ATM 500: Atmospheric Dynamics
Homework 2

Due Thursday September 14 2017

1. The *hypsometric equation* states that the vertical distance Δz between two surfaces of constant pressure is proportional to the average temperature of the layer:

$$\Delta z = \frac{R_d \bar{T}}{g} \ln \left(\frac{p_1}{p_2} \right)$$

Prove this result for an ideal gas in hydrostatic balance (where p_1, p_2 are the pressure levels with $p_1 > p_2$). As part of your derivation, state clearly how the average temperature $\bar{T}$ is defined when the temperature is not constant between the pressure surfaces.

2. a. Barometric pressure is measured continuously at surface-based stations all around the world, but maps of *surface pressure* (as directly measured at stations) are rarely plotted. Find one such map for the continental US. One possible source for current data is the High Resolution Rapid Refresh (HRRR) numerical weather prediction model. Also find a map of *sea level pressure* for (at least approximately) the same time as your surface pressure map. [Make sure that you cite your data sources.] Comment on why the two maps look so different! What are the dominant features in the surface pressure map? Why do you think this is?

   b. Find the surface pressures (*not* sea level pressures) at Denver, CO and at Springfield, MO. The raw stations observations are surprisingly hard to find! It is possible to obtain surface pressures from the raw sounding data reported by the upper-air sounding stations DNR and SGF. Alternatively you could just interpolate from your surface pressure map. Again, make sure you cite your data sources.

   The horizontal distance between these two locations is about 1060 km. Calculate the component of the *pressure gradient force* acting along a line between the surface at Denver and the surface at Springfield. What is its magnitude and direction? (Recall that we usually express forces per unit mass in $\mathrm{m\,s^{-2}}$). Clearly state any assumptions you need to make.

   c. "Sea level pressure" is a fictitious pressure that would be measured at zero elevation if that were possible at that location. Express in words why the sea level pressure must always be larger than the measured surface pressure for an atmosphere in hydrostatic balance, if the station elevation is above sea level.

   d. Now use the hypsometric equation from question 1 (which is always valid for a hydrostatic ideal gas layer) to "correct" the surface pressures at Denver and Springfield down to sea level (i.e. calculate the sea level pressure at those two locations). Describe clearly any assumptions you need to make. Do your values agree (at least approximately) with what is plotted on your map of sea level pressure?

e. Repeat part (b) but now for the *horizontal* component of the pressure gradient force at sea level. Express your result from part (b) as a vector sum of a horizontal component and a vertical component. One of the components is probably *much* larger than the other. Why do you think this is?

3. *Based on question 1.20 in Vallis*

The density of seawater depends non-linearly on temperature, salinity and pressure. However it is often useful to approximate with a linear equation of state. Here we will assume that

$$\rho = \rho_0 \left(1 - \beta_T \left(T - T_0\right)\right)$$

where ρ_0, T_0 are reference density and temperature, and β_T is known as the coefficient of thermal expansion. Values of β_T are determined empirically, and depend on choice of reference temperature. Here we will assume a constant value $\beta_T = 10^{-4}\,\mathrm{K}^{-1}$ appropriate for the cool abyssal ocean (β_T is larger for warmer water).

Assume that the ocean is a liquid sitting in a straight-sided container with $z = 0$ at the sea floor and $z = H$ at the free surface. Suppose that the temperature of the ocean rises uniformly by 4 K under global warming. Calculate the rise in sea level due to thermal expansion.

Speculate on the validity of your estimate. What other factors might you take into account for a more realistic estimate of sea level rise?