Problem Set 4
ATM 316: Dynamic Meteorology I
October 7, 2014
DUE Tuesday October 14 at beginning of class

1. Prove the vector identity

$$\nabla \cdot (A\,\vec{u}) = A(\nabla \cdot \vec{u}) + \vec{u} \cdot \nabla A$$

for any scalar field $A(x, y, z)$ and vector field $\vec{u}(x, y, z)$.

2. Suppose the horizontal wind field is

$$\vec{u} = \frac{U}{L}[(-ax - y)\hat{\imath} + (x - ay)\hat{\jmath}]$$

where $U > 0$ is a constant in units of m/s, $L > 0$ is a constant in units of m, and a is small positive constant number (roughly about 0.1).

a) Sketch the wind field (with arrows) in the vicinity of the origin. (Make your sketch nice and large because you'll be adding more to it later…)
b) Calculate the horizontal divergence and vorticity of the wind field. Make sure that your answers are consistent with what you sketched. What are units of divergence and vorticity?
c) Now assume that this flow occurs near the surface. Would you expect to find ascending or descending motion in the atmosphere above this surface feature? Explain you answer in words.
d) Now calculate the vertical motion at a height 1 km above the surface. You may assume that $U = 10$ m/s, $L = 1000$ km and $a = 0.1$. Make sure to state clearly any other assumptions you are making.
e) Now suppose that the specific humidity field (the amount of water vapor in the air) is given by

$$q = q_0 - q_1 y$$

with q_0, q_1 both positive constants. Sketch contours of constant q on the same sketch you drew in part (a) (use a different color). Indicate where the moister and drier air is located.
f) Suppose that specific humidity is conserved (remains constant) in each air parcel. Where do you expect that the specific humidity will increase with time, and where will it decrease? Sketch these regions (and the boundary between them) on your graph. Explain your reasoning. (For this question you may neglect the vertical motion, i.e. assume w = 0)