Problem Set 3
ATM 316: Dynamic Meteorology I
September 18, 2014
DUE Friday September 26 no later than 4:00 pm
(deliver to Prof. Rose's office ES 315 OR scan and submit by email)
NO LATE ASSIGNMENTS WILL BE ACCEPTED
Solutions will be posted on the class web page late Friday afternoon.

1. *This question will lead you through a useful application of the hypsometric equation.* Denver International Airport (DEN) is located in the Rocky Mountains at an elevation of 5414 feet. On the other hand, the elevation at Saint Louis (STL) on the Mississippi River is only 568 feet. The horizontal distance between these two stations is about 1200 km. Barometers measure air pressure at the surface at both stations. Call these observations the station pressures p_{sta}. Suppose that at some given time we observe $p_{sta} = 980$ mb at STL and $p_{sta} = 825$ mb at DEN. We also observe surface temperatures of 18ºC at STL and 10ºC at DEN.

The pressure gradient force (per unit mass) is given by $-\frac{1}{\rho}\nabla p$. We are frequently interested in knowing the magnitude and direction of just the horizontal component of this acceleration. As we have seen in class, we can estimate this as $\left|\frac{1}{\rho}\frac{\Delta p}{\Delta r}\right|$ where Δp is an observed pressure difference over a distance Δr, and the acceleration acts *from high to low pressure*.

- **a)** **Use the station pressure observations directly** to show that there is a pressure gradient force per unit mass of about 0.01 m s^{-2}, directed from STL to DEN. To make things simple, just assume that $\rho = 1.2$ kg m^{-3}. Show your work and be careful about units.
- **b)** Note that 0.01 m s^{-2} is a very large number for a horizontal acceleration. It's about 10x larger than we estimated in class for a moderate synoptic low pressure system, and about the same as Tomer estimated for Hurricane Sandy when it made landfall. **What is wrong with the estimate you just made, and why do think the value is so large?** (There was no hurricane present in the central US at the time these observations were made).
- **c)** To avoid these problems, station pressures are almost always converted to a fictitious "sea level pressure" – the pressure that would be measured at zero elevation if that were possible at that location. **Express in words** why the sea level pressure p_{slp} must always be larger than the station pressure p_{sta} for an atmosphere in hydrostatic balance, if the station elevation is above sea level.
- **d)** In class we derived the hypsometric equation

$$\Delta z = \frac{R_d \bar{T}}{g} \ln\left(\frac{p_1}{p_2}\right)$$

which gives the vertical thickness (for a hydrostatic atmosphere) of the atmosphere between two pressure levels (with level 2 above level 1 so that $p_1 > p_2$) in terms of the average temperature $\bar{T}$ of the layer. **Show mathematically** that

$$p_1 = p_2 \exp\left(\frac{\Delta z\, g}{R_d \bar{T}}\right)$$

(hint: recall that $\exp(\ln x) = x$ for any x)

e) We're going to use this expression to "correct" station pressures down to sea level. So we will take level 1 to be sea level (z=0) and level 2 to be the actual station elevation. We can therefore write $p_{slp} = p_{sta} \exp\left(\frac{z_{sta}\, g}{R_d \bar{T}}\right)$, where z_{sta} is the elevation above sea level. But what is $\bar{T}$ here? It's the average temperature of the slab of atmosphere extending from the surface at z_{sta} down to sea level. But of course there is no such atmosphere, so we have to make an assumption what temperature it would have. A reasonable assumption: $\bar{T}$ is the temperature at a location **halfway** between the station and sea level if the fictitious atmosphere had a standard constant lapse rate of $\frac{dT}{dz} = -6.5$ K / km (i.e. temperature decreases with altitude at a rate of 6.5 K / km). **Using this assumption, calculate $\bar{T}$ at STL and DEN.** Be careful about units.

f) Now **calculate the sea level pressure at STL and DEN.**

g) Now repeat your estimate of the magnitude and direction of the horizontal pressure gradient force per unit mass, this time using sea level pressures instead of station pressures. Comment on the difference between your new result and what you found in part (a).

2. *Same as Question 1.12 from the book by Martin – this will apply the concepts of Lagrangian and Eulerian time derivatives and require some vector calculations.*

A car is driving straight southward past a service station at 100 km / hour. A barometer in the car measures a decreasing pressure tendency of 50 Pa / 3 hours. The surface pressure decreases toward the southeast at 1 Pa / km. What is the pressure tendency (rate of change of pressure with time) at the service station? Make sure to indicate whether this is an increase or decrease.