# ATM 500: Atmospheric Dynamics
# Homework 10
### Due Wednesday December 1 2021

1. Here you will use the Ekman layer concept to make some inferences about how the ocean is set in motion by surface wind stress.

   The observed time-average surface winds $\vec{u_s}$ are westerly in the mid-latitudes, easterly in the tropics (the trade winds), and easterly near the poles. Let's represent this in a simple Cartesian domain with

   $$\vec{u_s} = U \cos\left(\frac{\pi y}{L}\right) \hat{i}$$

   where $U$ is a constant and the domain extends from $y = -L$ in the south to $y = +L$ in the north. Assume that the winds do not vary in the east-west direction, and that we are in the northern hemisphere. Also assume that the stress at the surface is given by

   $$\vec{\tau_s} = C\vec{u_s}$$

   where $C$ is a positive constant.

   a. If $\vec{u_s}$ is given in units of meters per second, what must be the units of the constant $C$?

   b. Calculate the mass transport in the oceanic Ekman layer.

   c. Calculate the Ekman pumping $w_{Ek}$ (the stress-induced vertical velocity at the bottom of the oceanic Ekman layer). Where (i.e. over what range of $y$) is the Ekman pumping upward and where is it downward? If you need to make any assumptions, make sure you state them clearly.

   d. Make a sketch that shows the wind stress, the direction of the Ekman mass transport and the direction of the Ekman pumping as functions of $y$. Clearly note the boundaries (if any) between the upward and downward Ekman pumping.

   e. *FOR BONUS POINTS, NOT REQUIRED.* Treating the ocean as a layer of shallow water with constant density and thinking about conservation of potential vorticity, how might the Ekman pumping you described above affect the ocean velocities (currents) below the Ekman layer?

2. *Based on question 5.6 from Vallis AOFD, 1st edition*

   Given the 1D (zonal) baroclinic Rossby wave dispersion relation

   $$\omega = \frac{-\beta k}{k^2 + 1/L_d^2}$$

   for what value of $k$ is the zonal component of the group velocity the largest (i.e. the most positive), and what is the corresponding value of the group velocity?

   Estimate the numerical values of the corresponding wavelength and group velocity for the mid-latitude atmosphere, explaining your reasoning.

3. *Vallis Essentials question 6.2*

   Consider barotropic Rossby waves obeying the dispersion relation

   $$\omega = Uk - \frac{\beta k}{k^2 + l^2}$$

   where $U$ and/or $\beta$ vary slowly with latitude.

   a. By re-arranging this expression to obtain an expression for the meridional wavenumber, or otherwise, show that the Rossby waves can only propagate in the meridional direction if

   $$0 < U - c < \frac{\beta}{k^2}$$

   where $c = \omega/k$ is the zonal phase speed.

   b. If the waves approach a latitude where $U = c$ (a 'critical latitude'), show that the meridional wavenumber $l$ becomes large but that the group velocity in the $y$ direction becomes small. Show that Rossby waves generated in the midlatitudes are unlikely to propagate into the tropics (where the mean flow is westward). *This is responsible for the phenomenon known as Rossby wave breaking in the subtropical troposphere*

   c. *FOR BONUS POINTS, NOT REQUIRED.* Suppose that the Rossby waves approach a latitude where $U - c = \beta/k^2$. Calculate the $x$- and $y$-components of the group velocity in this limit and infer that a wave will turn away from such a latitude.

4. Suggest at least one good question for the final exam. You do not need to write down the answer to your question here. The best questions just might show up on the exam.