# ATM 500: Atmospheric Dynamics
# Homework 11
**Due Tuesday December 6 2016**

1. *Holton and Hakim, problem 4.1*

   What is the circulation about a square of 1000 km on a side for an easterly (i.e. westward-flowing) wind that decreases in magnitude toward the north at a rate of 10 m $s^{-1}$ per 500 km? What is the mean relative vorticity in the square?

2. a. On an $f$-plane, show that the geostrophic velocity field on surfaces of constant pressure can be represented by a scalar streamfunction $\psi$ with

   $$u = -\frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \psi}{\partial x}$$

   and that the streamfunction is directly related to the geopotential $\Phi$ through

   $$\psi = \frac{\Phi}{f}$$

   b. Show that this relationship is not exact when $f$ is allowed to vary.

   c. Show that relative vorticity $\zeta$ of the geostrophic wind field can be written

   $$\zeta = \nabla^2 \psi$$

   d. Suppose we have a geostrophic flow on some pressure surface in the northern hemisphere described by a streamfunction

   $$\psi(x, y) = -\psi_0 \exp\left(-\frac{x^2 + y^2}{L^2}\right)$$

   with $\psi_0$ some positive constant and $L$ is a constant in length units.

   Draw a sketch of contours of $\psi$ and the associated geostrophic wind vectors. Make sure your sketch clearly indicates the horizontal scales in terms of $L$.

   e. Calculate the relative vorticity $\zeta(x, y)$. Is is positive or negative near the center of the feature?

   f. Draw a sketch of the resulting $\zeta$ field.

   *You might choose to plot this with a computer, which is fine. Just be sure to be clear about any assumptions you are making, e.g. the numerical value of $L$. You might also note that, since the field is isotropic (depending only on $x^2 + y^2$), you can treat this as a 1D field and just plot as a function of a single variable.*

   g. Comment on the different spatial scales of your sketches, and what that might imply about the information content of a map of geopotential height versus a map of relative vorticity.

3. An approximate vorticity equation for mid-latitude synoptic scale motion is

$$\frac{D}{Dt}\left(\zeta + f\right) = -\left(\zeta + f\right)\delta$$

where $\delta = \nabla \cdot \vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$ is the horizontal divergence. (This follows from scale analysis of the full vorticity equation, e.g. Holton and Hakim, section 4.3.3, which shows that the divergence term is a more important source of vorticity than either the tilting term or the solenoidal term for such scales).

   a. Suppose that $\zeta = 0$ initially, and ignore changes in latitude. Which one will generate *cyclonic* relative vorticity: horizontal *convergence* or horizontal *divergence*? Conversely, which one will generate *anticyclonic* relative vorticity? Does your answer depend on which hemisphere we are in? Draw sketches to illustrate your argument.

   b. Suppose that $\delta$ is constant in time. Does the rate of change of vorticity get larger or smaller with time? What happens when the magnitude of the relative vorticity approaches the magnitude of the planetary vorticity? Answer separately for the cyclone and anticyclone. (Again, suppose that there are no changes in latitude).

   c. Discuss the potential significance of your findings for the maximum intensity of cyclones versus anticyclones.

4. How should the absolute vorticity of an air parcel change if its static stability increases? Why?