ATM 500: Atmospheric Dynamics
Homework 3

Due Friday September 17 by 12 pm (noon)

Note special due date/time

1. a. Suppose that a sealed, insulated container consists of two compartments, and that one of them is filled with an ideal gas and the other is a vacuum. The partition separating the compartments is removed. How does the temperature of the gas change? (Answer: it stays the same. Explain.) Obtain an expression for the final potential temperature, in terms of the initial potential temperature of the gas and the volumes of the two compartments.

   b. A dry parcel that is ascending adiabatically through the atmosphere will generally cool as it moves to lower pressure and expands, and its potential temperature stays the same. How can this be consistent with your answer to part (a)?

   (Question 1.6 from Vallis Essentials)

2. Beginning with the expression for potential temperature for a simple ideal gas, $\theta = T(p_r/p)^\kappa$ where $\kappa = R/c_p$, show that

$$d\theta = \frac{\theta}{T}\left(dT - \frac{\alpha}{c_p}dp\right)$$

   and that the first law of thermodynamics may be written as

$$dQ = T d\eta = c_p \frac{T}{\theta} d\theta$$

   (here $d\eta$ is the change in entropy per unit mass)

   (Question 1.7 from Vallis Essentials)

3. The total energy equation for an inviscid, adiabatic fluid is (equation 1.73 in the text):

$$\frac{\partial E}{\partial t} + \nabla \cdot [\vec{v}\,(E + p)] = 0$$

   Why can we conclude from this that total energy is conserved for a closed domain with rigid boundaries? What is the role of the boundaries in this conclusion? Explain as clearly as you can.

4. Show that viscosity will dissipate kinetic energy in a compressible fluid.

   (Question 1.5 from Vallis Essentials)