Basic Equations for Deep Convective Motions:

Momentum:

$$\frac{du}{dt} = -c_p\overline{\theta}_v(z)\frac{\partial\pi}{\partial x} + fv + D_u \ , \tag{1}$$

$$\frac{dv}{dt} = -c_p\overline{\theta}_v(z)\frac{\partial\pi}{\partial y} - fu + D_v \ , \tag{2}$$

$$\frac{dw}{dt} = -c_p\overline{\theta}_v(z)\frac{\partial\pi}{\partial z} + B + D_w \ , \tag{3}$$

$$B \equiv g\left[\frac{\theta'}{\overline{\theta}} + .61(q_v - \overline{q}_v) - q_c - q_r\right] \ , \tag{4}$$

B is the buoyancy, f is the Coriolis parameter, θ is the potential temperature, q_v, q_c, and q_r represent the water vapor, cloud water and rainwater mixing ratios, respectively, and π represents a non-dimensionalized form of the pressure (Exner function).

$$\pi \equiv \left(\frac{p}{p_0}\right)^{R_d/c_p} \ . \tag{5}$$

$$\theta \equiv T\left(\frac{p}{p_0}\right)^{-R_d/c_p} \ . \tag{6}$$

$$\theta_v \equiv \theta\,(1 + 0.61q_v) \tag{7}$$

Continuity:

$$\frac{\partial\rho}{dt} + \nabla\cdot\rho v = 0 \tag{8}$$

Thermodynamics:

$$c_p\frac{dT}{dt} - \frac{1}{\rho}\frac{dp}{dt} = Q,$$

$$\frac{d\theta}{dt} = \frac{\theta}{c_pT}Q \tag{9}$$

Vorticity Equations:

$$\frac{d\omega}{dt} = (\omega + fk) \cdot \nabla v + \nabla \times (Bk) \ , \tag{10}$$

Vertical Vorticity:

$$\frac{d\zeta}{dt} = \omega_H \cdot \nabla_H w + (\zeta + f)\frac{\partial w}{\partial z} \ , \tag{11}$$

Vertical Vorticity (flux form):

$$\frac{\partial \zeta}{\partial t} = -\frac{\partial}{\partial x}\left[w\frac{\partial v}{\partial z} + u(f+\zeta)\right] - \frac{\partial}{\partial y}\left[-w\frac{\partial u}{\partial z} + v(f+\zeta)\right]. \tag{12}$$

Horizontal Vorticity:

$$\rho_0 \frac{d}{dt}\left(\frac{\eta}{\rho_0}\right) = -\frac{\partial B}{\partial x} \ , \tag{13}$$

$$\frac{\partial \eta}{dt} + \nabla \cdot \eta v = -\frac{\partial B}{\partial x} \ , \tag{14}$$

Potential Vorticity:

$$\frac{d}{dt}\left(\frac{\omega \cdot \nabla \theta_e}{\rho}\right) = 0 \ , \tag{15}$$

Diagnostic Pressure Equation:

$$\nabla \cdot (c_p \overline{\rho}\overline{\theta}_v \nabla \pi) = -\nabla \cdot (\overline{\rho} v \cdot \nabla v) + \frac{\partial B}{\partial z} \ , \tag{16}$$

$$\begin{aligned} \nabla \cdot (c_p \overline{\rho}\overline{\theta}_v \nabla \pi_{dn}) &= -\nabla \cdot (\overline{\rho} v \cdot \nabla v) \\ &= -2\overline{\rho}\left[\frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial w}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial w}{\partial y}\right] \\ &\quad - \overline{\rho}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2 - \frac{d^2 ln\overline{\rho}}{dz^2} w^2\right] \end{aligned} . \tag{17}$$

$$\nabla \cdot \left(c_p \overline{\rho}\overline{\theta}_v \nabla \pi_B\right) = \frac{\partial B}{\partial z} \ . \tag{18}$$