ATM 500: Atmospheric Dynamics
Homework 7
Due Thursday November 8 2018

1. In class we did a linear wave analysis for small perturbations in the x, z plane away from a state of rest ($\vec{v_0} = 0$) for the non-rotating simple Boussinesq system.

   a. For these waves, show that the phase speed in the *vertical* direction ($c^z = \omega/m$) is in the opposite direction of the vertical component of the group velocity, $c_g^z = \partial\omega/\partial m$. (This means that energy must travel downward wherever peaks and troughs are propagating upward and vice-versa). *Yes, we already covered this in lecture briefly, but please give a detailed answer and explain your reasoning clearly.*

   b. Consider now small perturbations away from a state with a constant background velocity in the x direction, $\vec{v_0} = U\hat{i}$ (where U is a constant). You may still assume that the motion is a function of x and z only as we did above. Show that the dispersion relation for these waves is

$$\omega = Uk \pm \frac{Nk}{\sqrt{k^2 + m^2}}$$

   c. How does the presence of a constant background velocity U change your answer in part (a)?