ATM 500: Atmospheric Dynamics
Homework 6
Due Thursday November 1 2018

1. Suppose that in a hydrostatic atmosphere in the southern hemisphere, temperatures increase northward on every pressure level between the surface and the tropopause. Also suppose that the pressure gradient force is zero on the 500 hPa surface (about halfway between the surface and tropopause). Make a sketch that shows how the slope of pressure surfaces and the geostrophic winds must vary with height. Make sure to explain your reasoning. What is the direction of the geostrophic wind near the surface? What about near the tropopause? (Use the meteorological convention: "westerly" = from the west, "northerly" = from the north, etc.)

2. In meteorological terminology, "veering" winds turn clockwise with increasing height, while "backing" winds turn anti-clockwise with increasing height. Show that for an atmosphere in thermal wind balance, veering winds must be associated with *warm air advection*. Also show that backing winds must be associated with cold air advection.

3. *Based on Question 2.17 from Vallis*

   Estimate the magnitude and direction of the zonal wind 5 km above the surface in the mid-latitude atmosphere in summer and winter using (approximate) values for the meridional temperature gradient in the atmosphere. Compare your estimated value to a typical climatological zonal wind at this same location. Make sure to cite your data sources and state any assumptions you make.

4. Consider a compressible ideal gas atmosphere. The environment is characterized by vertical profiles of density and potential temperature $\tilde{\rho}(z), \tilde{\theta}(z)$. A small parcel of fluid is adiabatically displaced upward a small distance δz.

   a. Show that the density difference $\delta\rho$ between the parcel and its new environment is given by
   $$\delta\rho = \frac{\tilde{\rho}}{\tilde{\theta}} \frac{d\tilde{\theta}}{dz} \delta z$$
   b. If the parcel obeys a linearized vertical momentum equation
   $$\frac{Dw}{Dt} = -g \frac{\delta\rho}{\tilde{\rho}}$$
   show that the resulting motion will be vertical oscillations at frequency N where
   $$N^2 = \frac{g}{\tilde{\theta}} \frac{d\tilde{\theta}}{dz}$$
   so long as $\frac{d\tilde{\theta}}{dz} > 0$

5. *Based on Question 2.18 from Vallis*

   Using approximate but realistic values for the observed stratification, what is the *buoyancy period* for (a) the midlatitude troposphere, and (b) the stratosphere. Make sure to cite your data sources and state any assumptions you make.