AN ESSAY ON THE GENERAL CIRCULATION OF THE EARTH'S ATMOSPHERE

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ABSTRACT

In this discussion of the general circulation the course of the normal transfer of absolute angular momentum from the belts of easterlies near the equator to the belts of surface westerlies in middle latitudes is studied. It is suggested that the horizontal transfer is brought about by the large-scale troughs and ridges in the mid-troposphere, which are adapted to perform this function by their departure from sinusoidal form. Also, the shapes of the subtropical circulations are found to be such as to produce a transport of angular momentum poleward. It is proposed that the downward flow of angular momentum in the westerly belts is effected by the presence of surface cyclones of the Bjerknes type in these regions. The upward flow in the easterly belts is assumed to be effected through some analogous mechanism, although the details are not clear owing to the scarcity of proper observational data.

The various processes which take place in the atmosphere and result in large-scale air motions are so extremely complex in their operation that up to the present time no rational theory approaching any degree of completeness has been devised to explain what we may call the general circulation. It is even doubtful whether all the physical processes which may have an ultimate influence on air motions have as yet been ascertained. It has, however, been realized for many years that there exist certain requirements which impose restrictions on these large-scale motions and impress upon them several characteristics which can be verified observationally. The purpose of this paper is to enquire further into the nature and consequences of some of these restrictions in the light of the added wealth of observational evidence which has been accumulated during the past several years, and to point out further possibilities for research.

In order to base the discussion on principles whose validity cannot be open to question and which are still not of a trivial nature, we shall first consider the fact that a dynamical system such as the atmosphere (or a portion of it) cannot change its absolute angular momentum about a given axis except through the addition or abstraction of angular momentum about that axis from or by external agencies. Considering the whole atmosphere, the only significant external interaction is with the earth's surface, almost entirely through friction. This general fact has been used as a basis for discussing the general circulation by Jeffreys [3] in a paper to which more reference will be made later. In reality, this essay may be construed as a further extension of the approach to the problem initiated by Jeffreys.

Before embarking upon the examination of details we shall review the simple qualitative picture of the conditions which normally prevail in the atmosphere from the standpoint of the distribution and transfer of absolute angular momentum about the earth's polar axis. In the regions between roughly 30°N and 30°S latitude there are present winds having a component from the east which cover most of this zonal belt. These so-called trade winds are probably the most steady and extensive air motions at the earth's surface. Since they are located in a region where the distance from the axis of the earth is large, the frictional effects at the surface are such as to produce a relatively large eastward (positive) torque upon the atmosphere. We thus have in this region a continuous and intense flow of absolute angular momentum from the earth to the atmosphere. This angular momentum can be removed only through the exertion of a negative torque by the earth upon the atmosphere at latitudes farther to the north and farther to the south mostly through friction at the surface in the regions of the prevailing westerlies. There must thus exist a horizontal flow of absolute angular momentum away from the equator in middle latitudes, diminishing poleward, however, as the surface frictional effects of the westerlies come into play. It is therefore a matter of great importance to study the details of the mechanism whereby this transfer is effected. Although we have made a tacit assumption that a steady average state is present so that no progressive accumulation or deficiency of angular momentum takes place, this limitation does not have to be imposed when relatively short periods of time are under consideration. We may also note at this point that a poleward flow of angular momentum does not necessarily imply a transfer of mechanical energy in the same direction.

In the vicinity of the poles, the north pole especially, there is often present a mass of cold air near the ground in which the wind has a component from the east. If such anticyclonic conditions persist for an appreciable length of time, it should be expected that an equatorward flow of angular momentum would set in from this region to the belt of westerlies. On rare occasions in the northern hemisphere this anticyclonic condition may become so exaggerated that for practical purposes the belt of westerlies vanishes temporarily at the surface. It is to be expected that during periods such as these the normal regime of transfer of absolute angular momentum is profoundly altered.

Since the zonal velocities relative to the earth which are present in actual wind systems are small as compared to the linear eastward velocity of the earth's surface itself, except in the vicinity of the poles, it follows that the distribution of absolute angular momentum in the atmosphere does not differ very much on a percentual basis from that corresponding to solid rotation at the angular speed of the earth. It is therefore apparent that there must normally exist a large gradient of absolute angular momentum northward and southward from the equatorial regions.

Although it is true that the combined system composed of the earth and the atmosphere cannot alter its total absolute angular momentum except for extremely slow secular changes resulting from tidal action as was pointed out by Darwin and others, still there is no reason to expect that the partition of angular momentum in the composite system should remain constant when seasonal and other short time-intervals are considered. Because of the great contrast in the moments of inertia of the two components, short-period anomalies of this kind represent major anomalies in the behavior of the wind systems, but, on the other hand, imply practically undetectible inequalities in the rate of the earth's rotation. From the standpoint of availability of observational material in regard to the motions of the atmosphere, we cannot, at the present time, follow these changes when the atmosphere is considered *in toto*. However, when restriction is made to the northern hemisphere alone, some approach to the problem could be made with existing data, and deductions might in this way result concerning the partition of atmospheric angular momentum between the hemispheres.

In view of the fact that about one-half of the mass of the atmosphere is found already below an elevation of five kilometers above sea level, it is possible to look upon the atmosphere as a two-dimensional film of approximately spherical form in many aspects of our study. This fact combined with the fact that it is not possible to deal with the actual exact values of the atmospheric angular momentum, but rather only with the variations in it and the processes tending to produce them, enables us to discuss the problem in a relatively simple fashion with sufficient accuracy for our purpose. Focusing attention on the portion of the atmosphere north of a particular latitude circle, we may think of its total absolute angular momentum as being the sum of two quantities. The first is the angular momentum due to the air motions relative to the earth's surface, while the second is due to the rotation of the same air about the polar axis with the constant angular speed of the earth itself. The second quantity depends not only upon the total mass of air north of the chosen latitude as expressed essentially by the surface pressure, but also upon the radial distribution of this mass with respect to distance from the polar axis. With the approximation as to the vertical extent of the atmosphere mentioned earlier, this radial distance is expressed by the latitude. Also, we may observe that the second quantity is far larger than the first, although this fact is not of direct concern to us. Rather, it is interesting to note that as far as synoptic and seasonal variations in the total absolute angular momentum are concerned, calculation shows that the contributions from the two quantities are of rougly the same order of magnitude. Furthermore, changes in the mass distribution are of greater significance in this sense, if they occur at relatively low latitudes.

We may next consider the processes which tend to alter the absolute angular momentum of this portion of the atmosphere. In the first place there can exist a horizontal transfer of such angular momentum across the selected latitude. Secondly, the interaction of the air with the earth's surface largely through friction can produce a flow of this angular momentum either from the atmosphere to the earth or *vice versa*. Taking the first of these transfer processes, it will be assumed that the air has negligible viscosity, so that there is no truly frictional interaction across the latitude circle. The actual horizontal transfer may also be thought of as being the result of two effects, one due to the advection of air having a positive or negative angular momentum relative to the earth across the latitude line, and the other due to the advection of angular momentum corresponding to the angular velocity of the earth's rotation. In order to secure a net contribution from the former effect, it is not necessary that there be a net transport of mass. Such a transport of mass is, however, necessary in order to secure a contribution from the latter effect, because the angular momentum per unit mass due to the earth's rotation is constant along the latitude so that a mere exchange of air produces no net result. Except for short-period variations and slow seasonal fluctuations, the latitudinal mass distribution of the atmosphere is constant, so that in all probability the significant north–south transport of absolute angular momentum is accomplished through the exchange of relative angular momentum.

The frictional interaction at the earth's surface is more or less proportional to the square of the surface wind speed, although the value of the constant of proportionality varies greatly with the local character of the surface. The tangential stress produced upon the earth is in the direction of the surface wind, and conversely, the earth's surface exerts a stress upon the air in a direction opposite to that of the surface wind. Corresponding to this stress there exists a flow of absolute angular momentum whose intensity depends upon the magnitude of the eastward component of the stress on the atmosphere and upon the distance from the polar axis. In addition to the frictional interaction, it is possible that at times differences of atmospheric pressure between the eastern and western sides of mountain ranges, especially those of great north–south extent, may produce significant torques upon the atmosphere. Thus the Rockies in North America and the Himalayas in Asia might produce sensible effects of this kind in the northern hemisphere, although, for simplicity in the first instance, we shall not take this phenomenon into account in what follows.

Returning to our principal task, namely the examination of the transfer of absolute angular momentum within the atmosphere, we have seen that this flow is brought about by appropriate exchange processes involving air motions relative to the earth's surface. Thus, if a latitude is selected in the belt of surface westerlies of the northern hemisphere, where normally the transfer is toward the north, the northward-moving individual masses of air should at the same time have a larger eastward component of motion than do the southward-moving ones. In agreement with the conclusion reached by Jeffreys, it seems reasonable to suppose that the necessary positive correlation between northward and eastward velocity components is brought about by the associated upper structures of the cyclones and anticyclones present, which thus form an integral part of the mechanism of the general circulation and constitute the individual eddies that bring about a turbulent transfer of angular momentum poleward on a grand scale. As contrasted with other studies of turbulence, we have in this case rather well-defined information concerning the structure of the eddies themselves, and hence we should be able to obtain a better grasp of the nature of the turbulent process in question.[1] To this end we shall attempt to analyze certain characteristic pictures of flow patterns which resemble those found on synoptic maps and which at the same time exhibit the necessary correlation between the horizontal velocity components.

The general character of the flow patterns observed in middle latitudes a short distance above the surface is a westerly motion on which are superposed troughs in the general vicinity of cyclonic disturbances, so that frequently the appearance of the streamlines at a fixed level resembles that shown in fig. 1. The important

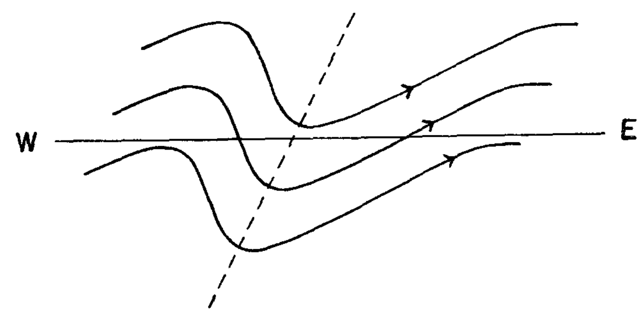

FIG. 1. Schematic picture of horizontal streamlines in a typical middle-latitude flow pattern in the upper westerlies.

feature is the departure from symmetrical sinusoidal form and the associated northeast–southwest tilt of the trough line. It is apparent that this characteristic produces the necessary velocity correlation, since the southward motions are associated with smaller eastward components than are the northward motions. Often the departure from sinusoidal form and tilt of the trough line are small at higher latitudes near the northern border of the surface westerly belt where the northward transfer of angular momentum is small, and become more and more pronounced at lower latitudes where a large transport of angular momentum is normally present. It thus seems that this typical atmospheric flow pattern, so common on meteorological maps, is a necessary automatic adjustment to provide for a poleward transfer of atmospheric angular momentum.

Examination of the rather rare cases when the belt of westerly winds at the surface is weak or absent has left the writer with the impression that many of the troughs present on the upper-level maps of the northern hemisphere for such periods display only small tilt or even a tilt from northwest to southeast. Also, during more normal periods with, however, rather well developed easterly surface winds near the pole, troughs extending to high latitudes seem to possess a reverse tilt north of the belt of surface westerlies as would be dictated by the probable southward flow of angular momentum at these high latitudes during such periods.

If the general scheme outlined above is correct, it becomes a matter of considerable interest to determine at what levels in the atmosphere the bulk of the poleward transport of angular momentum takes place. From preliminary statistical studies of daily hemispherical data it seems that at 45°N latitude the transport during winter is pronounced and almost always directed toward the north at an elevation of 10,000 feet above sea level. Further information must await much more extensive statistical study, although in the

[1] A legitimate question may be raised concerning the use of the word turbulence in the present connection. We shall however continued to use it because of the lack of a better term.

opinion of the writer, this transport is brought about mainly through the action of the large-scale upper troughs and ridges in the main body of the troposphere, at least within the confines of the surface westerly belt in each hemisphere.

Because of the basic importance of the transport process depicted by the streamline pattern sketched in fig. 1, it is instructive to examine it from other points of view than the one which has already been given. If a transport of angular momentum takes place across latitude circles in the general vicinity, it follows that such a transport must also exist across any other fixed curve oriented in a more or less west–east sense but not necessarily extending along a parallel of latitude. It is thus possible to speak of the instantaneous flow of angular momentum across a fixed curve which coincides with one of the streamlines in the figure. This flow is in general due to the torque exerted by the pressure forces acting across the curve, and also to any exchange of air masses of differing angular momentum across the curve. In the present instance no mass exchange takes place at the level under consideration, since the curve is a streamline, so that only the torque produced by pressure forces is operative. In order that a torque be exerted by the air to the south on the air to the north, it is essential that the pressure be greater on the west side of the southward bulge of the streamline than it is on the east side. It then follows that the horizontal streamline cannot at the same time be a line along which the pressure is constant.

Viewing the matter still otherwise, we may consider the instantaneous poleward flow of angular momentum across a fixed curve which coincides with an isobar in the given horizontal surface. Such isobars have more or less the same shape as the streamlines. In this case the pressure forces can produce no net torque, so that the transport must be due to an exchange of air masses of differing angular momentum across the curve. Qualitatively, the relation of the air motions to the isobar shown in fig. 2 could accomplish the transfer. A rela-

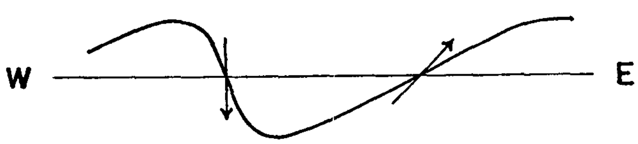

FIG. 2. Schematic picture showing the northward transport of angular momentum across an isobar. Arrows indicate the direction of air motions.

tion of the winds to the isobars similar to this has been noted from observational data by Houghton and Austin [2], and further statistical studies of the matter are at present in progress at the Massachusetts Institute of Technology.

One may ask finally how angular momentum is transported from the easterly belts at low latitudes to the belts of westerlies farther to the north and to the south. The important fact here is that the zone of the trade winds in each hemisphere is not continuous, but together with the equatorward portion of the westerlies forms a few large anticyclonic systems in the subtropics. Again using the northern hemisphere as an illustration, these anticyclonic circulations have horizontal streamlines of approximately the shape shown in fig. 3. It is evident that here also we have the neces-

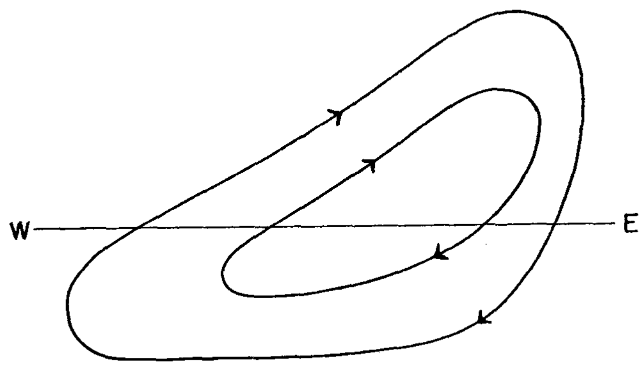

FIG. 3. Schematic picture of horizontal streamlines in a subtropical flow pattern.

sary correlation between the velocity components to accomplish the transfer, and the other facts as regards the pressure distribution in relation to the streamlines must follow a scheme similar to that which has been discussed in the case of the westerlies. We may then say that the separation of the high-pressure belts of the subtropics into individual cells is a necessary automatic adjustment in the atmosphere which provides for a poleward transfer of absolute angular momentum from the low-latitude easterlies.

We come now to the consideration of questions relating to the vertical transport of absolute angular momentum within the atmosphere. In view of the fact that at least a large part of the horizontal transport is probably accomplished at moderate elevations above the surface, it is obvious that there must exist some mechanism whereby angular momentum is communicated upward to these levels in the trade-wind zones, and also there must exist a mechanism for its downward transfer in the vicinity of the surface westerlies. Because the vertical dimension of the atmosphere is so small compared to the horizontal ones, it would be simple to invoke the virtual viscosity of the air due to small-scale turbulence in order to effect these vertical exchanges. On the other hand, but little is known concerning the efficacy of such friction above the surface turbulence layer, so that it is possible that friction in the free air, to the extent that it is present, may work together with other modes of angular momentum transfer in the vertical.

We have observed that in the case of horizontal transfer it is necessary that there exist a certain organization of the north–south air motions in reference to the absolute angular momentum of the air masses taking part in these motions. Similarly, we might expect to find an organization of the vertical air motions with respect to absolute angular momentum in the case of vertical transfer. We have noted also that in spite of the presence of air motions relative to the earth, there exists a strong poleward gradient of absolute angular momentum in the atmosphere. In the case of the downward transfer within the belts of surface westerlies, generally speaking, the upward branches of vertical circulations should occur closer to the poles than the descending branches of the same circulations. The magnitude of organized vertical velocities in the atmosphere is exceedingly small, so that it is difficult to secure measurements of such motions and consequently our information concerning them is still rather sketchy. We may nevertheless make the indirect inference that relatively vigorous upward motions take place over those regions where active precipitation is observed, since precipitation of sensible intensity requires an adiabatic cooling of the air produced by upward motions of air particles to levels of lower pressure. The areas of precipitation and therefore of ascending motion in middle latitudes are found more or less on the northeastern and the northern sides of the surface cyclones, using the northern hemisphere for purposes of illustration. It is commonly agreed that descending motions are to be found on the southwestern sides of these disturbances, in the relatively cold air which normally sweeps around to the south of the cyclone center. It thus would appear that the typical Bjerknes cyclones [1], as distinguished from the large troughs at upper levels, perform the function of turbulence units in the downward transport of absolute angular momentum from the upper levels in middle latitudes. We should therefore expect that the frequency of occurrence of Bjerknes cyclones should be greater on the eastern sides of the large troughs at upper levels than on the western sides, because these are the regions where vigorous northward transport of angular momentum takes place. This condition is common on meteorological maps.

The meteorological processes which take place in the trade-wind belts have not as yet been subjected to the same detailed scrutiny as those of middle latitudes. We are therefore at a disadvantage in attempting to trace the course of the upward flow of absolute angular momentum in these regions. It has nevertheless been observed that certain synoptic disturbances called easterly waves are present on the equatorward sides of the subtropical high-pressure cells. It is not inconceivable that these waves are accompanied by organized vertical motions of the type necessary to provide for an upward flux of angular momentum. The probability is, however, that the rather sporadic occurrence of tropical hurricanes in these regions cannot account for the normal upward transfer. Although the normal poleward gradient of absolute angular momentum is small in the trade-wind region, still its presence would favor the location of the upward branches of vertical circulations nearer to the equator than the descending branches. Such a distribution of vertical motions is not in conflict with the general climatological characteristics of the tropics and subtropics as indicated by the meridional distribution of rainfall within the subtropical high-pressure cells, the precipitation being more abundant nearer to the equator. On the present hypothesis the general selective effects of the phenomena of middle latitudes and of the trade-wind regions upon the location of descending motions evidently conspire to produce relatively arid zones in the intervening belts.

The reader has doubtless observed many gaps and shortcomings in the rough picture of the atmospheric motions which has been sketched, perhaps necessarily with a rather broad stroke. For example, questions relating to the sources and transfer of energy have not been touched upon, nor have the associated heat-transfer processes been treated. Likewise it is not immediately clear what role is played by frontal discontinuities in the scheme, and the characteristic phenomena of the atmospheric tropopause have not been related to the mechanics of the system. Nevertheless it is a matter of interest that a few pieces of the puzzle which the general circulation presents can be made to fit together, although the problem of why these pieces have the precise shapes they do is a far more profound and difficult subject. Efforts to deal with this latter question have recently been made by Rossby [4] and others [5].

Analytical representations of the concepts introduced have not been given, since it is felt that the first step in the treatment of a subject such as the present one is the formulation of a physical picture. Furthermore, the reader who is mathematically inclined can easily supply such representations where they are of obvious application.

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