Problem Set 1
ATM 316: Dynamic Meteorology I
September 2, 2014
DUE Tuesday September 9 at beginning of class

1. a) The following table gives wind measurements at various pressure levels taken by a radiosonde at Norman, Oklahoma at 5/20/13 18Z, a couple of hours before the devastating Moore tornado.

Pressure (mb)	u (m/s)	v (m/s)
1000	0.0	5.0
850	8.6	12.3
700	15.5	15.5
500	24.2	14.0

Forecasters are interested in the mean wind of the sounding because it affects how fast storms move. Calculate the mean wind vector and mean wind speed from the sounding data.

b) A hodograph is a useful way of displaying wind information from a sounding. To construct one, plot the wind vectors in order of increasing altitude (decreasing pressure), all emanating from the origin. Next, in the same order, connect the heads (arrows) of the vectors. Construct a hodograph using the wind information from the table above.

c) On your hodograph, mark the vector difference between the wind at 500 mb minus the wind at 1000 mb. What is the magnitude of this vector, i.e. the vertical wind shear magnitude? Forecasters typically look for vertical wind shear magnitudes between the lower and middle troposphere in excess of 15 m/s for severe weather along with large amounts of instability. Knowing there is ample instability on this day, was the data alerting forecasters to the imminent possibility of severe weather?

2. a) Show that a field of *pure deformation* (an arbitrary combination of the two components of deformation) is *irrotational* (vorticity is zero) and *non-divergent* (divergence is zero).

b) Given the following wind velocity,

$$\boldsymbol{u} = \frac{1}{x+y}\left(-y\,\hat{\imath} + x\,\hat{\jmath} + 0\,\hat{k}\right)$$

i) Where is the wind non-divergent?
ii) Where is the wind irrotational?

3. The following identity is true for any vectors $\boldsymbol{u}, \boldsymbol{v}$:

$$\boldsymbol{v} \cdot (\boldsymbol{u} \times \boldsymbol{v}) = 0$$

a) Derive this identify mathematically.
b) Give a geometrical explanation for this identity.
c) Derive the closely related identity $\nabla \cdot (\nabla \times \boldsymbol{u}) = 0$
d) State the identity in (c) in words for the case where $\boldsymbol{u}$ is the wind field.

4. Answer question 1.5 from Martin, "Mid-Latitude Atmospheric Dynamics" (a pdf copy of Chapter 1 is available from the class web site). Include "before and after" sketches of the isotherms in your answers.

5. Suppose temperature varies in three dimensions as follows:

$$T = T_0 \exp\left(-\frac{z}{L_z}\right) + T_1\left[\left(\frac{x}{L_x}\right)^2 + 1 - \cos\left(\frac{y}{L_y}\right)\right]$$

where L_x, L_y, L_z are length scales (in units of meters) in the x, y and z directions, and T_0 = 270 K, and T_1 is some other constant.

a) What are the units of T_1?
b) What is the temperature at the origin?
c) Derive an expression for the gradient of the temperature field at every point. What are the units?
d) As we will see, the *temperature advection* is defined as the negative of the dot product of the wind velocity and the temperature gradient, $-\boldsymbol{u} \cdot \nabla T$. Is this a scalar or a vector quantity? What are its units?
e) Suppose the wind field is $u = u_1\left(\frac{z}{L_z}\right)\hat{\imath} + u_2\,\hat{\jmath} + 0\,\hat{k}$, with u_1= 5 m/s and u_2 = 3 m/s. Derive an expression for the temperature advection at any point.
f) Evaluate the temperature advection numerically at the point $(x, y, z) = (L_x, 0, L_z)$. Do you have enough information to answer this question? Make an assumption if necessary.