

Note: These problems were taken from Ch. 6 of Holton (2004) with a subtle modification to the first problem.

1. Given the following expression for the geopotential field where Φ_0 is a function of p alone, c is a constant phase speed, k is a zonal wave number, and $p_0 = 1000$ hPa.

$$\Phi = \Phi_0(p_0) + cf_0\{y[\cos \pi p/p_0 - 1] + k^{-1} \sin k(x - ct)\}$$

- a) Obtain expressions for the corresponding geostrophic wind and relative vorticity fields.
- b) Obtain an expression for the relative vorticity advection.
- c) Use the quasi-geostrophic vorticity equation to obtain the horizontal divergence field consistent with this Φ field (assume that $\frac{\partial f}{\partial y} = 0$)
- d) Assuming that $\omega(p_0) = 0$, obtain an expression for $\omega(x, y, p, t)$ by integrating the continuity equation with respect to pressure.
- e) Sketch the geopotential fields at 750 hPa and 250 hPa. Indicate regions of maximum divergence and convergence, and cyclonic and anticyclonic vorticity advection.

2. For the geopotential distribution of Problem 1 obtain an alternative expression for ω by using the adiabatic form of the thermodynamic equation (equation 6.13b in Holton 2004). Assume that σ is a constant. For what value of k does this expression agree with that obtained in Problem 1

3. As an additional check on the results of Problems 1 and 2 use the approximate omega equation (eq 6.36 in Holton 2004) to obtain an expression for ω . Note that the three expressions for ω agree only for one value of k . Thus, the geopotential field $\Phi(x, y, p, t)$ of Problem 1 is consistent with quasi-geostrophic dynamics only for one value of the zonal wave number for a given value of σ and f_0 .