A STUDY OF THE FLOW OF ANGULAR MOMENTUM IN THE ATMOSPHERE

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ABSTRACT

A study is made of the flow of angular momentum in the atmosphere for the month of January 1946. The results generally confirm the pattern proposed by Starr on theoretical grounds. Angular momentum is transferred from the earth to the atmosphere in regions of surface easterly winds (chiefly the tropical and subtropical easterlies), transported upward, then horizontally poleward, and finally downward, being removed in regions of surface westerly winds. The torques due to surface friction are found to be of the same order of magnitude as those due to differentials of pressure across mountain ranges. During a period of the length of this study or less, it is found that the change and transport of angular momentum due to shifts of mass are of the same order of magnitude as the change and transport of relative angular momentum. If one accepts the method used for estimating the surface torques, there appears to be an excess of transfer of angular momentum to the atmosphere in the northern hemisphere. From a study of the normal January pressure profile, it would appear that this excess represents a flow of angular momentum to the southern hemisphere, where it is needed to balance accounts.

1. Introduction

One of the earlier mentions of the importance of angular momentum in any consideration of the general circulation was by Jeffreys [7]. In this paper, Jeffreys shows that, by considering the amount of angular momentum transported across a given latitude circle and the net loss of angular momentum by frictional torque north of this latitude, it is impossible to have a zonally symmetric distribution of wind and pressure if surface friction is present. He concluded that there must exist large-scale air streams extending through a major part of the troposphere with a strong meridional component of motion (of the same order of magnitude as the zonal component) and that the cyclones and anticyclones are a necessary part of the general circulation rather than being merely oscillations about a possible zonally symmetric steady state. It should be remembered that these conclusions were reached despite the absence of the extensive amount of upper-air data which is now available. Jeffreys' conclusions were discussed at some length in further papers by himself and others [3; 4; 8; 13], but without significant change in the conclusions presented above.

The importance of the angular-momentum concept in studies of the general circulation has been re-emphasized in a recent paper by Starr [11]. In his paper it is pointed out that, inasmuch as the earth and atmosphere may be considered as practically an isolated system, there must be a flow of angular momentum from the earth to the atmosphere in regions of surface easterly winds (the most important of these regions being those of the tropical easterlies or "trade" winds) and a reverse flow in regions of surface westerly winds (particularly in the prevailing westerlies of the temperate zones). There must then exist, on the average, a poleward flow of angular momentum. There must also exist an upward transport of angular momentum over the easterlies and a downward transport over the westerlies.

As over long periods there is no progressive net change in the distribution of atmospheric mass over the earth, the significant long-term meridional transport of angular momentum is accomplished by the meridional interchange of air masses with differing relative angular momentum. Starr has suggested that this interchange is effected principally by the upper-air trough and ridge systems with axes tilted from northeast to southwest. The transfer of angular momentum between the earth and the atmosphere[3] is effected by surface friction and by differentials of atmospheric pressure across mountain ranges.

The importance of considering the flow of angular momentum in any study of the general circulation should be obvious. Although even a complete knowledge of the angular-momentum transfer in the atmosphere cannot by itself furnish a solution of the problem of the general circulation, any proposed scheme for this circulation should include a means for securing the angular-momentum flows which are observed. For this reason, it is essential that all possible

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[3] Hereafter often referred to as the generation and removal (or similar terms) of angular momentum in the atmosphere.

observational knowledge concerning these processes be obtained. It is hoped that the study reported here represents a beginning in the accomplishment of such an aim.

At the suggestion of Prof. Starr it was decided to investigate quantitatively and as extensively as possible the generation and transport of angular momentum in the atmosphere for a period of one month. It was planned that this investigation should serve as a pilot project for similar later studies covering more extensive periods. The results reported here should therefore not be considered as final until confirmed by further studies.

2. Theoretical considerations

Since the force of gravity can exert no torque about the earth's axis, we may write the equation of zonal motion without approximation in the form

$$dM/dt = -\rho^{-1}r(\partial p/\partial x - D_u), \tag{1}$$

which states that the absolute angular momentum of an individual unit mass of air increases at a rate equal to the external torques exerted upon it by the pressure force and by friction. (See the table of symbols at the end of this article.)

Equation (1) may be rewritten with the aid of the following considerations:

$$\begin{aligned}\rho\, dM/dt &= \rho\, \partial M/\partial t + \rho \mathbf{c}\cdot\nabla M \\ &= \frac{\partial \rho M}{\partial t} + \nabla\cdot\rho M\mathbf{c} - M\left(\frac{\partial \rho}{\partial t} + \nabla\cdot\rho\mathbf{c}\right),\end{aligned} \tag{2}$$

where $\mathbf{c}$ is the absolute vector particle velocity. The quantity in parentheses in (2) vanishes identically because the general equation of continuity of mass states that this expression is zero. Accordingly (1) becomes

$$\frac{\partial \rho M}{\partial t} = -\nabla\cdot\rho M\mathbf{c} - \frac{\partial p r}{\partial x} + rD_u, \tag{3}$$

still without approximation. This equation states that the rate at which absolute angular momentum is increasing at a point fixed in space is equal to the negative divergence (convergence) of the transport of angular momentum plus the rate at which it is imparted to the air at the point by the external torques due to pressure and friction.

Noting that ρM is the angular momentum per unit volume, we may form a volume integral of (3) over a portion of the atmosphere bounded by the earth's surface and extending from the pole to a given latitude ϕ. The upper boundary is assumed to be the "top" of the atmosphere. The term on the left-hand side gives an integral which is the rate of increase of the total angular momentum within the region, while all the terms on the right give integrals which are expressible in terms of quantities measured at the boundaries of the space. Let us designate an element of volume by dV, and an element of the surface of this volume by dS. Further, let $d\sigma$ represent the projection of dS on the meridional plane passing through a given point and τ_x the eastward frictional stress acting at the boundary on the air inside. We may then write

$$\frac{\partial}{\partial t}\int \rho M\, dV = \int \rho M c_n\, dS + \int pr\, d\sigma + \int r\tau_x\, dS, \tag{4}$$

which is still rigorous and could have been written immediately from physical considerations.

In the first term on the right c_n is the inward component of the velocity and the quantity as a whole represents the rate at which angular momentum is brought into the region by air motions across the boundary. The only significant air motions across the boundary take place at the vertical surface at latitude ϕ. For all practical purposes we may assume that $r = R\cos\phi$, in all the discussion which follows, so that

$$M = uR\cos\phi + \omega R^2\cos^2\phi. \tag{5}$$

At this southern boundary $c_n = v$, and we may without sensible error say that $dS = dx\, dz$. The first term on the right side of (4) may then be written as the sum of two terms, namely,

$$R\cos\phi \iint \rho uv\, dx\, dz, \tag{6}$$

expressing the rate of the northward advection of *relative* angular momentum, and the term

$$\omega R^2\cos^2\phi \iint \rho v\, dx\, dz, \tag{7}$$

expressing the rate of northward advection of angular momentum due to the earth's rotation (hereafter referred to as *ω-angular momentum*). We note that the expression (7) cannot contribute unless there is a net flow of mass across the latitude circle.

The second integral on the right of (4) can give a contribution only because the lower boundary is not a smooth spherical one, but has imperfections in the form of mountain ranges at whose sides $d\sigma$ does not vanish. The measurement of this effect is discussed below.

The third term on the right side of (4) may be written in the form given because friction represents a mode of exchange of momentum and can give a net contribution only when the frictional interaction is present with the surroundings at the boundary of the region. The main effect of this nature results from the interaction at the earth's surface north of the latitude ϕ. Small-scale eddy friction appears to be far too small to contribute significantly to the flow of angular momentum across the vertical at latitude ϕ. We are thus left with the quantities (6) and (7) to account for the large meridional transfer of angular momentum for the maintenance of the general circulation.

In the long-run average the angular momentum of the atmosphere is constant and the left-hand side of (4) is zero. For shorter periods, however, this is not the case. By using (5) it follows that

$$\frac{\partial}{\partial t}\int \rho M\, dV = \frac{\partial}{\partial t}\int \rho Ru \cos\phi\, dV + \omega R^2 \frac{\partial}{\partial t}\int \rho \cos^2\phi\, dV. \quad (8)$$

Here the first term on the right is the rate of increase of relative angular momentum, while the second term is the rate of increase of the angular momentum due to the earth's rotation. This latter quantity (ω-angular momentum) can be changed only by net shifts of mass from one latitude belt to another. Such shifts of mass are measured in terms of surface pressure changes.

By use of (5), (6), (7), and (8), (4) may now be written as

$$\frac{\partial}{\partial t}\int \rho Ru \cos\phi\, dV + \omega R^2 \frac{\partial}{\partial t}\int \rho \cos^2\phi\, dV = R\cos\phi \int\int \rho uv\, dx\, dz + \omega R^2 \cos^2\phi \int\int \rho v\, dx\, dz + \int pr\, d\sigma + \int r\tau_x\, dS. \quad (9)$$

This investigation consists of an attempt to evaluate the six terms in (9) from actual atmospheric data.

3. General procedure

The month of January 1946 was chosen for this investigation, mainly because of the availability of the necessary data for this month and the fact that a general inspection of the maps for this period did not reveal any too outstanding abnormalities. It was found later that this month has a somewhat higher than normal zonal index (surface westerlies, 700-mb westerlies, and surface subtropical easterlies). What relation this may have to the results reported here must await investigation of other periods with differing indices.

The data for sea level and the 500-mb level were obtained from the Northern Hemisphere Historical Weather Maps [1]. The 700-mb level data were obtained from photostats of northern-hemisphere charts analyzed by the U. S. Air Force. On several days, only data for the western half of the northern hemisphere were available at 700 mb. There was one map per day at each level: 0400 GCT at 700 mb and 500 mb, 1230 GCT at sea level. In this study, the difference between the time of the maps at sea level and that at higher levels was neglected. In general, complete data were available from 80°N to 30°N at the upper levels and to 10°N at sea level. Computations for the upper levels south of 30°N were estimated from the available but incomplete data there.

Throughout this study, it was necessary to assume that the actual wind was sufficiently well approximated by the geostrophic wind, due to the lack of adequate actual wind data. Machta [9] has studied the validity of this assumption as it applies to the transport of relative angular momentum through computations based on a theoretical model of a trough with its axis tilted with respect to the meridians. This model suggests that fair agreement might be expected between the geostrophic and actual transport of relative angular momentum, aside from the effect of meridional circulations.

Lorenz[4] has also studied the validity of the assumption, using geostrophic deviation data gathered by Machta;[5] unfortunately but necessarily these data were limited to the United States. His results indicate that the geostrophic transport of relative angular momentum is of the same order of magnitude as the actual transport, but the geostrophic assumption gives transports away from the equator which are somewhat less than the actual transport.

Due to the fact that it was necessary to use geostrophic winds and that (as will be discussed later) the density was taken as constant for each level and latitude, it was not possible to compute directly the net transport of mass (and therefore of ω-angular momentum) across latitude circles.[6] The method of computation actually used for these quantities will be discussed later.

The surface frictional torque was computed from the sea level geostrophic wind, assuming the surface wind to be in the same direction as and 0.6 as great as the sea-level geostrophic wind. The transport and change of relative angular momentum as computed from the sea-level geostrophic winds were assumed to be representative of conditions at the geostrophic wind level. The geostrophic mean zonal winds were assumed equal to the actual mean zonal winds.

Due to the fact that neither the transport of relative angular momentum nor the surface frictional torque are linear functions of wind velocity, it was necessary to compute these quantities individually

[4] E. Lorenz, "Investigation of the general circulation of the atmosphere," Report no. 2, Contract W28-099 ac-406, between Watson Laboratories, AMC, and the Massachusetts Institute of Technology, 1948.

[5] L. Machta, "A study of the observed deviations from the geostrophic wind," Unpublished Sc.D. thesis, Massachusetts Institute of Technology, 1948 (and additional data not included in thesis).

[6] The use of the geostrophic wind assumption leads to a zero net transport of mass; *i.e.*, any contributions from circulation cells in a meridional cross section representing the mean condition around the earth are automatically neglected.

point by point and day by day, and then to sum or average as needed. Other quantities (change in relative angular momentum, transport and change of ω-angular momentum, torque due to mountains) are linear functions and could be calculated from pressure profiles or (in the case of the torque due to mountains) mean maps.

The winds were calculated on the basis of pressure (or contour height) data recorded for each 5 degrees of latitude and longitude within the limits of the analysis. The wind components at a given latitude and longitude were obtained from the pressures (contour heights) 5 degrees north and south (or east and west) of the given point. The wind components at each point were assumed to be representative within longitudes $2\frac{1}{2}$ degrees east and west of the point in computing the transport of relative angular momentum; for computing the surface frictional torque, they were assumed to be representative of the area $2\frac{1}{2}$ degrees east and west, and 5 degrees north and south, of the point.

The densities at each latitude and level were taken as the normal for January at that latitude and level. The 700-mb densities were computed from the normal contour and temperature map.[7] The 500-mb densities were extrapolated (assuming a moist adiabatic lapse rate) from the normal 20,000-ft pressure and temperature map [12]. The surface densities were computed on the basis of normal surface temperatures given by Haurwitz and Austin [5]; they were extrapolated to 900 mb to obtain the densities at the geostrophic wind level. The use of the normal, constant density introduces two sources of error: first, from the difference between the normal January densities and the mean January 1946 densities; second, from the fact that a correlation between density and wind direction would be expected. A preliminary investigation of the second point has indicated that it produces an error in the relative angular-momentum transport whose magnitude is of the order of 10 per cent.

4. Transport of relative angular momentum

This is the determination of the term,

$$R \cos \phi \int\int \rho uv \, dx \, dz,$$

in (9). In actual practice, the quantity

$$\int_0^{2\pi} \rho r^2 uv \, d\lambda, \qquad (10)$$

which gives the transport of relative angular momentum per unit time and per unit height across a given latitude, was determined at certain latitudes and levels.

For a constant-pressure level, under the procedure set forth above, the transport is given by

$$-\frac{9\rho g^2 \cos \phi}{4\pi\omega^2 \sin^2 \phi} \Sigma(\Delta z)_y(\Delta z)_x, \qquad (11)$$

where the summation is made for 72 points, each separated by 5 degrees of longitude, completely around the parallel of latitude. For a constant-level surface, this transport is equivalent to

$$-\frac{9 \cos \phi}{4\pi\omega^2 \rho \sin^2 \phi} \Sigma(\Delta p)_y(\Delta p)_x. \qquad (12)$$

In integrating the transport (and also the change) of relative angular momentum through height, it has been assumed that the transport at the geostrophic wind level is representative from the surface (assumed to be at sea level) to 1.5 km; that at 700 mb, from 1.5 km to 4.5 km; and that at 500 mb, from 4.5 km to 7.5 km. In the absence of data above 500 mb, no attempt was made to estimate the vertical distribution of the transport and change of relative angular momentum above 7.5 km. In integrating through time, it has been assumed that a quantity computed from a map is representative of the period from 12 hours before to 12 hours after map time.

Fig. 1 shows the net total transport of relative angular momentum in the three layers from map time of 1 January 1946 to map time of 31 January 1946.

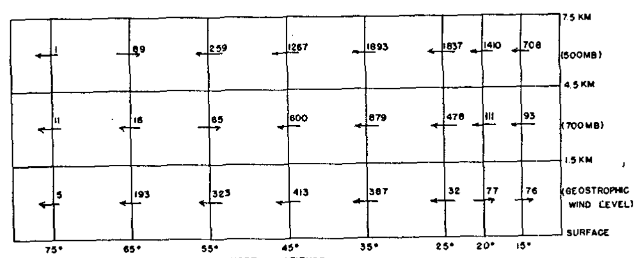

FIG. 1. Net amounts of relative angular momentum (in units of 10^{29} g cm^2 sec^{-1}) transported horizontally by geostrophic motion across entire latitude circles during January 1946 for the indicated horizontal layers.

It will be noted that there is generally a poleward transport of relative angular momentum and that the transport generally increases with height south of 50°N. The transport increases with increasing latitude up to 35°N, presumably due to the addition of angular momentum to the atmosphere in the zone of surface easterly winds. It decreases north of 35°N, presumably due to the removal of angular momentum in the surface westerlies. There is some evidence, particularly in the highest layer shown, at 65°N, of a comparatively minor flow of angular momentum generated in the polar easterlies, southward to the zone of the surface westerlies.

[7] U. S. Weather Bureau, "Normal 700-mb charts," Extended Forecasting Section, Washington, D. C. (unpublished photostats).

5. Change of relative angular momentum

The relative angular momentum of a given horizontal layer of air of unit thickness between two latitudes is

$$\int_0^{2\pi}\int_{\phi_1}^{\phi_2} \rho u r^2 R\, d\phi\, d\lambda. \tag{13}$$

For a constant-pressure level, this becomes

$$-\omega^{-1}18R^2\rho g\overline{(\Delta z)}_v\left[\cos\phi + \ln\tan\tfrac{1}{2}\phi\right]_{\phi_1}^{\phi_2}, \tag{14}$$

and for a constant-height level

$$-\omega^{-1}18R^2\overline{(\Delta p)}_v\left[\cos\phi + \ln\tan\tfrac{1}{2}\phi\right]_{\phi_1}^{\phi_2}. \tag{15}$$

In this study, it is chiefly the change in relative angular momentum over a period of time that is of interest. This corresponds to the term,

$$\frac{\partial}{\partial t}\int \rho R u \cos\phi\, dV,$$

in (9). The changes in the relative angular momentum within the three layers from the first to the last day of January 1946 are given in table 1. In general there

TABLE 1. Changes in relative angular momentum from map time of 1 January 1946, to map time of 31 January 1946 (in units of 10^{29} g cm^2 sec^{-1}).

Latitude belt	Levels 0–1.5 km	1.5–4.5 km	4.5–7.5 km
75–65	− 10	+ 16	− 7
65–55	+ 4	+ 38	+ 28
55–45	+101	+107	+174
45–35	− 38	+146	+ 10
35–25	−186	−331	−203
25–20	−123	− 9	0
20–15	− 78	−194	+112

was, during the month, an increase of relative angular momentum in the temperate westerlies and a decrease in the zone south of 35°N.

6. Change and transport of ω-angular momentum

This is the determination of the terms

$$\omega R^2 \frac{\partial}{\partial t}\int \rho \cos^2\phi\, dV,$$

and

$$\omega R^2 \cos^2\phi \int\int \rho v\, dx\, dz,$$

in (9) (the integrated effect from 1 January to 31 January 1946 being considered here). The first is the change of ω-angular momentum and is computed through computing the changes of mass within certain latitude belts in the three layers; surface to 10,000 ft, 10,000 ft to 18,000 ft, and 18,000 ft to infinity.[8] The second is the transport of ω-angular momentum which can be computed directly from the transport of mass. However, as was noted earlier, the use of the geostrophic assumption prevents any direct computation of the mass transport. The transport of mass has therefore been computed from the changes of mass and continuity considerations, starting at the north pole and assuming no net vertical transport of mass between layers within a latitude belt,[9] inasmuch as all net transport of mass into a polar cap must take place through the latitude circle at its southern boundary.

The change in mass within a latitude belt is computed from the changes in the pressures within that belt and is converted to change in ω-angular momentum by multiplying by

$$\frac{1}{\Delta\phi}\int_{\phi_1}^{\phi_2} \omega r^2\, d\phi = (R^2\omega/\Delta\phi)\left[\tfrac{1}{2}\phi + \tfrac{1}{4}\sin 2\phi\right]_{\phi_1}^{\phi_2}. \tag{16}$$

The transport of mass across a latitude circle is converted to transport of ω-angular momentum by multiplying by $r^2\omega$. Fig. 2 illustrates the net change and transport of ω-angular momentum from map time of 1 January 1946, to map time of 31 January 1946.

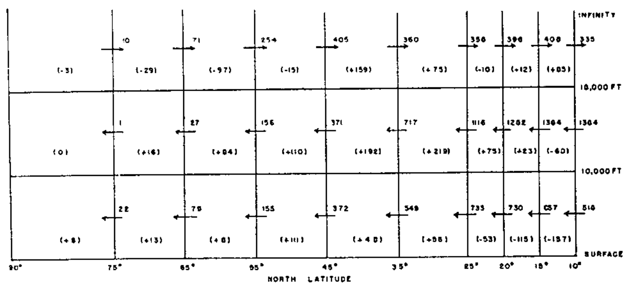

FIG. 2. Net horizontal transport across entire latitude circles during January 1946 (arrows) and change (figures centered in blocks in parentheses) of angular momentum due to the earth's rotation (ω-angular momentum) from map time 1 January 1946 to map time 31 January 1946, assuming no net vertical transport of mass (in units of 10^{29} g cm^2 sec^{-1}).

7. Surface frictional torque

The surface frictional torque corresponds to the term

$$\int r\tau_x\, dS,$$

in (9); its contribution per unit time within a latitude belt is [2]

$$\int_0^{2\pi}\int_{\phi_1}^{\phi_2} \kappa\rho u(u^2 + v^2)^{\frac{1}{2}} r^2 R\, d\phi\, d\lambda. \tag{17}$$

[8] The 700- and 500-mb contour heights were converted to pressures at 10,000 ft and 18,000 ft respectively.

[9] This assumption of no vertical transport of mass is admittedly questionable, but it is necessary to obtain any estimate of the vertical distribution of the transport of ω-angular momentum. It does not affect the total of the transport from the surface to infinity; the vertical distribution of this transport is in error to the extent that this assumption is in error.

Under the previously stated procedure, this is equivalent to

$$\frac{9\kappa R}{4\omega^2\pi\rho}\left[\phi + \cot\phi\right]_{\phi_1}^{\phi_2} \Sigma(\Delta p)_v \left[(\Delta p)_y^2 + \frac{(\Delta p)_x^2}{\cos^2\phi}\right]^{\frac{1}{2}}. \quad (18)$$

The determination of the surface frictional torque is perhaps the most questionable of any of the procedures used in this study. To begin with, the entire subject of the stresses exerted by a moving fluid on its boundary has not been satisfactorily determined and the relation used in this study (essentially that the force is equal to $\kappa\rho c^2$, where c is here the relative speed of the fluid) is not necessarily the best one. The value of κ, the coefficient of skin friction, varies with the type and topography of the surface, probably with wind speed, and very probably with other factors. It is hoped that the value used here (0.003) is a reasonable approximation of a satisfactory mean value. In addition, the values of u and v in the computation have been taken as 0.6 of the sea-level geostrophic wind with no correction for the commonly observed change in direction between the surface and the geostrophic wind level.

In both the surface frictional torque and the torques due to differentials of pressure across mountain ranges, the sign convention has been chosen so that a minus sign indicates transfer of angular momentum from the earth to the atmosphere and vice versa. This arbitrary convention was so chosen because the sign of the surface frictional torque is then the same as the sign of the eastward component of the surface wind velocity.

The values of the surface frictional torques within certain latitude belts in the period from 1 January 1946 to 31 January 1946, are given in table 2.

TABLE 2. Integrated effect of surface frictional torque from map time of 1 January 1946 to map time of 31 January 1946 (in units of 10^{29} g cm^2 sec^{-1}).

Latitude belt	Torque
80–75	− 86
75–70	− 158
70–65	− 122
65–60	− 188
60–55	+ 606
55–50	+ 875
50–45	+ 1251
45–40	+ 1842
40–35	+ 138
35–30	+ 205
30–25	− 1530
25–20	− 2470
20–15	− 4770
15–10	−10190

8. Torques due to differentials of pressure across mountain ranges

This corresponds to the term

$$\int pr\, d\sigma$$

in (9) and was computed by White [14]. The reader is referred to his report elsewhere in this issue for a discussion of the procedure used. The time-integrated values of the mountain torques are given in table 3.

TABLE 3. Integrated effect of torques due to differentials of pressure across mountain ranges from map time of 1 January 1946 to map time of 31 January 1946 (in units of 10^{29} g cm^2 sec^{-1}).

Latitude belt	Torque
65–60	− 411
60–55	− 221
55–50	+ 191
50–45	+ 870
45–40	+1550
40–35	+ 965
35–30	+ 194
30–25	− 681

The procedures used in all the calculations given above have many sources of error. In addition to those mentioned previously, perhaps one of the most serious is the smoothing of the pressure (and contour height) patterns, both in the original analyses and in the subsequent manipulations. It is felt that, in general, the values computed are correct at least as to direction and order of magnitude. It is probably not possible to improve the accuracy of the procedure in any major degree with the data available at the present time.

9. Conclusions

This study has confirmed the picture of the generation and transport of angular momentum which was proposed by Starr from theoretical considerations. This can be seen by reference to the various tables and diagrams which have been previously mentioned. Angular momentum is generated in the subtropical easterlies, is transported northward, and is lost to the earth in the prevailing westerlies. The polar easterlies act as a secondary, but rather minor, source of angular momentum. Inasmuch as the transport of ω-angular momentum can have no progressive net effect over long periods of time, fig. 1 perhaps represents, in a very general way, the long term horizontal transport of angular momentum in the atmosphere.

Starr stated without detailed discussion that the interchange of angular momentum between the earth and the atmosphere might be effected by the differentials of pressure across mountain ranges as well as by surface friction. This has turned out to be the actual existing condition (*cf.*, tables 2 and 3); the two processes are found to be of the same order of magnitude. Furthermore, for the month as a whole, the two effects have generally the same direction at the same latitude. White has found that the torque due to the mountains during January 1946 closely approximates the normal condition for the month of January.

Although over long periods of time, there can be no progressive net change or transport of ω-angular

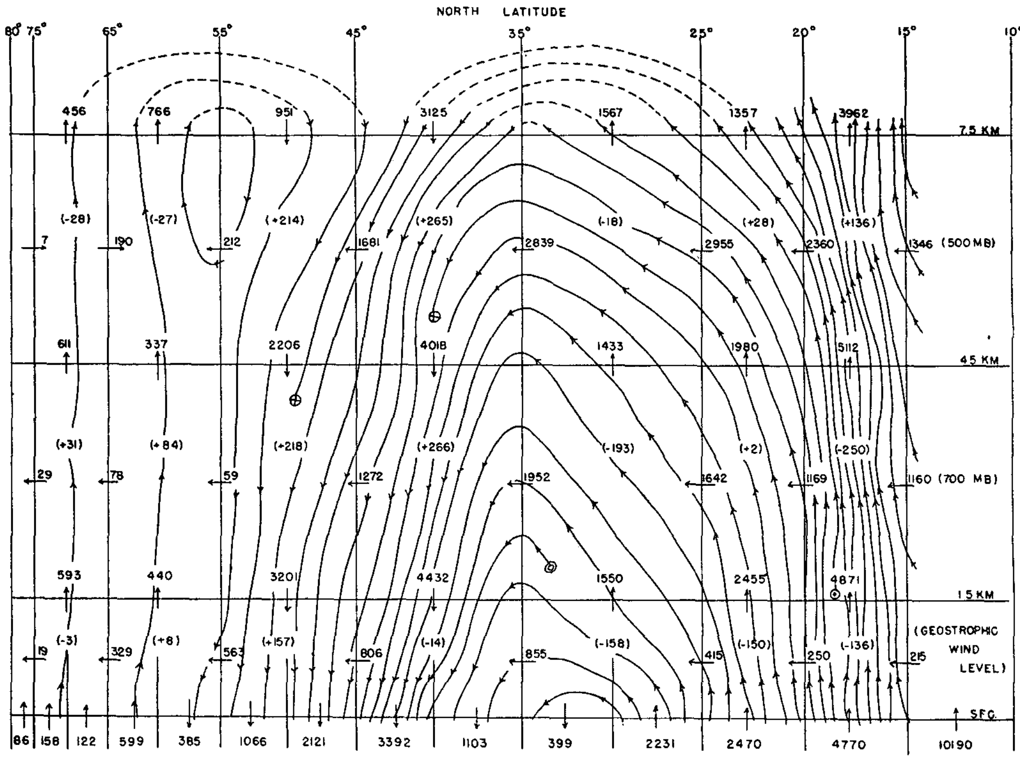

FIG. 3. Net total generation and transport of absolute angular momentum from map time 1 January 1946 to map time 31 January 1946 (in units of 10^{29} g cm^2 sec^{-1}). Small arrows indicate total flow of angular momentum between adjacent vertical or horizontal boundaries. Values in parentheses indicate total change of angular momentum within block. Each streamline indicates net flow of approximately 500×10^{29} g cm^2 sec^{-1} of angular momentum. Dotted streamlines indicate flow of relative angular momentum at undetermined heights above 7.5 km.

momentum, it has been found that during January 1946 the ω-angular momentum and the relative angular momentum terms were in general of the same order of magnitude, considering both individual days and the entire month. On some days, the ω-angular momentum term was larger than the relative angular momentum term. It would be expected that there should be a normal change and transport of ω-angular momentum for the month of January; this could be determined if normal maps for January 1 and January 31 were available. How closely January 1946 resembles this normal is not known.[10]

It is extremely desirable to construct an integrated picture of the contributions from all the terms of the angular momentum equation (9). In constructing this picture, it was found advisable to take account of the fact, previously mentioned, that the actual poleward transport of relative angular momentum is somewhat greater than the geostrophic transport. It has been assumed for this purpose, somewhat arbitrarily, that the actual transport is 1.5 times the geostrophic transport.

Such a picture is given in fig. 3, which illustrates the total generation and transport of absolute angular momentum during January 1946. All transport of ω-angular momentum is shown as occurring below 7.5 km; therefore, all the transport above this level (the vertical distribution of which is at present indeterminate) is in the form of relative angular momentum. The vertical transport of angular momentum was obtained from continuity considerations, beginning with the lowest layer.

It is possible, from this figure, to present the picture envisioned by Starr in somewhat greater detail. Not only does the horizontal transport of angular momentum increase in general up to the limit of the data, but, furthermore, apparently about one-half as much angular momentum is transported northward across 35°N above 7.5 km as below. The southward transport of the angular momentum generated in the polar easterlies would appear to occur chiefly above 7.5 km. The vertical transport of the angular momentum is concentrated mainly over the regions of generation and loss.

Inspection of fig. 3 reveals that apparently considerably more angular momentum was generated than can be accounted for by loss in the westerlies and increases in the angular momentum of the atmosphere (even if reasonably large increases are assumed to occur above 7.5 km). This excess of generation is even more pronounced when it is realized that the region between 10°N and the equator (not included on the

[10] Later preliminary investigations using normal maps for December, January, and February have indicated that the change and transport of ω-angular momentum during January 1946 was considerably larger than that which is normal for January.

figure due to lack of data) can also be assumed to be a region of generation. It would appear that the only area available to act as a sink for this excess momentum would be the southern hemisphere. A semiquantitative investigation of the plausibility of this assumption was made, using a mean January surface pressure profile for the years 1910–1934 extending from 70°N to 60°S. (This profile was prepared by the Extended Forecasting Project at Massachusetts Institute of Technology from data in [10].) The surface frictional force was approximated from the mean zonal wind.[11] As no estimate was possible of the torque due to the mountains in the southern hemisphere, the effect of the mountains was neglected throughout.

Although the results obtained probably have only a qualitative validity, they do appear to indicate that more angular momentum is generated in the northern hemisphere, particularly in the tropical and subtropical latitudes, than is dissipated in the northern-hemisphere temperate latitudes. The reverse is true in the southern hemisphere, due chiefly to the great surface intensity of the temperate latitude circumpolar vortex. There would appear to be, then, in the mean, an appreciable net transport of angular momentum from the northern to the southern hemisphere during January. In fact, a qualitative inspection of similar profiles for other seasons, indicates that while in some seasons both hemispheres may be substantially self-sufficient as to angular momentum, at no time, in the mean, would there apparently be an appreciable net transport of angular momentum from the southern to the northern hemisphere while the reverse might apparently often be the case. Investigations by White of the normal torque due to the mountains in the northern hemisphere indicate that this factor, although changing the magnitude of the excesses and deficits, does not eliminate the excess of angular momentum generated in the northern hemisphere.

The mechanism of the transport of angular momentum across the equator is far from apparent, and it will probably require an extensive study of equatorial air currents, based on actual rather than geostrophic wind data at all levels, even to begin to understand it. The above results would, nevertheless, seem to lend support to the opinions of those who believe that no final solution of the general circulation problem can be reached without considering the joint interaction of both hemispheres. Some further studies of these points and the interesting possibilities arising from them are now under way at the Massachusetts Institute of Technology.

The interchange of angular momentum between the earth proper and the atmosphere may, at times, produce a considerable net shift of angular momentum from one member of the system to the other. Due to the great difference in the masses of the two, relatively large changes in the atmospheric angular momentum would be expected to produce only small variations in the rate of rotation of the earth. To determine the order of magnitude of this change, it was decided to determine what the effect would be if the atmosphere were to lose all its relative angular momentum to the earth and the two were to rotate as a solid. The relative angular momentum of the earth proper was estimated from data on its surface density and mass [6], assuming a linear increase in density from the surface to the center. The relative and ω-angular momentum of the atmosphere was estimated from the January 1946 data, assuming the southern hemisphere to be a mirror image of the northern hemisphere.[12] It was found that even this change in the angular momentum distribution, extreme as it is, would decrease the length of a year (*i.e.*, 365 revolutions) by only 0.8 sec. It is understood that this change would be just noticeable from astronomical observations if it were to persist for an entire year. It would therefore appear that there is little hope of determining short period variations in the atmospheric angular momentum from changes in the speed of the earth's rotation.

It is interesting for purposes of comparison to compute the mean relative angular momentum of a major part of the atmosphere of the northern hemisphere. The average value of this quantity for the month of January 1946, for the portion of the atmosphere bounded by latitudes 35°N and 75°N, the surface and 7.5 km, is 2400×10^{29} g cm^2 sec^{-1}. The amount of angular momentum removed from within these boundaries, due to the surface frictional torque and the torque due to the mountains, during the thirty days under consideration was 7188×10^{29} g cm^2 sec^{-1}. It is clear that if no angular momentum were transported through the boundaries except at the surface and if the values of the torques were to remain constant in spite of the resultant decrease in the motion, the atmosphere within this region would cease to have any net relative angular momentum after approximately ten days. The necessity for a continual poleward transport to maintain the normally observed circulation is apparent.

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[11] The surface frictional force correlates with the mean zonal surface wind (around a latitude belt) to give a coefficient of +0.81 or better, on a daily basis.

[12] Angular momentum of solid earth = 52.4×10^{39} g cm^2 sec^{-1}. Relative angular momentum of the atmosphere = 12.82×10^{32} g cm^2 sec^{-1}. ω-angular momentum of the atmosphere = 1.021×10^{35} g cm^2 sec^{-1}.

given continual assistance as has been indicated, in part, above.

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TABLE OF SYMBOLS

R = mean radius of the earth
r = distance from the earth's axis
ϕ = latitude
λ = longitude
u = eastward component of the wind velocity (along a parallel of latitude)
v = northward component of wind velocity
g = acceleration of gravity
ρ = density
ω = angular speed of rotation of the earth
x = linear distance in eastward direction (along a parallel of latitude)
z = linear distance along the vertical
M = absolute angular momentum per unit mass
D_u = eastward component of frictional force per unit volume
p = pressure
t = time
$(\Delta z)_y$ = difference in contour height across a 10-degree interval northward
$(\Delta z)_x$ = difference in contour height across a 10-degree interval eastward
$(\Delta p)_y$ = difference in pressure across a 10-degree interval northward
$(\Delta p)_x$ = difference in pressure across a 10-degree interval eastward
κ = coefficient of skin friction (assumed 0.003)
$\overline{(\Delta p)}_y$ = difference in mean pressure between two given latitudes
$\overline{(\Delta z)}_y$ = difference in the mean contour height between two given latitudes.

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