Problem Set 7
ATM 316: Dynamic Meteorology I
November 25, 2014
DUE Thursday December 4 at beginning of class

1. a) We will study balanced forces and its relationship to the Rossby number using the rotating tank. We rotate a bucket filled with water with a hole in the middle. Water begins to drain out of the bucket, but the water spirals in cyclonically. Due to conservation of angular momentum, the azimuthal velocity of the spiraling water increases toward the center of the tank (see Marshall and Plumb, section 6.6.1). The azimuthal velocity at radial distance r from the center, measured relative to the rotating tank, is:

$$v_\theta(r) = \Omega \frac{({r_0}^2 - r^2)}{r}$$

where r_0 is the outer radius of the tank, and Ω is the rotation rate of the tank.

Approximate the Rossby number at three different radii: near the outer edge of the bucket, about halfway to the center, and near the center. Note that the Coriolis parameter for the rotating tank is $f = 2\Omega$.

b) Based on your Rossby numbers, what balanced flow (geostrophic, gradient, cyclostrophic) would be appropriate at each of the radii? What does this imply about the predominate balance of forces at each radii?

c) Consider the following winds in a strong hurricane at 20ºN:

Radius from Hurricane Center (km)	Wind speed (m/s)
20	50
500	25
1000	5

Calculate the Rossby number at each of the radii for this hurricane.

d) Relate your answers from part a) and b) to the hurricane, i.e. how does the balanced flow change as an air parcel spirals toward the center of a hurricane?

2. Another hurricane question! [Martin, question 4.2]
An Atlantic hurricane is located over the Carribbean Sea at latitude 26ºN. Balanced wind speeds of 60 m s^{-1} are found, at 950 hPa, at a station 200 km from the center of the storm. The 950 hPa geopotential height at the station is 367 m.

a) What is the sea level pressure at the eye of the hurricane? Show your work.

b) Calculate the ageostrophic wind speed at the station.

3. In this question we will consider the three-dimensional structure of the wind field in pressure coordinates, and the relationship between ageostrophic winds and vertical motion.

a) Prove that the geostrophic wind vector is non-divergent, so long as we consider the Coriolis parameter f to be constant.

b) Vertical motion in pressure coordinates is given by ω, measured in Pascals / second and defined as

$$\omega = \frac{Dp}{Dt}$$

(the rate of change of pressure following an air parcel). Is ω positive or negative for upward motion? Explain your answer.

c) Read section 3.1.2 from Holton and Hakim, which derives the continuity equation (conservation of mass) in pressure coordinates. Write down two forms of the continuity equation: in height coordinates, and in pressure coordinates. Comment on which form is mathematically simpler.

d) If the horizontal winds on an isobaric surface are convergent near the surface, do you expect upward or downward motion aloft? *[Recall that making this inference in height coordinates required making approximations such an incompressible atmosphere – in pressure coordinates, such assumption is required]*.

e) Which part of the total horizontal wind field determines the vertical motion: the geostrophic or ageostrophic part?

4. [Holton and Hakim problem 3.2]: The actual wind is directed 30º to the right of the geostrophic wind. If the geostrophic wind is 20 m s^{-1}, what is the rate of change of the wind speed? Let $f = 10^{-4}$ s^{-1}.

5. [Holton and Hakim problem 3.4]: Calculate the geostrophic wind speed on an isobaric surface for a geopotential height gradient of 100 m per 1000 km, and compare with all possible gradient wind speeds for the same geopotential height gradient and a radius of curvature of ±500 km. Let $f = 10^{-4}$ s^{-1}.

6. [Holton and Hakim problem 3.14]: Work out a gradient wind classification scheme equivalent to Table 3.1 for the Southern Hemisphere ($f < 0$) case.

7. a) Explain physically (with reference to horizontal forces) why the balanced wind speed is sub-geostrophic in a trough and super-geostrophic in a ridge.

b) Using the $\hat{n}$ component of the equation of motion, show that the relationship described in (a) is independent of hemisphere.