Hurricane Maximum Intensity: Past and Present

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ABSTRACT

Hurricane intensity forecasting has lagged far behind the forecasting of hurricane track. In an effort to improve the understanding of the hurricane intensity dilemma, several attempts have been made to compute an upper bound on the intensity of tropical cyclones. This paper investigates the strides made into determining the maximum intensity of hurricanes. Concentrating on the most recent attempts to understand the maximum intensity problem, the theories of Holland and Emanuel are reviewed with the objective of assessing their validity in real tropical cyclones. Each theory is then tested using both observations and the axisymmetric hurricane numerical models of Ooyama and Emanuel.

It is found that ambient convective instability plays a minor role in the determination of the maximum intensity and that the Emanuel model is the closest to providing a useful calculation of maximum intensity. Several shortcomings are revealed in Emanuel's theory, however, showing the need for more basic research on the axisymmetric and asymmetric dynamics of hurricanes. As an illustration of the importance of asymmetric vorticity dynamics in the determination of a hurricane's maximum intensity it is shown, using Ooyama's hurricane model, that the maximum intensity of a tropical cyclone may be diminished by convectively generated vorticity anomolies excited outside the primary eyewall. The vorticity anomolies are parameterized by adding a concentric ring of vorticity outside the primary eyewall that acts to cut off its supply of angular momentum and moist enthalpy. It is suggested that the generation of vorticity rings (or bands) outside the primary eyewall is a major reason why tropical cyclones fail to attain their maximum intensity even in an otherwise favorable environment.

The upshot of this work points to the need for obtaining a more complete understanding of asymmetric vorticity processes in hurricanes and their coupling to the boundary layer and convection.

1. Introduction

Tropical cyclones plague tropical and subtropical oceans as well as neighboring land areas around the world on an annual basis. Research into understanding these deadly vortices has been ongoing for many years. Much progress has been made into understanding some of the basic mechanisms that govern tropical cyclones, allowing for significant strides to be made in the forecasting of track. However, the associated forecasts of tropical cyclone intensity have not shown the same improvement (DeMaria and Kaplan 1999 and references). Understanding what controls hurricane intensity is vital in order to properly warn those in the path of storms, as well as in predicting the impacts of global climate change on the character of tropical cyclones (Knutson and Tuleya 1999 and references).

As a step toward understanding the tropical cyclone

intensity problem, numerous attempts have been made to compute an upper bound on tropical cyclone intensity for given atmospheric and oceanic conditions. For the purposes of the present study, the maximum intensity of a hurricane [hereafter called maximum potential intensity (MPI)] is defined as the maximum intensity (as determined by minimum surface pressure or maximum tangential winds) that a tropical cyclone may achieve for a given atmospheric and oceanic thermal structure. Generally, we assume that upper-level winds are favorable for development. Investigations into MPI have begun from different starting points leading to differing viewpoints on the structure of tropical cyclones. The primary goals of this paper are twofold. The first goal is to evaluate the current state of MPI theory and determine which proposed theory best correlates with the structure and behavior of observed tropical cyclones. The second is to suggest an answer to why most tropical cyclones fail to reach their MPI.

Kleinschmidt (1951), Miller (1958), and Malkus and Reihl (1960) conducted the first such studies aimed at determining the upper limit of tropical cyclone intensity.

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FIG. 1. Dependence of minimum sea level pressure (MSLP) on eyewall surface relative humidity (see text for details) predicted by Holland's (1997) (solid) and Emanuel's (1997) (dashed) MPI theories. Initial conditions: $T_s = 300$ K and $p_{env} = 1015$ mb.

Kleinschmidt (1951) modeled the tropical cyclone with a frictional boundary layer beneath a conditionally neutral outflow layer very similar, in many respects, to the recent work of Emanuel (1986) [see Gray (1994) for a comparison of these two approaches]. Malkus and Reihl (1960) examined parcel trajectories in the inflow layer of tropical cyclones, while Miller (1958) developed an MPI theory controlled by sea surface temperature (SST) and the height of the convective equilibrium level (EL). Camp (1999) provides a detailed review of each of these approaches.

In recent years, the most widely recognized investigations of MPI are those of Emanuel (1986, 1988a,b, 1991, 1995a, 1997) and Holland (1997). While the method used by Holland is similar to that of Miller, Emanuel takes a very different approach leading to a revised view of tropical cyclone thermodynamic structure. The MPI in both models is governed by SST and surface relative humidity. Additionally, each MPI formulation is also a function of the thermal structure of the upper troposphere, with Holland's MPI regulated by the EL, and Emanuel's MPI regulated by the average temperature of the outflow region. Figure 1 shows the MPI as a function of surface relative humidity at the eyewall calculated by Holland's and Emanuel's methods. The relative humidity in Holland's model is specified underneath the eyewall, presumably at the radius of maximum updraft, while the relative humidity in the Emanuel model is strictly valid at the radius of maximum winds. The calculations performed were made using unaltered code generously provided by K. Emanuel and G. Holland, respectively. With similar initial conditions, a sea surface temperature of 300 K, and a surface relative humidity of 86%, the two theories predict approximately the same MPI. However, as is shown by Fig. 1, this agreement is purely coincidental. While both theories are sensitive to the surface relative humidity near the eyewall radius, they are sensitive in opposite ways. The minimum sea level pressure given by Emanuel decreases with decreasing relative humidity such that a surface relative humidity of 100% at the eyewall does not support any circulation whatsoever. Conversely, the strongest storms in Holland's theory occur with 100% surface relative humidity under the eyewall, with only weak circulations (tropical depression strength) supported at 75% RH. This result is striking and suggests that the two models are fundamentally different. As we shall see, the hypothesized role of air–sea energy exchange and convective available potential energy (CAPE) lies at the heart of this difference.

The outline of this paper is as follows. We first review the basis of the MPI theories of Holland (1997) and Emanuel [1986, 1988a,b, 1989, 1991, 1995a,b, 1997, collectively referred to hereafter as Emanuel (1986-97)]. This is followed by a discussion of shortcomings present in each model, including a comparison of Emanuel's theory with the numerical models of Emanuel (1995b) and Rotunno and Emanuel (1987). The adverse effects of secondary eyewalls on tropical cyclone intensity are examined next using Ooyama's (1969) hurricane model. It is suggested that secondary eyewalls and the convectively generated vorticity anomalies that spawn them are the principle reason why most hurricanes fail to reach their MPI even under favorable environmental conditions based on SST, outflow temperature, vertical shear, etc. Finally, the main findings are summarized and paths for future research are discussed.

2. Holland (1997)

Nearly 40 years after Miller introduced his MPI theory, Holland (1997) introduced a similar, yet revised MPI theory. As with Miller (1958), Holland's theory relies heavily on the presence of ambient CAPE to determine the MPI. Both theories assume that surface air rises moist adiabatically in the eyewall before sinking dry adiabatically (with mixing from the eyewall) within the eye. The surface pressure fall due to the warming from moist-adiabatic ascent is termed the "one cell" theory. The surface pressure fall due to the warming from the adiabatic descent of parcels following moist adiabatic ascent is termed the "two cell" theory. Miller's (1958) and Holland's (1997) approaches are modifications of the one and two-cell theories. The primary change made by Holland (1997) to Miller's (1958) approach is to utilize the pressure dependence of moist entropy [proxied by equivalent potential temperature (θ_{α})], so that as convection warms the evewall and lowers the surface pressure, the boundary layer θ_{a} increases as well, allowing for a further warming in the eyewall and a further pressure drop at the surface. For most initial conditions, the pressure drops under the eyewall become smaller with each iteration, leading to a convergence of the surface pressure under the eyewall.

Once the eyewall surface pressure converges, an eye parameterization is used if the net pressure fall under the eyewall is more than 20 mb. It is argued that a system



FIG. 2. Skew *T*-Log*p* thermodynamic diagram showing Jordan (1958) mean "hurricane season" sounding (solid line), and associated Miller-type (dash) and Holland-type (dash-dot) synthesized eye soundings for $T_s = 300$ K and RH_s = 85%.

with a pressure fall less than 20 mb is unlikely to have the structure of a mature hurricane, thus there would be no eye. For such weak systems, the surface pressure calculated from the eyewall iteration is the MPI for the associated environment. A pressure drop under the eyewall of greater than 20 mb initiates the construction of an eye sounding. The dynamical processes responsible for determining the thermodynamic structure of the eye are not considered. Only the structure of the final state is evaluated. To determine this structure, the equivalent potential temperature throughout the eye is assumed equal to the surface saturation equivalent potential temperature under the eyewall once the iteration process is complete. Compared to the empirical eye of Miller (1958) (dashed line in Fig. 2), Holland's parameterization yields an eye structure that is in better agreement with observations (Hawkins and Rubsam 1968). The Holland eye, shown in Fig. 2 (dash-dot), generates a maximum temperature anomaly between 300 and 400 mb, higher than the 500-600-mb anomaly found using Miller.

Maximum potential intensity predictions using Holland's model are shown by the dotted line in Fig. 3. Analogous predictions from Miller's (1958) approach as well as the straight one- and two-cell approaches are shown for comparison. The relative humidity under the eyewall is assumed to be 90%, following Holland (1997), and the Jordan (1958) mean "hurricane season" sounding is used for the environment. Recall that the Holland model has little dependence on the relative humidity of environmental air since the air parcels are assumed to reach a relative humidity of 90% by the time they reach the eyewall. While the methods used by Holland and Miller are closely related, their predictions are dramatically different. The MPI predicted by each model is in fair agreement for surfaces temperature below



FIG. 3. Minimum sea level pressure from Miller's (1958) model (solid), and Holland's (1997) model (dot), as a function of sea surface temperature with $RH_s = 85\%$. Results from one-cell (dash) and two-cell (dash–dot) trajectories are also shown.

301 K. Above this temperature, however, the surface pressures predicted by Holland decrease rapidly, falling well below the values from even the two-cell model. Holland's surface pressures range from 960 mb at $T_s =$ 299 K to an incredibly low pressure of 770 mb at $T_s =$ 304 K. For a surface temperature of 305 K, the Holland model does not reach convergence. The surface pressure achieved with a surface temperature of 304 K is over 100 mb lower than the corresponding surface pressure from the Miller model. For temperatures of 302 K and higher, Holland predicts surface pressures below those of the strongest tropical cyclones ever observed, even though sea surface temperatures of 302 K and greater are common in tropical oceans. Since the eye parameterization does not differ substantially between the Miller and Holland models, the large difference in MPI must be a result of the eyewall iteration. This procedure has a small impact at lower surface temperatures, when the initial pressure drop is small and the procedure converges rapidly, but has an enormous impact at higher surface temperatures, when the initial pressure drop is large and the eyewall iteration converges slowly. The Miller and Holland model therefore give similar results for weaker systems, but dramatically different results for stronger systems.

The low pressures predicted by the Holland model bring to mind the hypercanes predicted by Emanuel (1988a) and Emanuel et al. (1995) (see section 3). In the present formulation, however, the extremely low surface pressures appear to be a product of raising the surface temperature while keeping the remainder of the sounding the same. This has the effect of producing a large amount of ambient CAPE, which leads to very large pressure falls under the eyewall. If a *convectively neutral* sounding with a surface temperature of 308 K, ambient surface relative humidity of 80%, ambient surface pressure of 1015 mb, and a tropopause at 100 mb is constructed, the Holland model predicts a minimum sea level pressure of only 907 mb. This predicted pressure is higher than the lowest pressures observed in the most intense tropical cyclones forming over oceans with temperatures less than 305 K. It then becomes clear how important an initially unstable environment is in producing tropical cyclones with very low central pressures in Holland's model. This is in sharp contrast to the Emanuel model, which, for low enough outflow temperatures, high enough sea surface temperatures, or a combination of both, predicts incredibly intense cyclones (hypercanes) in an *initially neutral* environment.¹

The unrealistically low surfaces pressures predicted by Holland (1997) raise concerns about the general validity of such an MPI formulation. An MPI of 800 mb is not useful in trying to assess the maximum intensity that a developing tropical cyclone may achieve. The assumptions used in the Holland model and the Miller model are now examined to assess their validity and applicability to real tropical cyclones.

The sensitivity to surface relative humidity in Holland's theory is similar to but not as dramatic as the sensitivity to surface temperature. The minimum surface pressure decreases with increasing relative humidity, as would be expected from a CAPE-based model since increased surface moisture yields a higher θ_e , thus a warmer column, and a lower surface pressure. The MPI ranges from 884 mb at 100% relative humidity to 1002 mb at 75% relative humidity for a surface temperature of 300 K (Fig. 1). The jump to lower surface pressures at relative humidities above 77% in the Holland model is due the eye parameterization being "switched on." The choice of 90% relative humidity under the eyewall in Holland (1997) is, in our opinion, an arbitrary choice for a quantity that has a large impact on the final results and for which observed values are scattered over a relatively large range of values. Holland (1997) cites relative humidities under the eyewall ranging from 80% to 95%, which corresponds to a range of MPI of 971-908 mb. Thus, the uncertainty in the eyewall relative humidity leads to estimates of MPI ranging from a moderate category 1 to a devastating category 5 hurricane on the Saffir-Simpson scale. Without a firm grasp of the moisture structure of the hurricane boundary layer, the MPI model of Holland (1997) appears to be of little use.

In order for the surface pressure to be calculated under the eyewall in Holland (1997), the moist-adiabatic profile associated with rising motion is used as the vertical temperature profile in the hydrostatic formulation. In other words, the eyewall is assumed to be vertical. In nature, to conserve angular momentum, the eyewall slopes outward with height. This has been observed in intense hurricanes such as Hurricane Gilbert (1988) (Dodge et al. 1999). Since the hydrostatic surface pressure under the eyewall is determined by the mass of the atmosphere directly above, such a vertical profile for a sloping eyewall would extend through the lower portion of the eyewall and then through the outer edge of the eye itself. Since the warmest temperature anomalies are found in the eye, the associated surface pressure under the sloped eyewall would be lower than the analogous surface pressure under a vertical eyewall. Consequently, a more realistic eyewall structure would tend to lower the minimum central pressure predicted by Holland even more.

Miller's and Holland's theories rely on a parameterized eye sounding to attain the minimum pressure. The Miller eye was arbitrarily set to match observed eye temperature and moisture profiles. As we have already indicated, however, the level of maximum warm anomaly produced with this particular method is well below the level of maximum warm anomaly observed in actual tropical cyclones. The maximum warm anomaly from Holland's parameterized eye is in better agreement with observations. But, since dynamics are not used to determine the eye sounding for either model, neither is able to resolve the characteristic temperature inversion in observed tropical cyclone eyes (Willoughby 1998). Whether or not the low-level inversion is important in determining the surface pressure is not yet clear, but the Holland eye in Fig. 2 bears little resemblance to the eye profiles from several intense tropical cyclones reported by Willoughby (1998), especially below 500 mb. A constant θ_{e} in the eye, used by Holland, is not observed in any of the eye profiles from Willoughby (1998). Without a clear understanding of the processes that control the thermodynamic structure of the eye, we think it is not insightful to apply an arbitrary temperature profile to obtain a surface pressure, since there are an infinite number of such profiles that would yield the same surface pressure. Conversely, there are an infinite number of profiles that would yield a different surface pressure. There is no dynamical basis for assuming the chosen profile is the correct one.

The Holland model has an indirect reliance on the ocean. The dependence of surface θ_{e} on surface pressure is the primary source of this reliance. Without any additional heat from the ocean, surface air flowing from the environment to the eyewall would adiabatically expand and cool as it moves to a region of lower pressure. Since Holland specifies the surface temperature to be a constant from the environment into the eyewall, sensible heat must be added to inflowing air to keep the parcels isothermal. This sensible heat is supplied by the ocean, which for practical purposes in this model can be considered an infinite heat source. The role of the ocean in supplying latent heat to the tropical cyclone is less clear. Without the addition of any water vapor, the mixing ratio of an air parcel remains the same as it travels isothermally down a pressure gradient and expands.

¹ Note added in proof: A recent paper by Schade (2000) argues that "the sensitivity of the MPI to the SST by Holland97 should be interpreted as the sensitivity of a tropical cyclone to local changes of the SST under its eye" (by ocean cooling). We suggest, however, that the sensitivity in Holland's formulation is an artifact of its over-reliance on CAPE.

Since the saturation mixing ratio increases along this same path, the relative humidity of air parcels traveling isothermally toward the cyclone center would decrease. Moisture added to the inflow from the ocean would allow the relative humidity to be maintained or increase on the inward path, while a loss of moisture would further decrease the relative humidity. Since the environmental relative humidity is not specified in the Holland model, it is, strictly speaking, unknown whether moisture is being added to or taken away from inflowing air to achieve the 90% relative humidity specified under the eyewall. Since the surface relative humidity of the ambient tropical atmosphere is generally lower than the value of 90% used by Holland (1997) one can infer that a latent heat transfer from the ocean to the atmosphere must take place in the Holland model. However, since the eyewall relative humidity is arbitrarily specified, the air-sea interaction may not be properly accounted for.

Finally, the lack of a dependence on surface friction in the Holland theory is troubling. Implicitly, surface friction is assumed to force the boundary layer radial inflow, the convergence of which produces the updraft at the base of the eyewall. However, the negative impact of surface friction in spinning down the vortex does not appear to be part of the formulation. As wind speeds increase, the effect of friction increases, such that its neglect in any theory of tropical cyclone structure and maintenance seems suspect.

3. Emanuel (1986–97)

Emanuel (1986, hereafter E86) introduced a theory of tropical cyclone structure and development that is fundamentally different from Holland's (1997) theory. There are two aspects of CAPE-dependent MPI theories that E86 takes issue with. The main disagreement concerns the role of CAPE in tropical cyclone development and maintenance. While the CAPE-dependent MPI theories rely heavily on the presence of ambient CAPE in the tropical atmosphere, E86 asserts that, even though CAPE does exist in the tropical atmosphere, it is questionable that tropical convection is able to utilize this source of energy due to entrainment, mixing, and other small-scale processes. In addition to the overemphasis on ambient CAPE in previous MPI theories, E86 argues that the energy exchange between the boundary layer and the ocean is much stronger than suggested by theories relying on ambient CAPE for energy production. This enhanced air-sea interaction has been recently termed wind-induced surface heat exchange (WISHE). Relationships governing the maximum tangential winds and minimum sea level pressure, based on the WISHE mechanism, are derived in two ways. The popularized derivation assumes that the tropical cyclone behaves as a Carnot heat engine (Emanuel 1986, 1988a,b, 1991, 1997). This is, however, Emanuel's secondary method of deriving the MPI. The primary method is based on a balance between frictional dissipation and energy production in the inflowing boundary layer. The latter formulation is the focus of the present paper.

To investigate the possibility of maintaining a hurricane-like vortex without a conditionally unstable environment, E86 developed a steady-state model of a tropical cyclone in a conditionally neutral environment. The model vortex is assumed axisymmetric, in hydrostatic and gradient balance, and reversible thermodynamics are assumed. The key feature of the model is that the CAPE is zero everywhere in the domain, such that the atmosphere is neutral to slantwise convection. The impact of the neutral constraint is that boundary layer air is neutrally buoyant when lifted along surfaces of constant angular momentum. Using the boundary layer closure from Ooyama (1969), the steady-state nondimensional entropy balance under the tropical cyclone eyewall is given by Emanuel (1995a, henceforth E95a) as

$$\psi_0 \frac{\partial \chi}{\partial r^2} = -\frac{C_k}{C_D} (1 + c|V|) |V| (\chi_s^* - \chi), \qquad (1)$$

where ψ_0 is the radial-mass streamfunction at the top of the boundary layer and V the tangential speed. Entropy is represented as a temperature-weighted deviation from an environmental value,

$$\chi \equiv (T_s - T_o)(s - s_{\rm ba}), \qquad (2)$$

where T_s is the surface temperature, T_o is the outflow temperature, s the moist entropy of the subcloud layer, and $s_{\rm ha}$ is the moist entropy of the ambient subcloud layer (Emanuel 1995b, henceforth E95b). The coefficients C_k and C_D are the sea-air exchange coefficients for moist entropy and angular momentum, respectively, and the empirical constant c determines the wind dependence of the surface fluxes. The left-hand side of (1) represents the radial advection of entropy, which is exactly balanced by the flux of entropy from the sea surface represented on the right-hand side. Note that the flux of entropy from the ocean is controlled by the amount of disequilibrium between the boundary layer and the sea surface. That is, a dry boundary layer allows for a greater entropy flux than a moist boundary layer, with zero entropy flux occurring if the boundary layer is saturated.

The nondimensional angular momentum balance under the eyewall is similarly given by

$$\psi_0 \frac{\partial R^2}{\partial r^2} = 2(1 + c|V|)|V|rV, \qquad (3)$$

where *R*, the potential radius, is analogous to angular momentum. This equation states that the radial advection of angular momentum is balanced by the flux of angular momentum into the ocean due to friction. In the previous two equations, the surface exchange has been parameterized using the standard bulk aerodynamic formulation; the justification for and a discussion of the attending uncertainties in such an approach are provided by E95a,b. Using thermal wind balance and the definition of moist entropy, Eqs. (1) and (3) can be combined to form a transcendental equation relating the tangential velocity and the moist entropy at the radius of maximum wind (RMW),

$$V^{2}\left(1 - \frac{1}{2}A\frac{C_{k}}{C_{D}}\right) \approx \frac{C_{k}}{C_{D}}\left[1 - \frac{1}{4}Ar_{0}^{2} - \chi(1 - A)\right], \quad (4)$$

where r_0 is the radius at which the winds decay to zero. In Eq. (4),

$$A \equiv \frac{T_s - T_o}{T_s} + \frac{\chi_s}{R_d T_s (1 - \mathrm{RH}_a)},$$
 (5)

where R_d is the gas constant for dry air and RH_a is the ambient relative humidity. Equation (4) is the MPI equation for a tropical cyclone that has developed in a conditionally neutral environment.² It can be solved for Vand χ at the RMW. Emanuel then assumes an "eye closure" in which the tangential flow inside the RMW is in solid-body rotation. This allows for the calculation of the minimum central pressure once the maximum tangential winds are known. We note, following E95a, that the maximum tangential winds are not dependent on the structure of the eye itself. The eye closure used in E95a is actually a revised version of that used in E86 to determine the minimum surface pressure. The original formulation resulted in a hydrostatic calculation somewhat similar to Holland (1997) and Miller (1958), which was independent of the ratio of surface exchange coefficients. The revised formulation now depends on the ratio of surface exchange coefficients, in better agreement with axisymmetric model results (E95a). Figure 4 presents MPI calculations for various sea surface temperatures and an environmental relative humidity of 85% using Emanuel's (E95a) formulation. Results using Holland's (1997) method are shown for comparison. The predicted minimum pressures from the Emanuel method are more reasonable than those from the Holland method. A minimum pressure of just below 900 mb at a sea surface temperature of 305 K is in good agreement with the minimum pressures observed in intense tropical cyclones.

Hypercanes

A unique feature of the transcendental equation given by (4) arises for high values of surface temperature, low values of outflow temperature, or a combination of both. A point is reached, at which no steady-state solution to the equations is possible for the given environmental



FIG. 4. Minimum sea level pressure from Emanuel's model (solid) as a function of sea surface temperature with $RH_{env} = 85\%$ and $p_{env} = 1015$ mb. Results from Holland's model are shown (dashed) for comparison.

parameters. During intensification, pressure falls lead to an increase in the surface mixing ratio, thereby increasing the moist entropy and allowing further intensification. Under normal conditions, the pressure falls lead to smaller and smaller increases in the mixing ratio, allowing the solution to converge to a steady state. Under extreme conditions, however, the change in mixing ratio increases more than is necessary to maintain the pressure falls, leading to runaway intensification. Emanuel (1988a, henceforth E88a) hypothesizes that storms forming under such conditions never reach an equilibrium intensity, with central pressures that spiral toward zero. Such "hypercane" solutions to the MPI equations occur for the combination of low outflow temperatures and high sea surface temperatures, conditions that do not exist under current climate conditions. For example (from E88a), conditions leading to the hypercane regime would require $T_s = 36^{\circ}$ C, $T_o = -110^{\circ}$ C, and surface relative humidity RH_a = 80%. Wind speeds in such storms could approach speeds of nearly 300 m s⁻¹ before internal dissipation limits the intensification (Emanuel et al. 1995). Emanuel et al. (1995) have suggested that the mass extinction at the end of the Cretaceous period may have been, in part, due to the presence of hypercanes triggered by an asteroid/meteor impact or extreme volcanic activity.

Limits to such hypercanes are hypothesized to exist by E88a, but such limits are not realized by the MPI equations due to the assumptions that underly their derivation. E88a hypothesizes three conditions that may limit the intensity of tropical cyclones in the hypercane regime. First, a nonisothermal boundary layer would limit the amount of heat available to the secondary circulation, limiting the intensity. In fact, observations of nonhypercane regime tropical cyclones under current atmospheric conditions indicate that the boundary layer is not exactly isothermal (Cione et al. 2000; Shay et al. 1992). The fact that nonisothermal boundary layers exist indicate that surface fluxes of sensible heat from the

² A revision to this formulation has been proposed by Bister and Emmanuel (1998) incorporating the effect of dissipative heating in the boundary layer. The only impact of this revision is to change T_s to T_o in the denominator of the first term in Eq. (5), leading to somewhat stronger storms. This modification has been used in all of the Emanuel MPI calculations presented here.



FIG. 5. Time evolution of maximum tangential wind predicted by the tropical cyclone model of E95b. Initial conditions: $T_s = 307$ K and $p_{env} = 1015$ mb, $T_o = 171$ K.

ocean are unable to fully keep up with adiabatic cooling even for tropical cyclones of present-day intensity, much less for tropical cyclones with tangential velocities of 300 m s⁻¹ or more. A second limit may occur due to strong internal dissipation (in the eyewall), which becomes important at high wind speeds, and therefore the assumptions that all dissipation occurs only in the boundary layer and at large radii in the outflow break down. Finally, the calculation of an outflow temperature from the environmental temperature profile may not be valid in the case of the hypercane, where outflow is so strong (supercritical or nearly so) that the hurricane vortex itself controls the outflow temperature. None of the previous arguments are included in the derivation of the MPI equations; thus, they may conceivably place strong constraints upon the maximum intensity of hypercanes, or prohibit them altogether.

Emanuel (1989, 1995b) developed a simple axisymmetric numerical tropical cyclone model based on the assumption of neutrality to slantwise moist convection. The model was obtained for the present study from K. Emanuel and several experiments using it have been conducted. Details of the model formulation can be found in Emanuel (1989, 1995b). For the present discussion, Emanuel's (1995a) numerical model was run for a particular case where the MPI equations do not converge to a solution. The input parameters are the sea surface temperature ($T_s = 307$ K) and the outflow temperature ($T_o = 171$ K). These values were chosen by examining Figs. 3 and 4 of E88 and choosing values that were well into the hypercane regime. All other model parameters were kept at the default values. The time evolution of the maximum tangential wind at the top of the boundary layer predicted by Emanuel's numerical model is shown in Fig. 5. The maximum tangential velocity reaches 283 m s⁻¹ at approximately t = 20 days. For the given surface temperature, this is about 80% of the speed of sound (\sim 351 m s⁻¹). The vortex appears to intensify in stages on its way to its peak intensity. After reaching the maximum intensity of 283 m s⁻¹, the vortex weakens rapidly with tangential winds dropping below 100 m s⁻¹ followed by a period of slower weakening. This rapid weakening is attributed, by Emanuel et al. (1995), to vortex breakdown resulting from internal dissipation, although the physics triggering the breakdown and the subsequent evolution is not yet well understood.

4. Limitations of Emanuel's MPI

There are two potential caveats in Emanuel's MPI theory, the assumption of constant relative humidity from the environment to the eyewall, and the assumption of neutrality to slantwise convection in the eyewall. The moist neutrality assumption is only strictly valid in the ambient environment and in the eyewall when the tropical cyclone is in a steady state. While the cyclone is intensifying, the moist neutral assumption is not exactly satisfied, since as θ_e increases at the surface, small amounts of CAPE are generated, leading to some positive buoyancy in the eyewall. In the axisymmetric theory, some buoyancy is necessary to warm the vertical column, as parcels moving neutrally through the eyewall would not produce any warming. Thus, Emanuel is not suggesting that the tropical cyclone is entirely absent of convective instability, just that any buoyancy that develops during the intensification process is quickly extinguished as the eyewall warms.

The steady-state assumption of neutrality to slantwise convection also deserves some discussion. Such an assumption implies that air moving along surfaces of constant angular momentum would not experience any upward acceleration. In fact, as air parcels move up through the eyewall, and outward into the outflow, their vertical velocity should decrease owing to the smaller and smaller vertical component of motion. Additionally, the radial velocity would be outward throughout the outflow layer. Without any inward component of velocity in the midlevels, there can be no inward advection of high angular momentum air. This raises the question of how the steady-state vortex maintains itself against frictional spindown without midlevel inflow. That a vortex can be maintained in a steady state without significant midlevel inflow above the boundary layer is supported by the hydrostatic model results of Emanuel (1989, 1995a). Figure 6 shows the r-z distribution of the tangential velocity field for a steady-state vortex calculated from the Emanuel's (1995a) hydrostatic model with maximum tangential winds of 76 m s⁻¹. Figure 7 shows the corresponding fields of radial velocity (panel a) and vertical velocity (panel b). Figure 8 shows the two-dimensional vector field of the transverse circulation corresponding to Fig. 7. Figure 7a clearly shows that all of the inflow is confined to the lowest 1.75 km of the model atmosphere, with outflow dominant in the middle and upper troposphere. Inside a radius of 50 km, no inflow occurs above 1.75 km. It appears then that a



FIG. 6. Steady-state two-dimensional tangential velocity field (m s⁻¹; contour interval, 5 m s⁻¹) from hydrostatic model of Emanuel (1995a).

sufficient amount of angular momentum is being advected inward near the top of the boundary layer to maintain the tropical cyclone against friction. Such a structure with highly concentrated inflow is not necessarily supported by observations. Analysis of the secondary circulation in extremely intense Hurricane Gilbert (1988) by Dodge et al. (1999) shows an inflow layer extending from the surface to 8 km near the eyewall.

A relatively deep inflow layer in the steady state is, however, predicted by the nonhydrostatic cloud model of Rotunno and Emanuel (1987, hereafter RE87). The steady-state fields from this model indicate an accelerating updraft in the lowest 10 km of the eyewall. This structure cannot be explained by neutrality to slantwise convection. Since weak midlevel inflow is present in the steady state, this suggests that entrainment is oc-



FIG. 8. Vector field of transverse circulation from hydrostatic model of Emanuel (1995a). Note that vectors are stretched in the vertical direction so that the vertical component of the vectors is exaggerated.

curring within the updraft, which is expected with an accelerating updraft. It appears then that some buoyancy in the eyewall is present in the steady state. While it is true that the isolines of angular momentum and moist entropy are nearly congruent in the core of the model tropical cyclone (Fig. 9 of RE87), only small deviations from this congruency would be enough to produce a buoyant updraft. The presence of a small amount of buoyancy in the steady-state nonhydrostatic tropical cyclone would indicate that the model of a slantwise neutral hurricane is not entirely correct. That the maintenance of the steady-state vortex is dependent on the presence of convective instability in the eyewall has been hypothesized recently by W. M. Gray (1997, unpublished manuscript). Gray argues that the intensity of a tropical cyclone is limited by a balance between friction and the maximum updraft generated by buoyancy



FIG. 7. Steady-state two-dimensional fields of (a) radial velocity (m s⁻¹; contour interval, 5 m s⁻¹) and (b) vertical velocity (m s⁻¹; contour interval, 2 m s⁻¹), from hydrostatic model of Emanuel (1995a).



FIG. 9. Radial profiles from Ooyama's (1969) numerical model: (a) tangential velocity (m s⁻¹), (b) relative vorticity (s⁻¹), (c) vertical velocity (m s⁻¹), and (d) radial velocity (m s⁻¹) without vorticity bump added at t = 100 h.

and frictional convergence. However, since the steadystate results of the Emanuel's time-dependent models (1987, 1995b) agree closely with the MPI predicted from the moist neutral theory and the results from RE87, one may argue that a large majority of the important dynamics needed to determine the MPI are captured in the simple theory.

The assumption of constant boundary layer relative humidity from the environment to the eyewall is also of some concern. E86 cites the composite analysis of Frank (1977) to support the claim that the relative humidity in the boundary layer is relatively constant with radius outside of the eyewall of tropical cyclones and that its value is approximately 80%. Frank (1977), however, does not suggest this is the case. The composite soundings from Frank (1977) show a relative humidity of 95% at the innermost compositing radius (78 km), decreasing to 90% at 222 km and 85% at 444 km. This suggests that the relative humidity in a tropical cyclone is far from constant with radius, with boundary layer values in the environment typically above 80%. In fact, the composite surface relative humidity at a radius exceeding 1300 km is 82%, still above the value assumed by Emanuel. Recent composite analysis of buoy and Coastal-Marine Automated Network data (Cione et al. 2000) similarly shows that relative humidity is not constant with radius. This study reveals relative humidities greater than 92% within 110 km of the center, with a maximum just over 96% at 56 km, dropping to roughly 84% outside of 200 km. It is clear then that the boundary layer approximation implemented by Emanuel is not ideal.

An illuminating situation develops, however, when the Emanuel boundary layer of constant surface temperature is compared with an observed hurricane boundary layer that is nonisothermal and the relative humidity increases inward. Consider Hurricane Mitch (1998). We can take the observed surface temperature of the environment from Mitch of 301 K and calculate the equivalent potential temperature for the observed pressures underneath the eyewall [\sim 969 mb from U.S. Air Force reconnaissance (Data obtained from National Hurricane Center's FTP site)] using an assumed value of relative humidity of 80% (following Emanuel). The resulting value of θ_e is 364 K. Calculating the equivalent potential temperature directly from the observed surface temperature, pressure, and relative humidity (298.75 K, 969 mb, 97%, respectively) from dropsondes deployed in the eyewall of Hurricane Mitch yields a value of θ_{a} averaging 364 K, the same as the θ_e calculated with the Emanuel boundary layer. This suggests that the observed boundary layer cooling and high relative humidity at the eyewall may tend to offset each other in terms of θ_{a} . Since it is the entropy deficit (proportional to θ_e deficit) that fuels the storm in Emanuel's model, the assumption of constant relative humidity and surface temperature in the boundary layer allows the entropy structure predicted by Emanuel to be roughly similar to that of observed tropical cyclones. In other words, the Emanuel boundary layer seems to be robust to realistic variations in relative humidity and surface temperature, at least for current conditions.

5. Internal limits to MPI: Convectively forced vortex Rossby waves and secondary eyewalls

Since Emanuel uses the basic assumption of axisymmetry, the effects of asymmetric processes cannot be simulated explicitly but must be parameterized. While the axisymmetry assumption captures many of the important processes that govern the evolution of tropical cyclones, it is nevertheless possible that asymmetric processes could, under certain conditions, limit the intensity of the vortex relative to its axisymmetric MPI. Results from Project STORMFURY have suggested the important role played by concentric eyewalls on hurricane intensity (Willoughby et al. 1985). The importance of the radial location of convection on hurricane intensity has also been investigated [e.g., Rosenthal and Moss (1971); see Anthes (1982) for a detailed review up to 1980 and more recently by Bister (2001), as well as Guinn and Schubert (1993) and Schubert and Hack (1982) who interpret the negative effect of outer core convection based on inertial stability]. Montgomery and Kallenbach (1997), Montgomery and Enagonio (1998), and Moller and Mont-

gomery (1999) showed that convectively generated vortex Rossby waves excited near or within the vortex RMW will act to spin up the vortex. If convectively generated vortex Rossby waves are excited too far from the RMW, however, then the outer tangential winds will be strengthened rather that the eyewall winds (see Table 3 of Montgomery and Enagonio 1998). The latter process, when coupled to the boundary layer and convection, may explain the formation of secondary eyewalls (Montgomery and Kallenbach 1997). Although further observations and full-physics modeling studies are needed to obtain a complete understanding of the physics responsible for their formation, we shall nevertheless suggest that the presence of outer vorticity bands or secondary eyewalls represent an important intensity limiting mechanism. The formation of an outer vorticity band or secondary eyewall will be parameterized here by impulsively adding a finite-amplitude vorticity ring (or "bump") to the vorticity field of an axisymmetric tropical cyclone model during the development phase of the storm.

We have chosen Ooyama's (1969) tropical cyclone model to examine the possible effects of outer eyewall formation on maximum intensity. Ooyama's (1969) model is a simple, axisymmetric, three-layer representation of the hurricane vortex with parameterized convection. The control run of the model begins with a 10 m s⁻¹ initial vortex possessing a 50-km RMW. The initial vortex intensifies, reaching hurricane intensity by 70 h and a maximum intensity of 64 m s⁻¹ at 130 h. After reaching peak intensity, the vortex spins down slowly due to radial diffusion and a boundary layer formulation that assumes the tangential wind is in gradient balance with the geopotential. During the evolution, the radius of maximum winds contracts to a radius of 40 km before expanding during the slow spindown phase. No secondary wind maximum develops during the control run. Although a true steady state is never attained, the model results prior to the weakening phase are believed to be representative of what a more accurate hurricane model would predict.

Figure 9 shows the radial profile of tangential velocity (Fig. 9a), relative vorticity (Fig. 9b), vertical velocity (Fig. 9c), and radial velocity (Fig. 9d) at t = 100 h of the control run. At this time, the vortex is intensifying rapidly. Each of the fields shown contains one distinctive peak near the RMW. To simulate the impact of outer vorticity bands or secondary eyewalls on intensification, a Gaussian bump is added to the vorticity profile at t = 100 h. The maximum vorticity of the bump is 10^{-3} s⁻¹ and has an *e*-folding radius from the center of the anomaly of 6 km. Such a bump would be representative of a symmetrical outer tangential wind maximum in a tropical cyclone. The character of the bump is shown in Fig. 10, which is identical to Fig. 9 except with the addition of the bump to the relative vorticity profile. In this particular experiment, the bump is added at a radius of 80 km. [Since the model is a balance model, the



FIG. 10. As in Fig. 9 except with vorticity bump added at t = 100 h.

associated mass field (geopotential) and meridional circulation is obtained directly upon inversion.] Adding the bump to the vorticity fields results in secondary maxima in the tangential, vertical, and radial velocity fields. The model is then integrated forward in time. The impact of the bump on the model evolution is quite dramatic, as summarized by Fig. 11 at t = 110 h. While the secondary maxima in the tangential wind has been suppressed, the intensification at the original RMW has been halted, and the maximum tangential winds have actually decreased to below 40 m s⁻¹. Also noteworthy is the fact that the secondary "eyewall" has become dominant in the vertical velocity and radial velocity fields. In effect, the outer vorticity maximum appears to have robbed the inner wind maximum of its inflow, leading to a spindown of the interior vortex. This interpretation is essentially the same as the STORMFURY hypothesis (see Anthes 1982, chapter 5) but intrinsic convective/vorticity dynamics are being called upon here for initiating the secondary eyewall.

The amplitude and radial position of the vorticity bump also plays an important role in the evolution of the vortex in Ooyama's model. Figure 12 shows the time evolution of the layer 1 maximum tangential winds for the control run as well as for several different radial placements of the vorticity bump. In each case, the intensity of the vortex decreases immediately following the insertion of the vorticity bump. The impact is found to be more substantial as the radius of insertion moves outward. When the bump is inserted at r = 80 km or



FIG. 11. As in Fig. 9 except at t = 110 h (10 h after vorticity bump introduced).

r = 100 km, the maximum intensity of the vortex is reached at the time of insertion. For the case with the bump inserted at r = 60 km, the vortex reaches maximum intensity approximately 30 h later. These experiments suggest that the internal dynamics of convectively generated vortex Rossby waves, their coupling to the boundary layer, and formation of vorticity bands or secondary eyewalls may play a crucial role in limiting the intensity of hurricanes, possibly providing the ultimate factor in prohibiting the formation of hypercanes. Further analysis along similar lines with more comprehensive axisymmetric and three-dimensional hurricane models with and without ocean coupling is needed for a more complete understanding of this limiting mechanism.

6. Conclusions

Over the years there have been numerous efforts to establish a useful theoretical limit on the maximum intensity of tropical cyclones. Such a limit would allow for strides to be made in the difficult task of intensity forecasting. With a perfect theory for MPI, the observed maximum intensities of tropical cyclones would reach the theoretical limit. That this is not the case, however, suggests that there are great strides yet to be made.

The theories of Miller (1958) and Holland (1997) appear to be the weakest of the MPI formulations. The heavy reliance on CAPE in the ambient atmosphere appears to be their main weakness. Modeling results using Ooyama's (1969) model, as well as the models of Ro-



FIG. 12. Evolution of maximum tangential winds in Ooyama's (1969) model with no vorticity bump (solid), vorticity bump at r = 60 km (dash-dot), vorticity bump at r = 80 km (dash), and vorticity bump at r = 100 km (dot).

tunno and Emanuel (1987), and Emanuel (1989, 1995a) suggest that ambient convective instability plays a minor role in the final intensity of the tropical cyclone. Thus, the physical basis for determining MPI used by Holland (1997) and Miller (1958) seems not well founded. Additionally, each of the latter formulations relies on the parameterization of the thermodynamic profile in the eye. Observations of the thermodynamic profiles through the full depth of tropical cyclone eyes are rare, so the actual thermodynamic structure is not yet well known. With such little knowledge of the structure of the tropical cyclone eye, any attempt to parameterize the thermodynamics that would correspond to the strongest tropical cyclones is quite arbitrary. The combination of the erroneous dependence upon ambient convective instability and the arbitrary eye sounding leads to the unrealistic predictions of MPI from Holland (1997) for sea surfaces temperatures greater than 302 K.

Modeling results showing the minor role of ambient convective instability in the tropical cyclone development indicate that MPI formulations that do not rely on such instability may be closer to reaching the desired result. The convectively neutral formulation proposed by Emanuel (1986) appears to generate a useful upper bound on tropical cyclone intensity for the range of sea surface temperatures examined. Additionally, although Emanuel's boundary layer parameterization in the outer regions of tropical cyclones is highly simplified, the predicted entropy structure of the boundary layer is roughly similar to the observed boundary layer entropy structure of observed tropical cyclones.

There are, however, some basic aspects of the tropical cyclone structure predicted by Emanuel's theories that are somewhat suspect. All of the inflow in the theoretical model is assumed to occur within or just above the boundary layer. This implies that the angular momentum imported in this relatively thin layer is sufficient to combat the detrimental effects of friction and maintain a steady state. Nonhydrostatic cloud model simulations (RE87) and observations of intense tropical cyclones (Frank 1977; Dodge et al. 1999), however, suggest that moderate inflow occurs through a relatively deep layer extending well above the boundary layer and that "near boundary layer" inflow, alone, may not be sufficient to maintain a tropical cyclone against friction. The deep inflow observed and modeled in a cloud resolving model indicates that the assumption of neutrality to slantwise convection breaks down to some extent in the steady state. But considering that the predicted MPI correlates well with the RE87 nonhydrostatic results, it would appear that the breakdown of the slantwise neutrality assumption is not entirely detrimental.

The WISHE model developed by Emanuel (1986–97) contains a unique instability that produces extraordinarily intense tropical cyclones with tangential winds approaching 300 m s⁻¹, corresponding to a Mach number of 0.86. This occurs when the rise in the surface mixing ratio as the pressure lowers under the eyewall is more than is needed to maintain the eyewall surface pressure, leading to runaway intensification. Such hypercanes are, in fact, realized in the axisymmetric cloud model of RE87 starting from an ambient atmosphere that is neutral to slantwise convection. The Holland model, however, does not appear to produce the same type of runaway vortex for a convectively neutral environment. Extremely low surface pressures in this model seem to be realized only with very high surface temperatures in a high-CAPE environment. This difference between the two theories indicates a fundamental difference in the underlying physics that maintains tropical cyclones. It is unclear, however, whether or not such hypercanes predicted by Emanuel's model can materialize under more realistic three-dimensional scenarios. Using Ooyama's (1969) tropical cyclone model, we have shown that vorticity bands or secondary eyewalls excited in an intensifying tropical cyclone may provide a powerful constraint on the actual vortex intensity and may ultimately prohibit the formation of hypercanes, despite favorable environmental conditions. While these results do not include the effects of vertical wind shear explicitly, the effect of wind shear is often to excite convection away from the center. Thus, we believe these results are insightful even though they are carried out in an axisymmetric framework. Further analysis of this issue awaits future work.

Despite the best attempts to formulate a limit on tropical cyclone intensity, current efforts still fall short of providing an effective prediction of MPI that can be used with confidence in an operational setting. While the WISHE theory of Emanuel appears to predict a storm structure that is close to reality (Emanuel 1999), there are potentially significant shortcomings that need to be examined more critically. There are numerous parameters that have yet to be accurately accounted for in the reviewed theories that should increase their usefulness. A better account of the role of asymmetric vortex dynamics in the intensification and maintenance of tropical cyclones is needed. The use of upper-ocean heat content and a better understanding of the role of ocean cooling may aid in producing an MPI formulation that is more representative of the intensities which storms actually achieve (Shay et al. 1992). In addition, observations of the actual boundary layer structure of tropical cyclones through the use of Global Positioning System dropsondes promise to add important information about the interaction between the ocean and the storm circulation and help quantify how much heat exchange is realized between the underlying ocean and atmosphere.

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