ATM 500: Atmospheric Dynamics
Homework 7
Due Thursday November 12 2015

1. Here you will use the Ekman layer concept to make some inferences about how the ocean is set in motion by surface wind stress.

   The observed time-average surface winds $\vec{u_s}$ are westerly in the mid-latitudes, easterly in the tropics (the trade winds), and easterly near the poles. Let's represent this in a simple Cartesian domain with

   $$\vec{u_s} = U \cos\left(\frac{\pi y}{L}\right)\hat{i}$$

   where U is a constant and the domain extends from $y = -L$ in the south to $y = +L$ in the north. Assume that the winds do not vary in the east-west direction, and that we are in the northern hemisphere. Also assume that the stress at the surface is given by

   $$\vec{\tau_s} = C\vec{u_s}$$

   where C is a positive constant.

   a. If $\vec{u_s}$ is given in units of meters per second, what must be the units of the constant C?

   b. Calculate the mass transport in the oceanic Ekman layer.

   c. Calculate the Ekman pumping w_{Ek} (the stress-induced vertical velocity at the bottom of the oceanic Ekman layer). Where (i.e. over what range of y) is the Ekman pumping upward and where is it downward? If you need to make any assumptions, make sure you state them clearly.

   d. Make a sketch that shows the wind stress, the direction of the Ekman mass transport and the direction of the Ekman pumping as functions of y. Clearly note the boundaries (if any) between the upward and downward Ekman pumping.

   e. *Bonus question, not required.* Treating the ocean as a layer of shallow water with constant density and thinking about conservation of potential vorticity, how might the Ekman pumping you described above affect the ocean velocities (currents) below the Ekman layer?

2. *Based on question 3.8 in Vallis*

   For the following questions, treat the fluid as a single layer of shallow water with constant density, and use the conservation of potential vorticity.

   a. A cylindrical column of air at 30° latitude with radius 100 km expands horizontally to twice its original radius. Assume the volume of air is conserved. If the air is initially at rest, what is the relative vorticity of the cylinder after the expansion? Draw a sketch of the resulting horizontal velocity field.

   b. An air column at 60°N with zero relative vorticity ($\zeta = 0$) reaches from the surface to the tropopause, which we assume is a rigid lid, at 10 km. The air column moves zonally onto an area of elevated topography 2.5 km high (this might represent the Tibetan Plateau, for example). What is its relative vorticity? Suppose it then moves southwards (with no change in fluid depth) to 30°N. What is its relative vorticity?