ATM 505 – Module 4 – Homework

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Due: April 24

Questions 1-3 pertain to warm cloud microphysics (corresponding to temperatures > 0 ^{o}C), covering cloud droplet formation, droplet growth by condensation and collision-coalescence. Questions 4-5 pertain to cold and mixed-phase cloud microphysics (corresponding to temperatures < 0 ^{o}C)

1. **Cloud Droplet Formation**
Use the Köhler curves shown in Figure 1 to estimate:

   (a) The diameter of the droplet that will form on a sodium chloride (NaCl) particle of diameter 100 nm in air that is 0.1% supersaturated with water vapor.

   (b) The relative humidity of ambient air adjacent to a droplet of diameter 300 nm that formed on a 50 nm ammonium sulfate ($(NH_4)_2SO_4$) particle.

   (c) The critical supersaturation required for an $(NH_4)_2SO_4$ particle of dry diameter 50 nm to grow beyond the haze state.

   (d) The mass fraction (and the number of molecules) of water within the haze droplet under the conditions described in (b) above. Assume the dry particle is spherical.

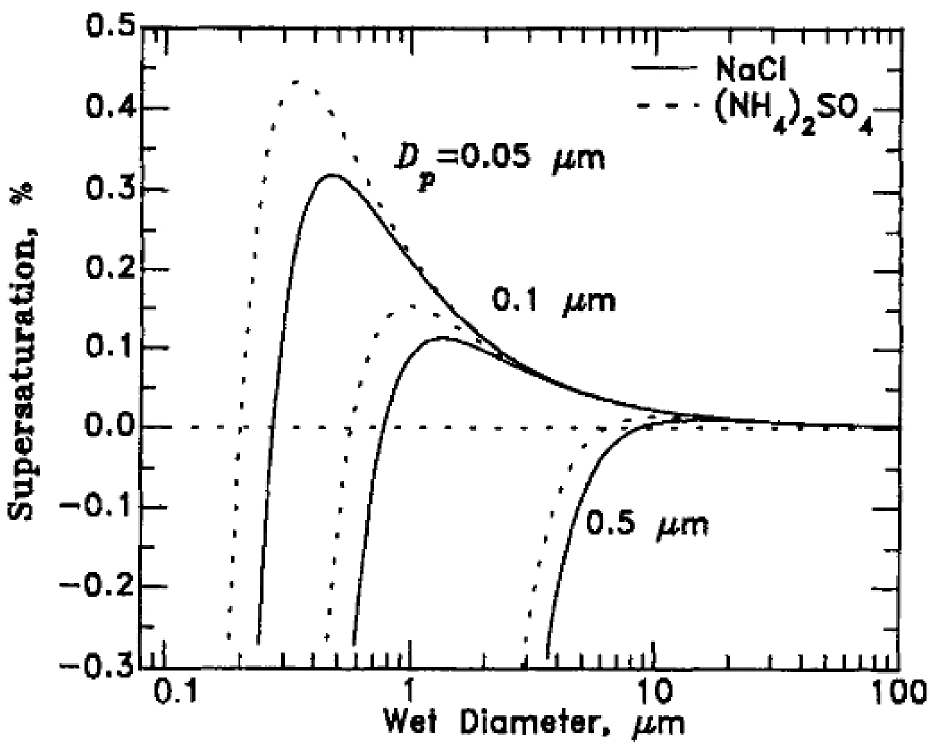


Figure 1: Köhler Curves for NaCl and $(NH_4)_2SO_4$ particles with dry diameters 0.05, 0.1, and 0.5 μm at 293 K (assuming spherical dry particles). The supersaturation is defined as the saturation ratio minus one. For example, a supersaturation of 1% corresponds to a relative humidity of 101%.

2. **Droplet Growth by Condensation**

   (a) Calculate the growth rate (dD_p/dt) of a 0.08 μm (80 nm) diameter cloud droplets from condensation of water vapor if the ambient relative humidity is 101% and the temperature is 30 ^{o}C. At this droplet size you should consider the effect of droplet curvature on growth. You may assume the droplet is pure and ignore latent heating.

   (b) How would the growth rate differ (higher or lower) if the droplet contained a dissolved salt. Explain why.

3. **Collision-Coalescence versus Condensational Growth**

   (a) Consider a large homogeneous layer of air with updraft velocity w and an ambient water vapor partial pressure of e, where $e >> e_s(T)$. Suppose at an altitude $z = 0$ that there is a steady state concentration N_d (droplets/cm^3) of small activated droplets with diameter D_p. Calculate the resulting LWC (liquid water content, g/m^3) as a function of z for $z > 0$ assuming that the droplets grow only by condensation and they have negligibly small fall velocities (i.e., derive an expression for $dLWC/dz$).

   Note that for a parcel moving upward with constant speed, w:

$$\frac{dD_p}{dt} = \frac{dz}{dt} \cdot \frac{dD_p}{dz} = w \cdot \frac{dD_p}{dz} \tag{1}$$

   Use Equation 1 and the equation for condensational growth (Eq. 8 given below for dD_p/dt), under the condition that:

$$(e_\infty - e_s(D_p)) \sim e_\infty \tag{2}$$

   Also note that:

$$LWC = N_d \cdot \left(\frac{\pi}{6} D_p^3\right) \cdot \rho_l \tag{3}$$

   (b) Consult the cloud parcel model results in Figure 2(a).How long does it take for a 60nm dry diameter particle (indicated by the thick black curve, corresponding to a 100nm diameter haze droplet at 98% RH at the start of the simulation) to activate and grow by condensation to a wet diameter of 10 μm? and 20 μm? At this rate, do you think the droplet could ever double again in size to 40 μm (a factor of 8 increase in mass) by condensation alone? How tall would the cloud have to be to sustain this rate of droplet growth for 40 min (with a constant 0.5 m/s updraft velocity)?

   (c) Consult the model output shown in Figure 2(b). Note that under some conditions droplets can be as large as 200μm after 1000s ($\sim$ 17 min) due to collision-coalescence, depending on the droplet size distribution and the cloud liquid water content. For a 0.5 m/s updraft velocity, 1 g/m^3 liquid water content, and cloud vertical thickness of at least 1200m, what maximum diameter drop do these simulations suggest could be produced at cloud base through collision-coalescenc?

Additional constants and info that you may need

H_2O(l) surface tension: $\sigma = 72.7$ dyn/cm
H_2O(l) density: $\rho_L = 1$ g/cm^3
H_2O(l) molar density: $n_L = \rho_L \ / \ M_{H2O} = 0.056$ mol/cm^3
H_2O(v) diffusivity: $D_v = 0.24$ cm^2/s
H_2O molecular weight: $M_{H2O} = 18$ g/mol
H_2O heat of vaporization: $L_v = 2.5\text{x}10^6$ J/kg
$(NH_4)_2SO_4$ density: $\rho_s = 1.77$ g/cm^3
$(NH_4)_2SO_4$ molecular weight: $M_s = 132$ g/mol
universal gas constant: $R = 8.31\text{x}10^7$ dyn cm/mol/K = 8.31 J/mol/K
specific gas constant for H_2O: $R_v = R/M_{H2O} = 461$ J/kg/K
ambient humidity = partial pressure of water vapor far from the droplet = e(∞)

units: 1 mb = 1000 dyn/cm^2 = 100 J/m^3
volume of a sphere: $(\pi/6)(2r)^3 = (4/3)\pi r^2$
mass: volume $*$ density
dry particle diameter: d_p
wet particle diameter: D_p

Clausius-Clapeyron Equation:

$$e_s(T) = 6.11\text{mb} \cdot \exp\left[\frac{L_v}{R_v}\left(\frac{1}{273K} - \frac{1}{T}\right)\right] \tag{4}$$

Kelvin Equation:

$$e_s(T, D_p) = e_s(T) \cdot \exp\left[\frac{4\sigma}{D_p R T n_L}\right] \tag{5}$$

Köhler Equation:

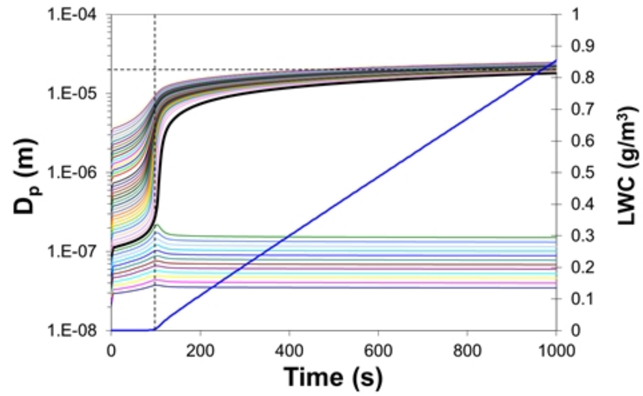
$$e_s(T, D_p, x_s) = e_s(T) \cdot \exp\left[\frac{4\sigma}{D_p R T n_L} - x_s\right] \tag{6}$$

with:

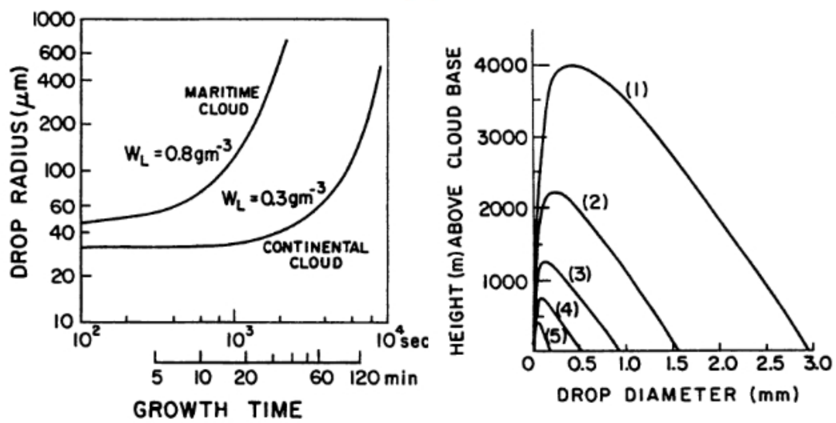
$$x_s = \frac{i \rho_s M_{H2O} d_p^3}{M_s \rho_{H2O} D_p^3} \tag{7}$$

Droplet Growth Equation (condensation only):

$$\frac{dD_p}{dt} = \frac{4D_v\left[e(\infty) - e_s(T, D_p, x_s)\right]}{D_p R T n_L} \tag{8}$$



(a)



(b)

Figure 2: (a) Growth of haze and cloud droplets inside an air parcel during its adiabatic ascent at a constant updraft velocity of 0.5 m/s, with liquid water content (LWC) plotted on the right axis (thick blue line). (b) Simulations of droplet growth rate by collision-coalescence under different conditions (left: maritime versus continental clouds with cloud top liquid water contents W_L of 0.8 and 0.3 g/m^3, respectively, and right: clouds of different updraft velocities: (1) 200, (2) 100, (3) 50, (4) 25, and (5) 10 cm/s with W_L set at 1 g/m^3), from Pruppacher and Klett, 2010. In the bottom right figure, droplets grow first during ascent, as smaller droplets (with relatively faster updraft velocity) collide with them. These relatively large "collector drops" continue growing as they fall back down through the cloud, first slowly and then rapidly as they continue growing by collision-coalescence, eventually becoming rain drops.

4. The effective densities of liquid and solid ice are $\rho_L = 1000$ kg m^{-3} and $\rho_i = 920$ kg m^{-3}, respectively.

   (a) Explain why this makes sense from a molecular point of view

   (b) What happens to the effective density of ice as it grows with an aspect ratio $\phi \neq 1$? Explain your answer physically.

5. When ice crystals grow from vapor, they tend to reach a point where the major axis (a for a plate, c for a column) continues to increase while the minor axis (c for a plate, a for a column) remains roughly constant in time.

   Remember from class that, for an oblate spheroid (plate), the mass growth equation can be described by Eq. 9 (including the influence from latent heating).

$$\frac{dm}{dt} = 4\pi a f_{ob}(\phi) G_i S_{ui} \tag{9}$$

   For a prolate spheroid (column), the mass growth equation can be described by Eq. 10.

$$\frac{dm}{dt} = 4\pi c f_{pr}(\phi) G_i S_{ui} \tag{10}$$

   (a) Derive two equations for (i) the evolution of the a-axis of a plate and (ii) the evolution of the c-axis of a column in time assuming all else remains constant. Also assume that the mass of the plate or column can be described by a spheroid: $m = \frac{4}{3}\pi a^2 c \rho_i$. Explain the physical dependencies described by the equations.

   (b) Using the derived equations, compute the final size of an ice crystal growing for 15 minutes in the following conditions: A temperature of -15°C, ice supersaturation of 15%, and an initial spherical particle radius of $r_o = 25$ μm. Assume that $\rho_i = 920$ kg m^{-3}, G_i is roughly constant at 1×10^{-8} kg m^{-1} s^{-1}, $f_{ob} = 0.6$, and $f_{pr} = 0.25$.