ATM 500: Atmospheric Dynamics
Homework 1
Due Thursday September 6 2018

Some of the questions here come from the book by Martin. Although you will find solutions to some problems listed at the end of the chapter, don't trust them! The book is unfortunately riddled with errors. Just make sure that your own answer makes sense.

1. Suppose temperature varies in three dimensions as follows:

$$T = T_0 \exp\left(-\frac{z}{L_z}\right) + T_1 \left(\left(\frac{x}{L_x}\right)^2 + 1 - \cos\left(\frac{y}{L_y}\right)\right)$$

where L_x, L_y, L_z are length scales (in units of meters) in the x, y, z directions, and $T_0 = 270$ K, and T_1 is some other constant.

   (a) What are the units of T_1?

   (b) What is the temperature at the origin?

   (c) Derive an expression for the *temperature gradient* at every point. What are the units? Is it scalar or vector?

   (d) Suppose the wind field is

$$\vec{u} = u_1 \frac{z}{L_z}\ \hat{i} + u_2\ \hat{j} + 0\ \hat{k}$$

   with $u_1 = 5$ m s^{-1} and $u_2 = 3$ m s^{-1}. Derive an expression for the *temperature advection* at any point. Is this a scalar or a vector quantity? What are its units?

   (e) Evaluate the temperature advection numerically at the point $(x, y, z) = (L_x, 0, L_z)$. If you do not have enough information to answer this question, make a clearly-stated assumption.

2. *Based on Question 1.2 from the book by Martin. We will be using all of these identities regularly in our derivations of fluid properties.*

   Prove the following vector identities, letting $\vec{v} = u\hat{i} + v\hat{j} + w\hat{k}$:

   (a) $\nabla \cdot (\nabla \times \vec{v}) = 0$

   (b) $(\vec{v} \cdot \nabla)\,\vec{v} = \frac{1}{2}\nabla\,(\vec{v} \cdot \vec{v}) - \vec{v} \times (\nabla \times \vec{v})$

   (c) $\nabla \cdot (f\vec{v}) = f\,(\nabla \cdot \vec{v}) + \vec{v} \cdot \nabla f$

3. Newton's law of universal gravitation states that any two elements of mass in the universe attract each other with a force proportional to both their masses and inversely proportional to the square of the distance between their centers of mass.

   Mathematically, we write the gravitational force of mass M on mass m as

$$\vec{F}_g = -\frac{G\ M\ m}{|\vec{r}|^2}\ \frac{\vec{r}}{|\vec{r}|}$$

   where $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2} is a universal constant, and $\vec{r}$ is a vector distance from the center of M to the center of m.

   Assume that Earth is a perfect sphere with mass $M = 5.97 \times 10^{24}$ kg and radius 6.37×10^6 m. Calculate g, the acceleration due to gravity on a small mass m at two different locations: the surface of the Earth, and at a height 30 km above the surface (somewhere in the stratosphere). Express your answer as a percentage difference,

$$\frac{g_{30km} - g_{surface}}{g_{surface}} \times 100\%$$

   Based on your answer, do you think it's reasonable to assume that g is a constant for most meteorological purposes?

4. *Same as Question 1.12 from the book by Martin. This will apply the concepts of Lagrangian and Eulerian time derivatives and require some vector calculations.*

   A car is driving straight southward past a service station at 100 km / hour. A barometer in the car measures a decreasing pressure tendency of 50 Pa / 3 hours. The surface pressure decreases toward the southeast at 1 Pa / km. What is the pressure tendency (rate of change of pressure with time) at the service station? Make sure to indicate whether this is an increase or decrease.