SUPPLEMENT

A VENTILATION INDEX FOR TROPICAL CYCLONES

BY BRIAN TANG AND KERRY EMANUEL

This document is a supplement to "A Ventilation Index for Tropical Cyclones," by BRIAN TANG AND KERRY EMANUEL (*Bull. Amer. Meteor. Soc.*, **92,** 1901–1912) • ©2012 American Meteorological Society • *Corresponding author:* Brian Tang, ES 324, 1400 Washington Ave., Albany, NY 12222 • E-mail: btang@albany.edu • DOI:10.1175/BAMS-D-11-00165.2

OVERVIEW. The ventilation index derived from the theoretical results of Tang and Emanuel (2010) (hereafter TE10) used an axisymmetric, steady-state, slantwise neutral framework to study the symmetric response of a tropical cyclone (TC) to ventilation. Ventilation is introduced as an eddy flux of entropy through a control surface that surrounds the inner-core of the TC. Low-entropy air is fluxed through this control surface, reducing the entropy through a deep-layer through convective motions. Consequently, the reduction in entropy destroys available potential energy supplied by surface fluxes of enthalpy and reduces the TC intensity.

The steady-state intensity of a ventilated TC can be expressed as the solution to the equation

$$-u_m^3 + u_{\mathrm{PI}}^2 u_m - \frac{T_s(T_s - \overline{T}_o)}{C_D \overline{T}_o}\lambda = 0, \tag{S1}$$

where u_m is the maximum wind speed, u_{PI} is the potential intensity, T_s is the surface temperature, $\overline{T}_o$ is the mean outflow temperature, C_D is the drag coefficient, and λ is the ventilation. The ventilation is defined as

$$\lambda = \frac{1}{2\pi\rho r_m \delta r}\int_M \rho\overline{u's'}\cdot d\sigma, \tag{S2}$$

where ρ is the density, r_m is the radius of maximum wind, δr is a characteristic eyewall width, u' is the radial wind perturbation, s' is the entropy perturbation, and the overbar is an azimuthal average. The integral is taken through an angular momentum surface, M, with surface element $d\sigma$. Hence, the ventilation is the integrated eddy flux of entropy through an angular momentum surface scaled by the eyewall area. Readers interested in the derivation of (S1) and (S2) are referred to TE10.

There exists a maximum ventilation beyond which a steady TC cannot exist. We define the maximum ventilation value as the "ventilation threshold." Mathematically, the ventilation threshold occurs where the discriminant of (S1) vanishes and can be expressed as

$$\lambda_{\mathrm{thresh}} = \frac{2}{2\sqrt{3}}\frac{C_D \overline{T}_o u_{\mathrm{PI}}^3}{T_s(T_s - \overline{T}_o)}. \tag{S3}$$

It is advantageous to nondimensionalize (S1) by normalizing the maximum wind speed by the potential intensity and normalizing the ventilation by the ventilation threshold:

$$\begin{aligned} u_m &\to \tilde{u} u_{\mathrm{PI}}, \\ \lambda &\to \tilde{\lambda}\lambda_{\mathrm{thresh}}, \end{aligned} \tag{S4}$$

where $\tilde{u}$ is the normalized intensity (maximum wind speed) and $\tilde{\lambda}$ is the normalized ventilation. Substituting (S4) into (S1) results in the nondimensional form of the intensity equation for a ventilated TC:

$$-\tilde{u}^3 + \tilde{u} - \frac{2}{2\sqrt{3}}\tilde{\lambda} = 0. \tag{S5}$$

Equation (S5) is the cornerstone result of TE10 and the basis for comparison to TC intensity observations in the main portion of the manuscript.

DERIVATION OF THE VENTILATION INDEX. The key parameter in (S5) is the normalized ventilation and is the basis for the ventilation index. To derive the ventilation index, we apply several simplifications.

First, we assume the ventilation, given by (S2), occurs at a single level, p_m. We choose p_m to be at midlevels (600 hPa), where the entropy perturbations are typically the largest. Furthermore, we group the other variables in to a single constant, α. As a result, (S2) becomes

$$\lambda \approx \alpha \overline{u's'}\Big|_{p_m}. \tag{S6}$$

The eddy flux of entropy can be simplified by considering the system sketched in Fig. S1. Consider a circular disc of high saturation entropy, s_m^*, surrounded by a lower value entropy, s_m, at midlevels. The inner disc represents the inner-core eyewall and eye of the TC, while the surroundings represent the environment. Now, a wavenumber-1 flow perturbation is imposed on the boundary of this disc. Advection by these perturbations induces a wavenumber-1 perturbation in the entropy in phase with the radial flow perturbations, such that

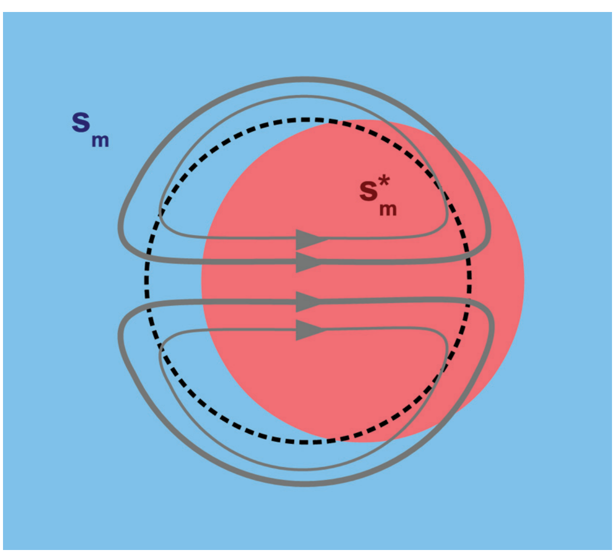

FIG. S1. Illustration of simplified system used to calculate the eddy entropy flux. The dotted circle represents a disc around the center of the storm. The light red region has relatively high (saturation) entropy, s_m^*, and the light blue region has relatively low entropy, s_m. A wavenumber-1 perturbation in the flow (gray arrows) displaces the region of high entropy to the right, resulting in an eddy entropy flux given by (S7).

$$u's' = \frac{1}{2}|u'|(s_m^* - s_m). \tag{S7}$$

The radial flow perturbations represent, for instance, the response of the TC to environmental vertical wind shear forcing. Vertical wind shear tilts the vortex, exciting vortex Rossby waves. The tilt of sheared TCs has been studied in both numerical simulations and observations (Reasor et al. 2000; Rogers et al. 2003; Braun and Wu 2007; Reasor et al. 2009). The studies show that the tilt magnitude is positively correlated with the magnitude of the vertical wind shear. Since the tilt projects on to asymmetries in the potential vorticity field and their quasi-balanced flow perturbations (Reasor et al. 2004), the vertical wind shear must be correlated with the flow perturbations as well. As a first step, we assume a linear relationship between the vertical wind shear and radial flow perturbations. However, one should keep in mind an exact dependence between the vertical wind shear forcing and radial flow perturbation is undoubtedly more complicated and involves the characteristics of the vortex itself. Applying the linear relationship assumption in (S7) results in

$$u's' = \frac{1}{2}ku_{\text{shear}}(s_m^* - s_m), \tag{S8}$$

where k is a proportionality constant between the environment vertical wind shear, u_{shear}, and the radial flow perturbations.

Next, the ventilation threshold is written in a more convenient form by substituting the definition of the potential intensity,

$$u_{\text{PI}}^2 = \frac{T_s(T_s - \overline{T}_o)}{\overline{T}_o}\frac{C_k}{C_D}(s_{\text{SST}}^* - s_b), \tag{S9}$$

into (S3) to yield

$$\lambda_{\text{thresh}} = \frac{2}{2\sqrt{3}}C_k u_{\text{PI}}(s_{\text{SST}}^* - s_b), \tag{S10}$$

where C_k is the enthalpy exchange coefficient, s_{SST}^* is the saturation entropy at the sea surface temperature, and s_b is the boundary layer entropy. The variables in (S10) are evaluated at the radius of maximum wind for a TC at its potential intensity.

The normalized ventilation can be formed by dividing (S6) by (S10) and using (S8):

$$\tilde{\lambda} = \frac{\sqrt{3}\alpha k u_{\text{shear}}(s_m^* - s_m)}{2C_k u_{\text{PI}}(s_{\text{SST}}^* - s_b)}. \tag{S11}$$

Using the definition of the nondimensional entropy deficit,

$$\chi_m = \frac{s_m^* - s_m}{s_{\mathrm{SST}}^* - s_b}, \tag{S12}$$

and dropping the constants results in a ventilation index, Λ, that is proportional to (S11):

$$\Lambda = \frac{u_{\mathrm{shear}} \chi_m}{u_{\mathrm{PI}}}. \tag{S13}$$

The ventilation index is used to assess the effects of ventilation on the statistics of TC genesis and intensification and is also used as a forecast and model diagnostic in the main manuscript.

CALCULATION FROM GRIDDED DATA. The 850–200-hPa environment vertical wind shear must be separated out from the full shear, which includes the TC circulation. The component of the vertical wind shear due to the TC is removed following the method of Davis et al. (2008), which removes the differential divergence and vorticity of the primary and secondary circulations. The resultant environmental shear is then interpolated to the TC position.

For the numerator of the nondimensional entropy deficit, the midlevel (600 hPa) saturation entropy is averaged over a 100-km disc centered on the TC position, and the environmental midlevel entropy is averaged over a 100–300-km annulus from the center of the invest position. These two regions represent characteristic scales associated with the steady flow topology of sheared TCs (Riemer and Montgomery 2011), albeit the exact nature of mixing of air between the environment and the TC is undoubtedly more complicated than the simple average suggests.

For coarse-resolution data, such as reanalyses, the averaging areas only encompass a handful of data points. Reanalysis data are not capable of resolving the full structure of the TC, particularly intense ones. This poses a problem for accurately estimating the saturation entropy of the TC. Simple models of the radial profile of saturation entropy with radius can be used as a correction (Emanuel 1986; Emanuel and Rotunno 2011). However, these models require knowledge about the storm structure itself that are neither globally archived nor robust. Hence, when using raw reanalysis or other coarse-resolution data, one must keep in mind that the entropy deficit is likely underestimated, especially for strong TCs.

The potential intensity is calculated from the algorithm of Bister and Emanuel (2002) under pseudoadiabatic assumptions. The potential intensity is then interpolated to the TC position, but at a three-day lead. The three-day lead is necessary because the TC itself will reflect a local minimum in the potential intensity that is not representative. Since the potential intensity is a slowly varying quantity, a three-day lead provides an estimate of the potential intensity along the TC track that is more representative.

Finally, the denominator of the nondimensional entropy deficit, or the air–sea disequilibrium, is evaluated at the radius of maximum wind for a TC at its potential intensity using the same algorithm from Bister and Emanuel (2002).

INTENSITY CHANGE. The framework that results in (S5) describes the steady-state intensity of a ventilated TC. However, (S5) can easily be altered in order to indirectly measure the intensification as a function of the normalized intensity and ventilation.

By substituting values of $\tilde{u}$ and $\tilde{\lambda}$ into (S5), one obtains a residual that represents the nondimensional power surplus or deficit. Assuming all the power surplus or deficit corresponds to changing the power of the wind, such that

$$\delta\tilde{u}^3 = -\tilde{u}^3 + \tilde{u} - \frac{2}{2\sqrt{3}}\tilde{\lambda}, \tag{S14}$$

an estimate of the resulting normalized intensity change can be obtained. However, the time interval over which the intensity change occurs is ambiguous. Fig. 7b in the main manuscript shows the resulting normalized intensity change as a function of the normalized intensity and normalized ventilation.

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