ATM 500: Atmospheric Dynamics
Homework 7

Due Thursday November 9 2017

1. In class we did a linear wave analysis for small perturbations in the x, z plane away from a state of rest ($\vec{v_0} = 0$) for the non-rotating simple Boussinesq system.

   a. For these waves, show that the phase speed in the *vertical* direction ($c^z = \omega/m$) is in the opposite direction of the vertical component of the group velocity, $c_g^z = \partial\omega/\partial m$. (This means that energy must travel downward wherever peaks and troughs are propagating upward and vice-versa). *Yes, we already covered this in lecture briefly, but please give a detailed answer and explain your reasoning clearly.*

   b. Consider now small perturbations away from a state with a constant background velocity in the x direction, $\vec{v_0} = U\hat{i}$ (where U is a constant). You may still assume that the motion is a function of x and z only as we did above. Show that the dispersion relation for these waves is

$$\omega = Uk \pm \frac{Nk}{\sqrt{k^2 + m^2}}$$

   c. How does the presence of a constant background velocity U change your answer in part (a)?

2. Here you will use the Ekman layer concept to make some inferences about how the ocean is set in motion by surface wind stress.

   The observed time-average surface winds $\vec{u_s}$ are westerly in the mid-latitudes, easterly in the tropics (the trade winds), and easterly near the poles. Let's represent this in a simple Cartesian domain with

$$\vec{u_s} = U \cos\left(\frac{\pi y}{L}\right)\hat{i}$$

   where U is a constant and the domain extends from $y = -L$ in the south to $y = +L$ in the north. Assume that the winds do not vary in the east-west direction, and that we are in the northern hemisphere. Also assume that the stress at the surface is given by

$$\vec{\tau_s} = C\vec{u_s}$$

   where C is a positive constant.

   a. If $\vec{u_s}$ is given in units of meters per second, what must be the units of the constant C?

   b. Calculate the mass transport in the oceanic Ekman layer.

   c. Calculate the Ekman pumping w_{Ek} (the stress-induced vertical velocity at the bottom of the oceanic Ekman layer). Where (i.e. over what range of y) is the Ekman pumping upward and where is it downward? If you need to make any assumptions, make sure you state them clearly.

d. Make a sketch that shows the wind stress, the direction of the Ekman mass transport and the direction of the Ekman pumping as functions of y. Clearly note the boundaries (if any) between the upward and downward Ekman pumping.