Problem Set 2
ATM 316: Dynamic Meteorology I
September 11, 2014
DUE Thursday September 18 at beginning of class

1. Calculate the acceleration due to gravity g at two locations: the surface, and at a height 30 km above the surface (somewhere in the stratosphere). Assume the Earth is a perfect sphere with radius a = 6.37 x 10^6 m and mass M = 5.97 x 10^{24} kg. Express your answer as a percentage difference:

$$\frac{g_{30\,km} - g_{surface}}{g_{surface}} \times 100\%$$

Based on your answer, do you think it's reasonable to assume that g is a constant for most meteorological purposes?

2. a) Use the ideal gas law $p = \rho\, R_d\, T$ (where R_d= 287 J kg^{-1} K^{-1} is the gas constant for dry air) to rewrite the pressure gradient force in terms of a temperature gradient, assuming the density is constant.

b) On a hot summer day in southern California, the temperature in downtown LA is 35°C and is 20°C in Santa Monica. The distance between downtown LA the Santa Monica is about 25 km. Calculate the magnitude of the pressure gradient force that results from this temperature difference. Make sure to include the units. Is the pressure gradient force pointing from Santa Monica to downtown LA or vice versa?

c) Hot summer days like this are often accompanied by a sea breeze, which blows inland. For this to occur, a pressure gradient force pointing from Santa Monica to downtown LA is needed to accelerate the wind inland. Is this consistent with your results from part b)? If not, the assumption of constant density in part a) is not valid. What must actually happen to the density of a parcel inland relative to one along the coast in order to give the correct direction for the pressure gradient force?

3. Suppose that we have a two-dimensional temperature field given by

$$T(x,y) = T_0 \cos\left(\frac{x}{L_x}\right)\cos\left(\frac{y}{L_y}\right)$$

a) Calculate the temperature gradient ∇T. Make sure to indicate whether it is a scalar or a vector.
b) Calculate the Laplacian $\nabla^2 T$. Again, is it a scalar or a vector?
c) Now suppose that this temperature field is subject to a diffusion process governed by the heat equation $\frac{\partial T}{\partial t} = D\nabla^2 T$. What units must the constant D have? (Assume that lengths, time and temperature are all measured in standard SI units).
d) Draw a sketch showing the direction of the heat flux in the area near the origin (0,0).
e) Will the temperature increase or decrease with time at the origin? What about at the point $\left(0, \frac{\pi L_y}{2}\right)$?