

ATM 408/508 Review of Saturated vs. Unsaturated Concepts:

Darcy's Law (saturated conditions) $\bar{q} = -\bar{K} \cdot \bar{\nabla}h = -\bar{K} \cdot \nabla \left(z + \frac{p}{\rho_w g} \right)$ where $|\bar{K}| = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$

- The pressure head term is always positive.
- Elevation head and pressure head are usually similar magnitudes.
- Elevation head relies on an arbitrary datum (usually the lowest point of interest).
- Saturated hydraulic conductivity $|\bar{K}| = \frac{Cd^2\gamma}{\mu}$ is the product of the properties of the medium (Cd^2) and the properties of the fluid (γ/μ) and represented in 3 dimensions by a tensor of constants.
- However, it is often safe to assume that the properties of the medium and the fluid are relatively uniform in our area of interest, so our 3D tensor reduces to a single constant.
- This assumption is the *isotropic assumption* (isotropy means “same everywhere”).

We can approximate Darcy's Law with the isotropic assumption as $\bar{q} = -K\bar{\nabla}h$

Darcy-Buckingham Law (unsaturated conditions) $\bar{q} = -\bar{K}(\theta) \cdot \bar{\nabla}h(\theta) = -\bar{K}(\theta) \cdot \bar{\nabla}(z + \psi(\theta))$

- Unsaturated hydraulic conductivity and tension head are both functions of water content.
- Tension head is always negative to represent the forces of attraction between water and soil holding the water at tension in the mineral matrix.
- The water table is a convenient datum for elevation head.
- The unsaturated hydraulic conductivity tensor elements are functions of the water content ($K_{xx}(\theta)$, $K_{xy}(\theta)$, etc.) rather than constants.
- But we can still apply the isotropic assumption and use one function to represent the unsaturated hydraulic conductivity.
- We know from observation that water flow in the vadose zone is largely *downward*.
- While horizontal flow does occur, it is typically orders of magnitude smaller than vertical flow. (Think about why this would occur!)

We can approximate the Darcy-Buckingham Law as $q_z = -K(\theta) \frac{\partial h(\theta)}{\partial z} = -K(\theta) \left(1 + \frac{\partial \psi(\theta)}{\partial z} \right)$

After calculating q_z , we can use the following table to determine the direction of water flow:

	downward	upward
Unsaturated	+	-
Saturated	-	+

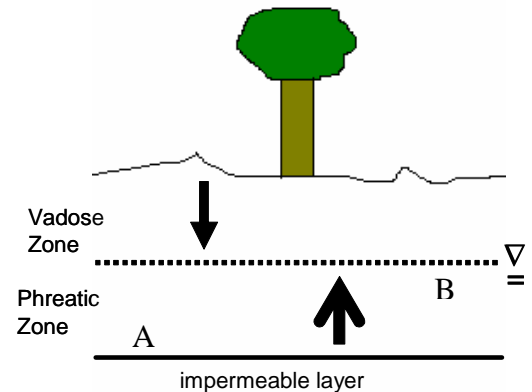
- We know from observations that water in the phreatic zone truly moves in 3D, and it is equally probable to find upward or downward flow. Wells and springs would not exist otherwise!
 - The direction of vertical flow where up is *positive* is intuitive.
- We must be very careful defining our datum and calculating the hydraulic gradient so that the sign of our z-result tells us whether our flow is upward or downward.
 - We can rely on Darcy's Law to steer us in the right direction, so to say.
- It is a mathematical fact that the way we've defined tension head being negative that Darcy-Buckingham usually yields a positive result in 95% of the vadose zone.
 - On the face of it, this is non-intuitive and leaves us wondering...
- Negative results do occur in the capillary fringe and in the root zone (both less than 5% of the vadose zone), but we'll leave those details for the unit on evapotranspiration.

There is a good physical picture you can keep in mind to help you think about flow in the sub-surface in both saturated and unsaturated conditions, and it is consistent with typical results from both Darcy and Darcy-Buckingham:

Let's rewrite the above table by referencing whether water is moving towards or away from the water table.

	Towards water table (+)	Away from water table (-)
Unsaturated	downward	upward
Saturated	upward	downward

The arrows on the figure demonstrate positive flow towards the water table.



A note on horizontal flow: Terms like “east” and “west” are meaningless in the sub-surface. In the sub-surface we want to know whether the water is flowing away from (+) or towards (-) a point we define as the origin ($x = 0, y = 0$). If we define A to be the origin so that $x_1 = 0$, and B is 100 m from A, then $x_2 = 100\text{m}$. If our resulting calculation of q_x is positive (negative), it implies that a component of \bar{q} is flowing parallel to the impermeable layer from A to B (B to A).

A hint on doing Darcy's Law problems: Choose the lowest point (A) to be your z-direction datum. Then, calculate the hydraulic head. Choose the point with the highest hydraulic head to be your origin. Then calculate your three q-components from Darcy's Law.

FAQ: Why does water move from pore to pore?

At the pore level, there are three fundamental forces we have to account for: 1. hydraulic head of incoming water (a function of the water pressure, weight per unit area); 2. the attraction due to bonding between water and minerals; and 3. the attraction of water to itself.

- The strongest force is force 2. Force 2 is a maximum at the surface of the mineral.

- Water directly bonded to the surface of the minerals is what we call hygroscopic water.
- With each layer of water molecules out from the mineral surfaces, the less they are attracted to the minerals (force 2), the more they are attracted to each other (force 3).
- In saturated conditions, there is plenty of water within and above every pore, so force 1 is large because the mass of the water is large, and the pressure exerted by that mass is also large.
 - More importantly, force 1 is larger than force 3 for the pore water.
- So the pressure exerted by water at higher hydraulic head will displace pore water at lower hydraulic head.
- In unsaturated conditions, even a small loss of water from a pore causes force 1 to decrease, and the difference between forces 1 and 3 becomes much smaller.
 - Infiltrating water will then seek to hydrogen bond with remaining pore water and to the soil matrix rather than go anywhere.
 - This is why $K(\theta)$ changes much faster with decreasing θ than tension head.
- Once the pore water is mostly gone, then force 2 is more noticeable, and tension head becomes more negative much quicker.
 - The less water in the pore, the stronger force 2 is compared to force 1.
- \bar{q} approaches zero when the unsaturated hydraulic conductivity approaches zero asymptotically. (See the slides on soil water characteristic curves).

If this is still unclear to you, try the following thought experiment with two sponges:

- Take a completely dry, thick sponge and hold it under a faucet. Turn on the water at a fairly low flow rate.
- Observe what happens at the top and bottom of sponge. Ignore the water dripping off of the sides.
- Note how long it takes for water to flow through the sponge.
- Take another sponge, and completely saturate it. Do not wring it.
- Hold the saturated sponge under the faucet and observe what happens.
- How long did it take the water to flow through the saturated sponge vs. the initially unsaturated sponge?