ATM 500: Atmospheric Dynamics
Homework 8
Due Thursday November 16 2017

1. *Based on question 3.8 in Vallis*

   For the following questions, treat the fluid as a single layer of shallow water with constant density, and use the conservation of potential vorticity.

   a. A cylindrical column of air at 30° latitude with radius 100 km expands horizontally to twice its original radius. Assume the volume of air is conserved. If the air is initially at rest, what is the relative vorticity of the cylinder after the expansion? Draw a sketch of the resulting horizontal velocity field.

   b. An air column at 60°N with zero relative vorticity ($\zeta = 0$) reaches from the surface to the tropopause, which we assume is a rigid lid, at 10 km. The air column moves zonally onto an area of elevated topography 2.5 km high (this might represent the Tibetan Plateau, for example). What is its relative vorticity? Suppose it then moves southwards (with no change in fluid depth) to 30°N. What is its relative vorticity?

2. Read section 3.7.3 in Vallis. The *Kelvin wave* is a special kind of gravity wave that exist in the presence of a lateral boundary. The wave is in geostrophic balance in the direction perpendicular to the boundary, but unbalanced (and propagating at phase speed $\sqrt{gH}$) in the direction along the boundary. (These waves are particularly important for our understanding of tropical phenomena such as El Niño).

   Why did we not find the Kelvin wave when we did our standard wave analysis of the rotating shallow water equations in class?

3. Starting from the conservation of potential vorticity $Q = (f + \zeta)/h$ for parcels in the shallow water system, derive the linear result

$$\frac{\partial q}{dt} = 0, \qquad q = \zeta' - f_0 \frac{\eta'}{H}$$

   for small perturbations of the height and velocity field on an f-plane with a flat bottom.