CORRESPONDENCE

Meridional circulations and the transfer of angular momentum in the atmosphere

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In a recent letter published in this JOURNAL, C.-G. Rossby and Victor P. Starr[1] discuss the maintenance of the zonal circulation of the earth's atmosphere. They show that under a steady state, the conditions for the constancy of the total angular momentum contained in an annular zonal strip reduce to

$$\int r\rho u c_n \, dS + \int r^2\omega\rho c_n \, dS = \int T \, dV. \qquad (1)$$

Furthermore, if the contribution of the second integral is not zero, there must exist *mean meridional circulations*. The authors then agree that it is the importance of this term in (1) which they call into question.

For convenience, extend the annular zonal strip northward to the pole and consider the flux of angular momentum into this layer across the southern boundary. The normal velocity c_n becomes v, the northward component of velocity, which may be considered as the sum of a geostrophic component v_g and a nongeostrophic component Δv. The transport of angular momentum northward across this boundary is then

$$\int r\rho u v_g \, dx + \int r\rho u \, \Delta v \, dx + \int r^2\omega\rho \, \Delta v \, dx, \qquad (2)$$

the geostrophic contribution to the ω-integral vanishing. Furthermore, u can be considered as geostrophic in these calculations.

Computations of the first of these three integrals (which depends upon a correlation between u and v_g) have been made by Widger.[2] The maximum value found by him was about 190×10^{30} cgs units per month for the layer 4.5–7.5 km at 35°N. It is interesting to compute the value of Δv necessary to give the third integral in (2) the same value. A simple computation shows that the third integral has a value of about $31 \times 10^{30} \times \Delta v$ cgs units per month for the same layer. A mean value for Δv of only 6 cm sec^{-1} would thus transport as much absolute angular momentum across the zonal surface as was accomplished by the correlation between the components of the geostrophic wind.

If the annular zonal strip is extended vertically to include all of the atmosphere north of a given latitude, the contribution of the second integral in (1) must practically vanish if there is to be no net transfer of mass across the latitude. However, mean meridional circulations can still give a net transport of angular momentum across the latitude if the mean Δv is positively correlated "in the vertical" with the mean west wind. This effect has been treated by the writer in a paper to be published by the Finnish Society of Science. A mean Δv of +50 cm sec^{-1} in the upper layers of the troposphere at 40°N and a mean of −50 cm sec^{-1} in the lower layers will, when combined with the normal increase of u with height, transport as much angular momentum northward across 40°N as is lost to the earth by skin friction north of that latitude.

If we could momentarily stop the relative rotation of the atmosphere it is obvious that only the second integral in (1) could produce a transfer of angular momentum when the circulation begins again because of the pressure gradient associated with the average meridional temperature gradient. However, this process would also produce a redistribution of mass, so that $\int \rho c_n \, dS$ would not vanish. The strongest influx of momentum per unit mass would then occur at the level where the isobaric surfaces are most inclined, *i.e.*, at the tropopause. The result would be the development of a new quasistationary zonal motion characterized by strong vertical wind shear in the troposphere. If the integration is extended to the upper and lower limits of the atmosphere the contribution of the first term in (1) would then gradually increase and the contribution of the second term would decrease with time, while for the ultimate steady state the second integral would vanish. In the lower troposphere (excluding the surface frictional layer) there would be a southward flux of momentum in the zone of westerlies, in the upper troposphere a northward flux, and to this process both terms in (1) could contribute. However, *to maintain the wind distribution necessary for the first integral in (1) to operate, it is essential that the second integral be different from zero if computed for layers of limited thickness.*

Therefore the conclusion made by Rossby and Starr that it is the first term in (1) which is of prime importance in the mechanics of the general circulation cannot, in the writer's opinion, withstand a critical analysis of the whole problem. The term does not involve the rotation of the earth and thus the theory for momentum transfer would be the same on a nonrotating earth, which seems paradoxical. The solution of the dilemma lies in the explanation given above: *The distribution of westerly relative momentum which can be used for the evaluation of the first integral in (1) is ultimately the result of the second integral.*

It is not clear whether the authors intended to question the necessity of meridional circulations for the maintenance of the *kinetic energy* of the atmosphere. Such an intention would mean a complete change of the whole foundation of dynamic meteorology, and I doubt strongly that the genius of the authors, recognized by all meteorologists, will be sufficient for that goal.

[1] C.-G. Rossby and Victor P. Starr, "Interpretations of the angular-momentum principle as applied to the general circulation of the atmosphere," *J. Meteor.*, 6, 288, 1949.

[2] William K. Widger, Jr., "A study of the flow of angular momentum in the atmosphere," *J. Meteor.*, 6, 291–299, 1949.