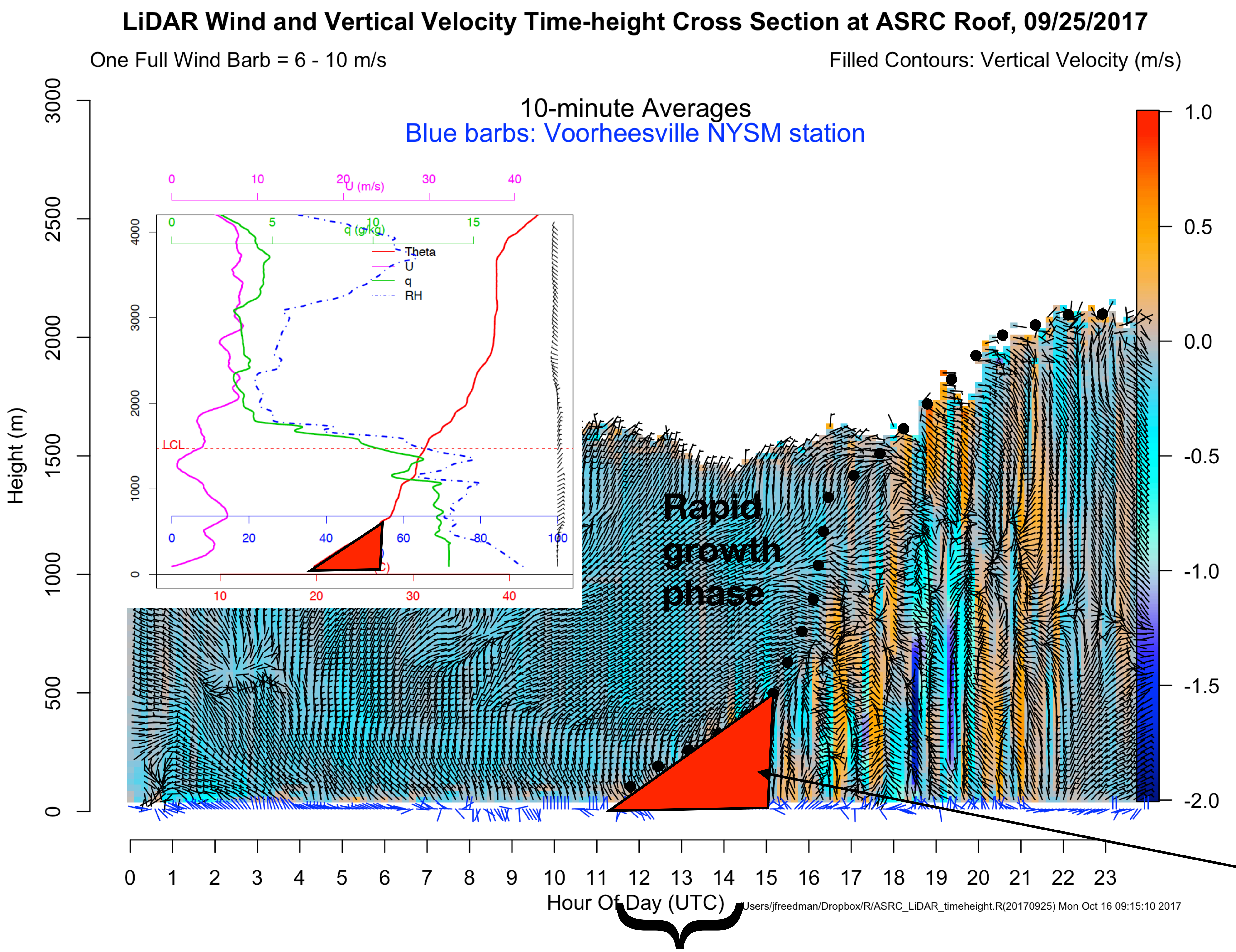


Tennekes 1973

$$t = \left[\frac{2\Upsilon h_0 \Delta_0}{(w'\theta')_n} \right]^{1/2}$$

Note: most flux stations in grassy areas—using Schuylerville here (previous slide)

Can you predict when surface inversion will erode? A homework problem!



Tennekes 1973

$$t = \left[\frac{2\Upsilon h_0 \Delta_0}{(w'\theta')_n} \right]^{1/2}$$

Assume $\mathcal{T} = 10^4$ s, h_0 = thickness of the inversion, $(w'\theta')_n$ = midday surface flux (in kinematic units) and Δ_0 = inversion strength, what is *t?*

Note: most flux stations in grassy areas—using Schuylerville here (previous slide)