ATM 500: Atmospheric Dynamics
Homework 5
Due Friday October 4 2019

1. Consider motion under Coriolis force alone (e.g. a ball on a rotating parabolic turntable, or a fluid parcel on a quasi-horizontal geopotential surface in the absence of pressure gradients). The equations of motion for such a system are

$$\frac{Du}{Dt} = 2\Omega v \qquad\qquad \frac{Dv}{Dt} = -2\Omega u$$

   where Ω is the rotation rate.

   Show that a parcel can move in a *steady circular path*, completing exactly 2 circles for every 1 rotation of the system. State clearly any assumptions you are making.

   (This is a kind of motion known as inertial circles, which we can easily observe on the turntable, and is sometimes observed in the ocean with freely drifting buoys.)

2. In class (and in HW3) we studied the total energy budget of a fluid in the inertial reference frame. Thinking about the energetics of a fluid in the *rotating frame*, show that the Coriolis force cannot change the kinetic energy of a fluid parcel.

3. Section 2.5.4 of the *Essentials* text discusses the total energy budget of the Boussinesq system.

   a. Starting from the simple inviscid, adiabatic Boussinesq equations (2.66), give a more detailed derivation of the total energy equation (2.69). Show the steps and explain your reasoning.

   b. Compare this result (2.69) to the case of a *constant density fluid* (1.78), for which total *kinetic* energy is conserved. In both cases the continuity equation is $\nabla \cdot \vec{v} = 0$ (i.e. both flows are considered incompressible or nearly so). Explain in your own words why the energy conservation is not the same for a Boussinesq fluid and a constant density fluid.