Keep in mind that wind is defined by the direction the air is coming from. A wind direction of 0° is northerly, 90° is easterly, 180° is southerly, and 270° is westerly.

1. Temperature is decreasing to the north at 5°C per 100 km. The wind is southerly at 5 m/s. The local temperature change at our location is +1°C per hour (warming). What is the heating or cooling rate, in K/hr, a parcel must be experiencing as it travels in our direction?

   **Ans:** The equation we need (removing terms that are zero) is
   \[
   \frac{dT}{dt} = \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y},
   \]
   where \(\frac{\partial T}{\partial t} = +1 \text{ K h}^{-1}\), \(\frac{\partial T}{\partial y} = -\frac{5}{1 \times 10^5} \text{ m}^{-1}\), and \(v = +5 \text{ m s}^{-1} = +18000 \text{ m h}^{-1}\). Thus \(\frac{dT}{dt} = 1 \text{ K h}^{-1} - 0.9 \text{ K h}^{-1} = +0.1 \text{ K h}^{-1}\). There is warm advection, but insufficient to explain the observed local T change, so some warming of the air along its motion is also needed.

2. The temperature at a point 50 km north of a station is 3°C cooler than at the station. If the wind is northeasterly at 10 m/s and the air is being heated by radiation at a rate of 0.5°C/hr, what is the local time rate of change of temperature at the station?

   **Ans:** The wind is from the northeast, but since the temperature only varies in the north-south direction, only the north-south component of the wind is doing any temperature (here, cold) advection. The \(v\) component is \(-7.07 \text{ m s}^{-1} = -25452 \text{ m h}^{-1}\), negative since it is directed southward. The cold advection is being opposed by some amount by radiative heating.

   Again, the Lagrangian temperature tendency is (with zero terms removed):
   \[
   \frac{dT}{dt} = \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y}
   \]

   We need to compute the local temperature derivative at the station, so solve for that term. The total derivative describes the radiative heating following the parcel; the other term is temperature advection. **Watch the minus signs.** Thus:
   \[
   \frac{\partial T}{\partial t} = \frac{dT}{dt} - v \frac{\partial T}{\partial y} = +0.5 \text{ K hr}^{-1} - (-25452 \text{ m h}^{-1}) \left[ \frac{-3 \text{ K}}{50000 \text{ m}} \right] = +0.5 \text{ K hr}^{-1} - 1.03 \text{ K hr}^{-1} = -1.03 \text{ K hr}^{-1}
   \]

   The station’s temperature is falling about 1°C hr⁻¹.

---

ATM 316 Homework #2 Answers

Fall, 2017 – Fovell
3. A perfectly dry, insulated air parcel would cool at the dry adiabatic lapse rate (DALR) upon ascent, owing to expansion cooling. For this problem, we can round the DALR to $10^\circ C/km$ of vertical displacement. Suppose at every point, air is being warmed by radiation at $1^\circ C/hr$. You observe an air parcel ascending at constant vertical velocity, but its temperature following the motion is not changing. What is the ascent rate necessary for expansion cooling to offset radiation warming? Express the answer in meters per second.

**Ans:** Again, we start with the total derivative for temperature, removing terms that are zero. In this case, that leaves the vertical term:

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z}.$$  

The total derivative on the left-hand side describes the temperature change following the air parcel. For this problem, it is zero. The local time rate of change of temperature owing to radiation is specified to be $+1 K/hr$. The only “tricky” bit (if it is even that) is to recognize that the concept of lapse rate presumes a minus sign, so $\frac{\partial T}{\partial z} = -10 K/km$. So, we have

$$\frac{\partial T}{\partial t} = -w \frac{\partial T}{\partial z} + 1 K/hr = -w[-10 K/km].$$  

$w = \frac{1 K}{1000 m} \frac{1000 m}{3600 s} \approx 0.03 m/s$.

If you missed the minus sign on the vertical temperature gradient, you should realize you had a sign error by your understanding of the problem. The air parcel is experiencing both radiative warming and expansion cooling, such that the two contributions cancel and its temperature is not changing. To experience expansion cooling in this situation, the parcel must be rising, towards lower pressure. Thus, $w$ has to be positive.

Suppose you get a negative value for $w$ and you do not realize where you missed the sign. In this case, for maximum credit you should provide an explanation or justification for why the sign you calculated must be incorrect.

4. The horizontal geostrophic wind is $\vec{V}_g = u_g \hat{i} + v_g \hat{j}$. Work backwards from

$$\vec{V}_g = \hat{k} \times \frac{1}{\rho_f} \nabla p$$

and recover $u_g = -\frac{1}{\rho_f} \frac{\partial p}{\partial y}$ and $v_g = \frac{1}{\rho_f} \frac{\partial p}{\partial x}$.

**Ans:**

$$\vec{V}_g = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{\rho_f} \frac{\partial p}{\partial x} & \frac{1}{\rho_f} \frac{\partial p}{\partial y} & \frac{1}{\rho_f} \frac{\partial p}{\partial z} \end{vmatrix} \approx \left[ -\frac{1}{\rho_f} \frac{\partial p}{\partial y} \right] \hat{i} + \left[ \frac{1}{\rho_f} \frac{\partial p}{\partial x} \right] \hat{j}$$
5. Taylor series question, in the context of NWP. The current time is $t_0$, forecast time is $t$, and the unknown time between those two is $t^*$. The true value at time $t$ is $f(t)$, while the forecast for that time is $f_{est}(t)$. Let forecast error be defined here as the actual value minus the forecasted value [i.e., $f(t) - f_{est}(t)$].

Suppose the temperature error for a forecast is $+5$ K. The estimated first derivative at the current time is $f'(t_0)$ is $5$ K/h, but the true value at the unknown, intermediate time is $f'(t^*)$ is $6$ K/h. What is the forecast time step, $\Delta t = t - t_0$?

**Ans:** Set up the problem. We have

\[
\begin{align*}
  f(t) &= f(t_0) + f'(t^*) \Delta t \\
  f_{est}(t) &= f(t_0) + f'(t_0) \Delta t.
\end{align*}
\]

I wrote the latter expression with an equal sign (instead of an approximation) because that is how we are defining the forecast, and we are admitting we will have error. Subtract the second from the first equation. The left hand side is the forecast error, given as $5$ K. The right hand side is the difference between the true and approximated derivatives, or $1$ K/h. Solve for the time step.

\[
\begin{align*}
  f(t) - f_{est}(t) &= f(t_0) + f'(t^*) \Delta t - f(t_0) - f'(t_0) \Delta t \\
  5 \text{ K} &= [f'(t^*) - f'(t_0)] \Delta t \\
  \Delta t &= 5 \text{ h}.
\end{align*}
\]