ATM 500: Atmospheric Dynamics
Homework 7
Due Thursday November 7 2019

1. *Based on question 4.1 in Vallis Essentials*

   Using the shallow water equations:

   a. A cylindrical column of air at 30° latitude with radius 100 km expands horizontally (shrinking in depth accordingly) to twice its original radius. If the air is initially at rest, what is the mean tangential velocity at the perimeter after the expansion?

   b. An air column at 60°N with zero relative vorticity ($\zeta = 0$) reaches from the surface to the tropopause, which we assume is a rigid lid, at 10 km. The air column moves zonally onto an area of elevated topography 2.5 km high (this might represent the Tibetan Plateau, for example). What is its relative vorticity? Suppose it then moves southwards to 30°N, staying on the plateau. What is its relative vorticity?

2. Read section 4.3.3 of Vallis *Essentials* (or Vallis *AOFD* section 3.8.3 (2nd edition) or 3.7.3 (1st edition)). The *Kelvin wave* is a special kind of gravity wave that exist in the presence of a lateral boundary. The wave is in geostrophic balance in the direction perpendicular to the boundary, but unbalanced (and propagating at phase speed $\sqrt{gH}$) in the direction along the boundary. (These waves are particularly important for our understanding of tropical phenomena such as El Niño).

   Why did we not find the Kelvin wave when we did our standard wave analysis of the rotating shallow water equations in class?

3. Starting from the conservation of potential vorticity $Q = (f + \zeta)/h$ for parcels in the shallow water system, derive the linear result

$$\frac{\partial q}{dt} = 0, \qquad q = \zeta' - f_0 \frac{\eta'}{H}$$

   for small perturbations of the height and velocity field on an f-plane with a flat bottom.