ATM 500: Atmospheric Dynamics
Homework 2
Due Tuesday September 22 2015

1. The *hypsometric equation* states that the vertical distance Δz between two surfaces of constant pressure is proportional to temperature of the layer:

$$\Delta z = \frac{R_d \bar{T}}{g} \ln \left(\frac{p_1}{p_2} \right)$$

Prove this result for an ideal gas in hydrostatic balance (where p_1, p_2 are the pressure levels with $p_1 > p_2$). As part of your derivation, state clearly how the average temperature of the layer $\bar{T}$ is defined.

2. *This question will lead you through a meteorological application of hydrostatic balance*

Denver International Airport (DEN) is located in the Rocky Mountains at an elevation of 5414 feet. The elevation at Saint Louis (STL) on the Mississippi River is only 568 feet. The horizontal distance between these two stations is about 1200 km. Barometers measure air pressure at the surface at both stations. Call these observations the station pressures p_{sta}. Suppose that at some given time we observe $p_{sta} = 980$ mb at STL and$p_{sta} = 825$ mb at DEN. We also observe surface temperatures of 18°C at STL and 10°C at DEN.

The pressure gradient force (per unit mass) is given by $-\frac{1}{\rho}\nabla p$ (always acting from high to low pressure). We are frequently interested in knowing the magnitude and direction of just the *horizontal component* of this acceleration.

a. One might naively try to estimate the magnitude of the horizontal pressure gradient as $\Delta p / \Delta r$, where Δp is an observed pressure difference over a distance Δr. Using the station pressure observations directly, show that there is a pressure gradient force per unit mass of about 0.01 m s^{-2} directed from STL to DEN.

b. Note that 0.01 m s^{-2} is a very large number for a horizontal acceleration. It is about 10× larger than accelerations experienced in a moderate synoptic cyclone, and similar to accelerations in a strong hurricane. What is the logical flaw in the estimate you made in part (a), and why do you think the value is so large? (There was no hurricane present in the central US at the time these observations were made).

c. To avoid these problems, station pressures are almost always converted to a fictitious "sea level pressure" the pressure that would be measured at zero elevation if that were possible at that location. Express in words why the sea level pressure p_{slp} must always be larger than the station pressure p_{sta} for an atmosphere in hydrostatic balance, if the station elevation is above sea level.

d. Now use the hypsometric equation from question 1 (which is always valid for a hydrostatic ideal gas layer) to "correct" the station pressures down to sea level. Specifically, calculate the sea level pressure p_{slp} at STL and DEN. Describe clearly any assumptions you need to make (particularly about temperature).

e. Now repeat your estimate of the magnitude and direction of the horizontal pressure gradient force per unit mass, this time using sea level pressures instead of station pressures. Comment on the difference between your new result and what you found in part (a).

3. *Based on question 1.5 in Vallis*

   What amplitude of sound wave is required for the nonlinear terms in the momentum equation to become important? Is this achieved at a rock concert (120 dB), or near a jet aircraft that is taking off (160 dB)?

   To answer this, you may want to read section 1.11 on scaling the equations of motion. Here, choose velocity and length scales consistent with the dispersion relation for sound waves. The decibel unit (dB) is defined on page 37 of the text.

4. *Based on question 1.9 in Vallis*

   a. Suppose that a sealed, insulated container consists of two compartments, and that one of them is filled with an ideal gas and the other is a vacuum. The partition separating the compartments is removed. How does the temperature of the gas change? (Answer: it stays the same. Explain.) Obtain an expression for the final potential temperature, in terms of the initial temperature of the gas and the volumes of the two compartments.

   b. A dry parcel that is ascending adiabatically through the atmosphere will generally cool as it moves to lower pressure and expands, and its potential temperature stays the same. How can this be consistent with your answer to part (a)?

5. *Based on question 1.20 in Vallis*

   The density of seawater depends non-linearly on temperature, salinity and pressure. However it is often useful to approximate with a linear equation of state. Here we will assume that

   $$\rho = \rho_0 \left(1 - \beta_T \left(T - T_0\right)\right)$$

   where ρ_0, T_0 are reference density and temperature, and β_T is known as the coefficient of thermal expansion. Values of β_T are determined empirically, and depend on choice of reference temperature. Here we will assume a constant value $\beta_T = 10^{-4}$ K^{-1} appropriate for the cool abyssal ocean (β_T is larger for warmer water).

   Assume that the ocean is a liquid sitting in a straight-sided container with $z = 0$ at the sea floor and $z = H$ at the free surface. Suppose that the temperature of the ocean rises uniformly by 4 K under global warming. Calculate the rise in sea level due to thermal expansion.

   Speculate on the validity of your estimate. What other factors might you take into account for a more realistic estimate of sea level rise?