ATM 210 Chapter 1 and 2 practice problems

Fall, 2023 – Fovell

Some equations [p = pressure (Pa); ρ = density (kg/m^3); T = temperature (K); $\bar{T}$ = layer mean temperature (K); Z = height (m); E = energy per unit area; λ = wavelength (microns)]:

- $p = \rho RT$
- $\frac{dp}{dz} = -\rho g$
- $Z_2 - Z_1 = \frac{R\bar{T}}{g} \ln\left(\frac{p_1}{p_2}\right)$
- $E = \sigma T^4$
- $\lambda_{max} \approx \frac{3000}{T}$

Some constants/conversions:

- $R = 287$ J/kg/K
- $g = 9.81$ m/s^2
- 1 mb = 1 hPa = 100 Pa
- 1 Pa = 1 N/m^2
- 1 J = 1 N $\cdot$ m
- 1 N = $\frac{\text{kg m}}{\text{s}^2}$

1. A dry air parcel has T = 300 K and density 1 kg/m^3. What pressure is it at? Express in mb for hPa.

   Start with the IGL $p = \rho RT$. Plug in.

$$
\begin{aligned}
p &= \rho RT \\
&= (1\ \text{kg/m}^3)(287\ \text{J/kg/K})(300\ \text{K}) \\
&= 86100\ \text{N/m}^2 \\
&= 86100\ \text{Pa} \\
&= 861\ \text{mb}
\end{aligned}
$$

2. Dry air parcel A is at 800 mb with temperature 0°C. Dry air parcel B has the same temperature but is at 700 mb. Which parcel is less dense? For full credit, calculate and compare the densities.

   Qualitative response: The IGL is $p = \rho RT$. We are comparing dry air parcels at the same temperature, so both T and R are constant in this problem. Parcel B has a lower p so its density also has to be smaller.

Quantitative response: Parcel A has $p = 80000$ Pa and $T = 273$ K, so

$$\begin{aligned} p &= \rho R T \\ \rho &= \frac{p}{RT} \\ &= \frac{(80000 \text{ Pa})}{(287 \text{ J/kg/K})(273 \text{ K})} \\ &= 1.02 \text{ kg/m}^3 \end{aligned}$$

Parcel B has $p = 70000$ Pa and $T = 273$ K, so

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{(70000 \text{ Pa})}{(287 \text{ J/kg/K})(273 \text{ K})} \\ &= 0.89 \text{ kg/m}^3 \end{aligned}$$

Parcel B has lower density.

3. Layer A extends between 800 and 600 mb, and layer B extends between 300 and 100 mb. Which layer contains more mass?

   They're the same. Pressure is proportional to mass; the pressure depths of the layers (200 mb) are the same so the mass is the same.

4. A hot sidewalk has $T = 40°$C. An ice cube has $T = 0°$C. How much more radiation per unit area is the sidewalk producing?

   Sidewalk $T = 40°\text{C} = 313$ K. Ice cube $T = 0°\text{C} = 273$ K. Take the ratio of the Stefan-Boltzmann laws for the two objects:

$$\begin{aligned} \frac{E_{sidewalk}}{E_{icecube}} &= \frac{\sigma T^4_{sidewalk}}{\sigma T^4_{icecube}} \\ &= \left(\frac{T^4_{sidewalk}}{T^4_{icecube}} \right) \\ &= \left(\frac{313 \text{ K}}{273 \text{ K}} \right)^4 \\ &= (1.15)^4 \\ &= 1.73 \end{aligned}$$

   The sidewalk is producing 1.73 times more radiation per unit area than the ice cube.

5. Suppose the density of air near the surface is constant with height and is 1 kg/m^3. What is the rate of change of pressure with height, in mb/km, presuming the atmosphere is hydrostatic (i.e., the hydrostatic balance applies)?

   Start with the hydrostatic equation, the left hand side of which is the rate of pressure change with height, dp/dz, and plug in for ρ and g:

$$
\begin{aligned}
\frac{dp}{dz} &= -\rho g \\
&= -(1 \text{ kg/m}^3)(9.81 \text{ m/s}^2) \\
&= -9.81 \text{ N/m}^3 \\
&= -9.81 \text{ Pa/m} \cdot (1 \text{ mb})/(100 \text{ Pa}) \cdot (1000 \text{ m})/(1 \text{ km}) \\
&= -98.1 \text{ mb/km}
\end{aligned}
$$

Pressure decreases by roughly 100 mb per kilometer in the lower troposphere. So, if your surface pressure is 1000 mb, about 1 km above you the pressure is 900 mb, a reduction of 10%.

6. The hypsometric equation tells us the thickness $Z_2 - Z_1$ of the $p_1 - p_2$ layer – where $Z_2 > Z_1$ and $p_1 > p_2$ – is:

$$Z_2 - Z_1 = \frac{R\bar{T}}{g} \ln \frac{p_1}{p_2}.$$

Say the mean temperature of the 1000-500 mb layer is 0°C. What is its thickness?

$$
\begin{aligned}
Z_2 - Z_1 &= \frac{R\bar{T}}{g} \ln \frac{p_1}{p_2} \\
&= \frac{(287 \text{ J/kg/K})(273 \text{ K})}{9.81 \text{ m/s}^2} \ln\left(\frac{1000}{500}\right) \\
&= 7986.9 \text{ J/kg s}^2\text{/m } (0.693) \\
&= 5536 \text{ m}
\end{aligned}
$$

7. Today, the 1000-200 mb layer mean T is 255 K. Tomorrow, it will be 260 K. How much thicker will the layer be tomorrow than today?

Straightforward application of the hypseometric equation similar to the question above. Today's thickness is 12007 m, tomorrow's will be 12242 m, so the thickness increase is 235 m.

8. Betelgeuse is a star in the constellation Orion with an outer surface temperature of T = 3600 K. What is its wavelength of maximum emission? What color is the star?

Wien's law: $\lambda_{max} = \frac{3000}{T} = 3000/3600 = 0.83$ μm. This wavelength is outside of the visible range (0.4-0.7 μm), so the star is red. Among the colors visible to us, more red than others is being produced, so we see red.