ATM 500: Atmospheric Dynamics
Homework 2
Due Wednesday September 8 2021

1. The *hypsometric equation* states that the vertical distance Δz between two surfaces of constant pressure is proportional to an *average* temperature of the layer:

$$\Delta z = \frac{R_d \bar{T}}{g} \ln \left(\frac{p_1}{p_2} \right)$$

Prove this result for an ideal gas in hydrostatic balance (where p_1, p_2 are the pressure levels with $p_1 > p_2$). Do not assume that the temperature $T(p)$ is constant. $\bar{T}$ represents a weighted average of $T(p)$ over the layer. Show how this average is defined.

2. a. Barometric pressure is measured continuously at surface-based stations all around the world, but maps of *surface pressure* (as directly measured at stations) are rarely plotted. Find one such map for the continental US. One possible source for current data is the High Resolution Rapid Refresh (HRRR) numerical weather prediction model. Also find a map of *sea level pressure* for (at least approximately) the same time as your surface pressure map. [Make sure that you cite your data sources.] Comment on why the two maps look so different! What are the dominant features in the surface pressure map? Why do you think this is?

   b. Find the surface pressures (*not* sea level pressures) at Denver, CO and at Springfield, MO. The raw stations observations are surprisingly hard to find! It is possible to obtain surface pressures from the raw sounding data reported by the upper-air sounding stations DNR and SGF. Alternatively you could just interpolate from your surface pressure map. Again, make sure you cite your data sources.

   The horizontal distance between these two locations is about 1060 km. Calculate the component of the *pressure gradient force* (PGF) acting along a line between the surface at Denver and the surface at Springfield. What is its magnitude and direction? (Recall that we usually express forces per unit mass in $\mathrm{m\,s^{-2}}$). Clearly state any assumptions you need to make.

   c. "Sea level pressure" is a fictitious pressure that would be measured at zero elevation if that were possible at that location. Express in words why the sea level pressure must always be larger than the measured surface pressure for an atmosphere in hydrostatic balance, if the station elevation is above sea level.

   d. Now use the hypsometric equation from question 1 (which is always valid for a hydrostatic ideal gas layer) to "correct" the surface pressures at Denver and Springfield down to sea level (i.e. calculate the sea level pressure at those two locations). Describe clearly any assumptions you need to make. Do your values agree (at least approximately) with what is plotted on your map of sea level pressure?

e. Repeat part (b) but now for the *horizontal* component of the pressure gradient force at sea level. How does the result differ from what you found in part (b), and why? Describe the direction and magnitude of the full PGF acting on air parcels between Denver and Springfield (a qualitative answer is sufficient here).

3. *Based on Vallis question 1.3*

   Newton's 2nd law for a body of mass M is

   $$\frac{d\vec{m}}{dt} = \vec{F}$$

   where $\vec{m} = M\vec{v}$ is the *momentum* of the body, $\vec{v}$ is the velocity of the body, and $\vec{F}$ is the net force acting on the body. If it is momentum, not velocity, that responds when a force is applied, then why is the momentum equation for a moving fluid (in the absence of gravity and viscosity) given by $\rho\frac{D\vec{v}}{Dt} = -\nabla p$ and not $\frac{D(\rho\vec{v})}{Dt} = -\nabla p$?