ATM 316, Fall 2014
Hint for Problem Set 5, question 1 (a):

In class we derived an expression for the height h of the free surface of the water in a rotating tank:

$$h(r) - h(0) = \frac{\Omega^2 r^2}{2g}$$

where $h(r)$ is the height at a radial distance r from the center and $h(0)$ is the height at the center.

The problem here is that we don't know what $h(0)$ is after the tank starts rotating. All we know is that $h = z_0$ everywhere before it starts rotating. You need to express $h(0)$ in terms of known parameters: Ω, r_0, z_0, g. To do this, you need to use the fact that **the total volume of water in the tank does not change**.

You can work out the total volume of water in a cylindrical tank by dividing the tank up into lots of thin concentric cylindrical shells and adding them up. The volume of each thin shell would be

$$dV = 2\pi r\, h\, dr$$

since $2\pi r$ is the circumference of the circle of radius r, h is the height of the shell, and dr is the radial width of the thin shell.

So the total volume of water in the tank can be calculated by integrating from the center to the edge:

$$V = \int_0^{r_0} dV = \int_0^{r_0} 2\pi\, r\, h(r)\, dr$$