Problem Set 3  
ATM 316: Dynamic Meteorology I  
October 7, 2015  
**DUE:** October 19, 2015 in class

1) The equation of state for the ocean is a function of both the temperature and salinity. The following figure gives the density (g/cm$^3$) as a function of the water temperature (°C) and salinity (psu or ppt). Typical salinity values in the ocean are around 35 psu.

![Temperature-Salinity Diagram](image)

In class, we did an experiment where one side of the tank was filled with salty water and the other side was filled with fresh water. The salty water has a salinity of 35 psu and a temperature of 10°C. The fresh water has a salinity of 0 psu and a temperature of 10°C, yielding a density of 1.0 g/cm$^3$.

a) Determine the pressure at the bottom of each side of the tank, assuming the depth ($h$) of 30 cm on each side, by using the following formula:

\[ p = \rho gh, \]

where $\rho$ is the density of the salty/fresh water and $g = 9.81$ m/s$^2$.

b) Calculate the pressure gradient force at the interface between the two sides. Assume a distance of 10 cm between the center of the salty and fresh sides of the tank. (Hint: Use the average density when calculating the pressure gradient force. Assume a one-dimensional system, i.e. only in the x-direction.)

c) A parcel of water initially has zero velocity. Following the parcel, what is its final zonal velocity if it is subjected to the pressure gradient force you calculated for a duration of 10 seconds.

\[ \frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \]
d) What would the height of the fresh water side have to be in order for the pressure gradient force to disappear.

2) The atmospheric surface layer extends about 100 m from the Earth’s surface during the day. Viscous and turbulent forces are important in this layer. Assume the zonal wind profile can be described as a logarithmic profile:

\[ u = a \log \frac{z}{b}, \quad (z > 0) \]

where \( a \) and \( b \) are constants.

a) Derive an expression for the zonal component of the viscous force, assuming \( u \) does not vary with \( x \) and \( y \).

b) A zonal pressure gradient force balances the viscous force exactly such that there is no acceleration of air parcels. Derive an expression for the pressure as a function of \( x \) and \( z \), assuming the density in the surface layer is constant.

c) As we move away from the surface, does the magnitude of the pressure gradient force increase or decrease in order to maintain balance with the viscous force? Physically explain why this must be the case.

3a) We typically combine the effects of gravity and the centrifugal force in a frame of reference rotating with the earth. We assume that gravity doesn’t change much from a constant value. One way to evaluate this assumption is to calculate the ratio of the magnitude of the centrifugal force to the magnitude of the gravitational force (9.81 m/s\(^2\)). Plot this ratio as a function of latitude from 0 to 90 degrees. Calculate this ratio by assuming that the earth is spherical and has a mean radius of \( 6.37 \times 10^6 \) m and has an angular velocity of \( 7.292 \times 10^{-5} \) 1/s. Is the assumption good based on your answer?

b) What would the Earth’s rotational period have to be (in hours) in order for the magnitude of the centrifugal force to equal the magnitude of the gravitational force at the Equator?