ATM 612 Homework 2: Due Monday, October 9

1) Using the diagnostic pressure equation,

$$\begin{aligned}\nabla \cdot (C_p \overline{\rho} \overline{\theta}_v \nabla \pi_{dn}) &= -\nabla \cdot (\overline{\rho} v \cdot \nabla v) \\ &= -2\overline{\rho} \left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial w}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial w}{\partial y} \right] \\ &\quad - \overline{\rho} \left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 - \frac{d^2 ln\overline{\rho}}{dz^2} w^2 \right]\end{aligned}$$

along with a mathematical representation of the simple flow types listed below, describe the pressure perturbation field that would be associated with:

a) a two-dimensional (XY) flow in solid-body rotation
b) a two-dimensional diverging (converging) flow
c) a two-dimensional shear flow

2) Can a supercell exist in 2 dimensions (i.e. in the XZ plane)? (Hint: Compare the 2D versus 3D forms of the vorticity and diagnostic pressure equations, and consider whether the primary dynamic processes needed for a supercell can exist in 2D)

3) Using the approximation of a Rankine type vortex along with the equation for a cyclostrophically-balanced pressure field,

$$\frac{v^2}{r} = \frac{1}{\rho_0}\frac{\partial p}{\partial r}$$

estimate the vertical acceleration that would be produced at 1.5 km AGL by a 10 km wide axisymmetric mesocyclone located at 3 km AGL, assuming that the mesocyclonic flow and associated pressure deficit decreases linearly to zero at the surface: for a cyclone with a 10, 20 and 40 ms-1 maximum tangential wind.

How much buoyancy (potential temperature excess) must a parcel have to equal this "dynamic" vertical pressure gradient forcing.

4) Given a 2D circular vortex of radius, R, and using the Circulation Theorem, calculate the average tangential velocity that would be observed at R, given an average vorticity within each vortex of .001 and .01 s-1 for vortex radii of 1 and 10 km.

Now, assuming no vorticity at radii greater than R, calculate the tangential velocity that would be induced by such vortices at radii of 20 and 40 km.