ATM 500: Atmospheric Dynamics
Homework 5
Due Thursday October 11 2018

1. a. A kicker misses a game-winning field goal and blames the Coriolis force. He kicks the football a horizontal distance of 50 m in 5 s. The football field is at latitude 45°N. If he misses the right upright goalpost by 0.1 m, is his claim valid?

   b. At what speed would the kicker have to kick the football in order to actually miss the upright by 0.1 m due to the Coriolis force?

   To answer these questions, suppose the Coriolis force is the only force acting on the ball.

2. Consider again motion under Coriolis force alone (e.g. a ball on a rotating parabolic turntable, or a fluid parcel on a quasi-horizontal geopotential surface in the absence of pressure gradients). Show that a parcel can move in a steady circular path, completing exactly 2 circles for every 1 rotation of the system. State clearly any assumptions you are making.

   (This is a kind of motion known as inertial circles, which we can easily observe on the turntable, and is sometimes observed in the ocean with freely drifting buoys.)

3. In Homework 3 you looked at the *total energy* budget (kinetic plus internal plus potential) per unit volume for an inviscid, adiabatic, compressible fluid, and you worked out the details of the derivation in section 1.10.2 of Vallis.

   a. Thinking about the energetics of a fluid in the *rotating frame*, show that the Coriolis force cannot change the kinetic energy of a fluid parcel.

   b. When we derived the *primitive equations* we threw away some terms that might affect the total energy budget. So we would like to verify what, if any, form of energy is conserved in a fluid obeying the primitive equations. Answer question 2.13 in Vallis (1st ed):

   > *Show that the inviscid, adiabatic, hydrostatic primitive equations for a compressible fluid conserve a form of energy (kinetic plus potential plus internal), and that the kinetic energy has no contribution from the vertical velocity. You may assume Cartesian geometry and a uniform gravitational field in the vertical direction.*

   Last question on the other side of the page!

4. Consider a layer of fluid between two solid boundaries at $z = 0$ and $z = H$ (an arbitrary height, not necessarily the scale height!). Suppose that the flow is horizontally convergent (i.e. $\nabla_z \cdot \vec{u} < 0$) in some thin layer near the lower boundary, and horizontally divergent in a thin layer near the upper boundary. Assume that the horizontal flow is non-divergent elsewhere. What can you infer from the continuity equation about the vertical structure of vertical motion $w(z)$?

   a. Assume a Boussinesq fluid. Be as quantitative as you can, and draw a sketch of $w(z)$.

   b. Now calculate and sketch $w(z)$ for an anelastic fluid. For the reference profile $\tilde{\rho}(z)$ assume an *isothermal ideal gas.*

   c. Give a physical explanation (in words) for the differences between your two sketches.