ATM 612 Homework 3: Due Monday, Nov. 20

1) a) Show that the vertical vorticity equation,

$$\frac{d(\zeta + f)}{dt} = \omega_H \cdot \nabla_H w + (\zeta + f)\frac{\partial w}{\partial z}$$

can be rewritten in the following "flux" form:

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial}{\partial x}\left[w\frac{\partial v}{\partial z} + u(f+\zeta)\right] - \frac{\partial}{\partial y}\left[-w\frac{\partial u}{\partial z} + v(f+\zeta)\right].$$

b) Given this flux form of the vertical vorticity equation, show that the rate of change of circulation about the northern half of an asymmetric convective system can be written as follows, assuming that w and ζ are negligible at far distances from the convective system and that a,b represent the western, eastern end points of the southern end of the circuit that passes through the convective system:

$$\frac{\partial C}{\partial t} = \iint_A \frac{\partial \zeta}{\partial t} = \int_a^b [-w\frac{\partial u}{\partial z} + v\zeta]dx + \oint f\mathbf{v} \cdot \hat{n}\, dl$$

2) Using the vertical vorticity equation,

$$\frac{d(\zeta + f)}{dt} = \omega_H \cdot \nabla_H w + (\zeta + f)\frac{\partial w}{\partial z}$$

along with the Boussinesq continuity equation and conveniently ignoring the tilting term, one can integrate the resulting equation along a parcel's path to gage the relative contributions from stretching of relative plus Coriolis rotation over various time scales. Using this derived integrated equation, plot the magnitude of vertical vorticity that a parcel would obtain in the allotted times for the following specified constraints:

a) between 0 and 60 min., given an initial relative vertical vorticity of 0, Coriolis forcing of .0001 s-1, and an average total magnitude of horizontal convergence along the parcel path of 10 ms-1 over 10 km and 20 ms-1 over 10 km.

b) Repeat the above calculation given an initial relative vertical vorticity of .0004 and Coriolis forcing of .0001 s-1.

c) between 0 and 240 min., given an initial relative vertical vorticity of 0, Coriolis forcing of .0001 s-1, and an average total magnitude of horizontal convergence along the parcel path of 10 ms-1 over 100 km.

3) Assuming hydrostatic conditions and ignoring contributions from water or ice, estimate the change in pressure that would be produced at 4 km AGL underneath a stratiform precipitation region within a mature, upshear-tilted convective system, for average warming in the stratiform region of 2 K and 6 K, respectively. For this calculation, assume that the warm stratiform layer extends from 4 km to 10 km AGL.

4) Using the circulation theorem, estimate the winds that would be induced between two 20 km wide, equal strength, counter-rotating Rankine vortices on a horizontal (XY) plane, considering maximum tangential winds of **a)** 20 ms-1 and **b)** 40 ms-1, for vortices spaced 60, 120, and 180 km apart. (HINT: Consider question 4 from previous homework)