Newton’s Second Law: \( \frac{1}{m} \sum \vec{F} = \vec{a} = \frac{du}{dt} \hat{i} + \frac{dv}{dt} \hat{j} + \frac{dw}{dt} \hat{k} \)

Frames of Reference
- Inertial Frame: observe motions from a non-accelerating frame. Fundamental forces.
- Noninertial Frame: observe motions from an accelerating frame. Fundamental + apparent forces.

Fundamental Forces

**Pressure Gradient Force (per unit mass)**
Arises when there is a difference in pressure across space.

\[ \frac{\vec{F}_{\text{pressure gradient}}}{m} = -\frac{1}{\rho} \nabla p \]

**Gravitational Force (per unit mass)**
Arises between any two bodies with mass.

\[ \frac{\vec{F}_{\text{gravity}}}{m} = -\frac{GM}{a^2_E} \hat{k} \approx -9.81 \text{m s}^{-2} \]

**Viscous/Frictional Force (per unit mass)**
Arises when there is a resistance to flow, causing shearing stresses in a viscous fluid

\[ \frac{\vec{F}_{\text{viscous}}}{m} = \nu \left( \nabla^2 u \hat{i} + \nabla^2 v \hat{j} + \nabla^2 w \hat{k} \right) \]

Forces in Rotating Systems

**Centrifugal Force (per unit mass)**

*Apparent* force that acts equal and opposite to the centripetal force.

Combined with true gravity to yield an apparent gravity. Alters shape of Earth to be oblate spheroid.

\[ \frac{\vec{F}_{\text{centripetal}}}{m} = \Omega^2 \vec{R} \]

**Coriolis Force (per unit mass)**

*Apparent* force that acts on parcels moving relative to the ground.
Two effects: conservation of angular momentum \( M = R(u + \Omega R) \) and centrifugal imbalance.

\[
\vec{F}_{\text{Coriolis}} = \frac{(2\Omega v \sin \phi - 2\Omega w \cos \phi)}{m} \hat{i} - (2\Omega u \sin \phi) \hat{j} + (2\Omega u \cos \phi) \hat{k}
\]

Deflects objects to the right of the motion in the NH and to the left of the motion in the SH. Only changes direction, not speed.

**Structure Static Atmosphere**

**Ideal Gas Law**

\[
p = \rho RT
\]

**Hydrostatic Balance**

Balance between vertical pressure gradient and gravitational forces.

\[
\frac{\partial p}{\partial z} = -\rho g
\]

**Hypsometric Equation**

Thickness between two pressure surfaces. Proportional to layer mean temperature.

\[
\Delta z = \frac{R}{g} \int_{p_2}^{p_1} T d\log p = \frac{R T}{g} \log \left( \frac{p_1}{p_2} \right)
\]

where \( p_2 < p_1 \)

**Altimeter Equation**

Used to correct the station pressure to sea level.

\[
p_1 = p_2 \exp \left( \frac{g \Delta z}{RT} \right)
\]

**Conservation of Mass**

\[
\frac{\partial \rho}{\partial t} = -\nabla \cdot (\vec{u} \rho)
\]

\[
\frac{d\rho}{dt} = -\rho \nabla \cdot \vec{u}
\]

For an incompressible fluid, \( \nabla \cdot \vec{u} = 0 \).

**Integrated Form of Conservation of Mass**

\[
\frac{\partial m}{\partial t} = - \int \int \vec{u} \rho \cdot \vec{n} ds
\]
1a) Write an equation down describing the acceleration of a parcel in the meridional direction given the following information:

- The planet is not rotating.
- There is only flow in the meridional direction ($\vec{u} = v\hat{j}$) and that the flow is only a function of $z$.
- The pressure only varies as a function of $y$.
- Friction is acting on the parcel.

Simplify this equation to the fullest extent. (Hint: Your answer should only include $\hat{j}$ components.)

1b) Now modify the situation in 1a). The flow is not accelerating. Additionally, you know that pressure is increasing to the north. Everything else remains the same. Sketch a plausible picture of what the forces acting on the parcel may look like.
2a) Consider an air parcel on Earth that travels to the north at a speed of $10 \text{ m s}^{-1}$ at a latitude of $30^\circ \text{S}$ and a longitude of $20^\circ \text{W}$. What is the magnitude of the Coriolis force acting on this parcel initially, ignoring the curvature terms? Which direction is it pointing?

2b) Derive an equation for the zonal velocity as a function of time, assuming the Coriolis force you found in 2a) is the only force acting on the parcel and ignoring any variations in latitude.

2c) Calculate what the meridional velocity must be after 1 hour (3600 seconds) given that the initial velocity at $t = 0$ is $v = 10 \text{ m s}^{-1}$. (Hint: There are two ways to calculate this. One of which is much easier.)
3) Two observation stations, A and B, record the same surface pressure of 1000 hPa, but the mean temperature over station A is 10 K cooler than the mean temperature over station B. Draw a sketch of the slope of the 800 hPa pressure surface and explain your reasoning.