ATM 500: Atmospheric Dynamics
Homework 5
Due Thursday October 15 2015

Corrected Friday October 9

1. *Question 2.14 from Vallis*

   Consider the Boussinesq equations including *viscous* and *diabatic* effects:

$$\frac{D\vec{v}}{Dt} = -\nabla\phi + \hat{k}b + \nu\nabla^2\vec{v}$$
$$\nabla \cdot \vec{v} = 0$$
$$\frac{Db}{Dt} = \dot{b} + \kappa\nabla^2 b$$

   where ν is the kinematic viscosity, κ is a diffusivity for buoyancy, and $\dot{b}$ is an external source of buoyancy (e.g. heating). (We are ignoring rotation, since we know by now that it does not influence the *energetics* of the flow).

   a. Obtain an energy equation similar to eq. (2.112) in the text, but now with the terms on the right-hand side that represent viscous and diabatic effects. Over a closed volume, show that the dissipation of kinetic energy (i.e. changes associated with viscosity) is balanced by a buoyancy source.

   b. Also show that, in a statistically steady state, the heating must occur at a lower level than the cooling if a kinetic-energy dissipating circulation is to be maintained.

2. *Question 2.19 from Vallis* Consider a dry, hydrostatic, ideal-gas atmosphere whose lapse rate is one of constant potential temperature. What is its vertical extent? That is, at what height does the density vanish? Is this a problem for the anelastic approximation discussed in the text?

3. Consider a layer of fluid between two solid boundary at $z = 0$ and $z = H$. Suppose that the flow is horizontally convergent (i.e. $\nabla_z \cdot \vec{u} < 0$) in some thin layer near the lower boundary, and horizontally divergent in a thin layer near the upper boundary. Assume that the horizontal flow is non-divergent elsewhere. What can you infer from the continuity equation about the vertical structure of vertical motion $w(z)$?

   a. Assume a Boussinesq fluid. Be as quantitative as you can, and draw a sketch of $w(z)$.

   b. Now calculate and sketch $w(z)$ for an anelastic fluid with an isothermal ideal gas reference profile.

   c. Give a physical explanation (in words) for the differences between your two sketches.

   d. How would your sketch in part (b) differ if you used an isentropic (constant potential temperature) profile as in Question 2 instead of an isothermal profile? Here a qualitative answer is sufficient.

4. *The dancing student problem*

   Suppose that the number of students taking ATM 500 is so large, and the size of each student so small, that they can be treated as a continuum. Each student takes up a certain infinitesimal volume in the classroom. Suppose the mass of each student is constant.

   Since the UAlbany administration was not willing to provide infinitesimally small chairs for the students, they move freely about the classroom during the lecture. Also, some students arrive late for the lecture and others leave early.

   ATM 500 is a very enjoyable class for these students, so rather than just simply wander around the classroom, the students are dancing during the lecture. The volume occupied by each student is a function of how wildly they are dancing, since neighboring students don't want to get whacked by flailing arms and legs.

   A long time ago, smart people figured out the "Law of Conservation of Students", which states that students cannot be created or destroyed.[1]

   a. Write a differential equation that expresses the physical principle of conservation of students in the framework of the fixed classroom.

   b. Students dance harder (wilder) when exposed to candy. There is a source of candy located somewhere in the classroom. Candy causes local variations in dance amplitude from a background state that we can assume is uniform throughout the classroom.

      Consider the case where the lecture is scheduled for the mid-morning, when the energy level of the students is naturally quite high. In this case, the candy-induced dancing perturbation is very small compared to the mean dancing amplitude.

      Using a scaling argument, show that the law of conservation of students reduces to the statement that the flow of students is non-divergent. How might this change if the lecture were rescheduled to late afternoon, when the students tend to be very tired?

   bonus Suppose that parcels of students exert forces on neighboring parcels proportional to their dancing amplitude (more bumping gives a stronger force). Further suppose that the students are blind and mute, and communicate with their neighbors only through dance-related interactions. Construct an argument (consistent with what you have already shown) the candy should not affect the motion of students far from the candy source under the "morning lecture regime", whereas it might under the "afternoon lecture regime".

5. Suggest a good question for the mid-term exam (Oct. 22). You don't need to provide an answer here. The best questions just might show up on the exam.

[1] Later, some even smarter people realized students do occasionally graduate, and so a correction factor needs to be applied to the conservation law, but the correction factor is vanishingly small except when students are moving VERY quickly through the school system, so we can safely ignore it.