1. If the rotating Earth were a perfect sphere, at most latitudes apparent gravity would not point directly towards the center of mass. Succinctly explain why. Draw picture(s) and label them completely. I expect to see vector algebra in your answer.

\[ \vec{g} = \vec{g}^* + \Omega^2 \vec{R} \]

**Ans:** True gravity always points towards the center of mass (CM). On a perfect sphere, as depicted above, true gravity \( \vec{g}^* \) would also point to the center of the sphere, and be perpendicular to the local horizontal (and thus in line with the local vertical). This is illustrated with the black dashed line being normal to the horizontal (red line). However, the centrifugal acceleration \( \Omega^2 \vec{R} \) points outward from the center of spin. Apparent gravity, defined as \( \vec{g} = \vec{g}^* + \Omega^2 \vec{R} \), is thus displaced away from the CM and also makes an angle (grey dashed line) with the local vertical.

In the case of the Earth, this created a stress that was mitigated by the deformation of the sphere into an oblate spheroid, one that is flatter at the poles and wider at the equator. This had the effect of making apparent gravity perpendicular to the horizontal and aligned with the local vertical.

2. The wind velocity (m/s) at the location of the star in the figure below is \( \vec{U} = 3\hat{i} + 3\hat{j} \). There is no vertical wind component. Compute the temperature advection at the location of the star, in K per second. Show your work. Be sure to specifically state whether this is cold advection, warm advection, or no advection.

**Ans:** First, we try to anticipate the answer. In this case, it’s not immediately clear. The wind is blowing with a component from cold to warm (zonally) and from warm towards cold (meridionally). However, while the wind components are the same magnitude, the meridional gradient is larger, which means we should anticipate the net result being warm advection.
Using finite differencing, we can compute

\[
\frac{\partial T}{\partial x} = \frac{8^\circ C - 3^\circ C}{200 \text{ m}} = \frac{1\text{ K}}{40 \text{ m}}
\]

\[
\frac{\partial T}{\partial y} = \frac{0^\circ C - 10^\circ C}{200 \text{ m}} = -\frac{2\text{ K}}{40 \text{ m}}
\]

Temperature advection is \(-\vec{U} \cdot \nabla T\). Don’t forget the minus sign!

\[
-\vec{U} \cdot \nabla T = -(u\hat{i} + v\hat{j}) \cdot \left[ \frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} \right]
= - \left[ \frac{3\text{ m}}{\text{s}} \left( \frac{1\text{ K}}{40 \text{ m}} \right) - \frac{3\text{ m}}{\text{s}} \left( \frac{2\text{ K}}{40 \text{ m}} \right) \right]
= \frac{3\text{ K}}{40 \text{ s}}
= 0.075\frac{\text{ K}}{\text{s}}
\]

The result is warm advection, as anticipated.

3. A rocket on the spherical, rotating Earth is initially located at latitude \(\phi_0 = 40^\circ \text{N}\) is launched westbound at a speed of \(u_0 = -2000 \text{ m/s}\). Its motion remains strictly horizontal. By what distance, and in which direction, is the rocket deflected by the Coriolis acceleration after it travels for 250 seconds?

**Ans:** This is solved in the same manner as the example from class. The rocket is heading west in the northern hemisphere, so Coriolis will deflect it to the north relative to our point of view. The relevant acceleration component based on initial velocity \(u_0\) is

\[
\left( \frac{dv}{dt} \right)_{\text{Coriolis}} = -fu_0.
\]
where \( f = 2\Omega \sin \phi_0 = 0.94 \times 10^{-4} \text{ s}^{-1} \). If we assume the acceleration is constant, we’re essentially using the familiar equation from physics

\[ y = y_0 + \frac{1}{2}at^2, \]

where \( y_0 = 0 \) and \( a = \left( \frac{dv}{dt} \right)_{\text{Coriolis}} \). Thus, we have

\[ y = \frac{1}{2}(0.94 \times 10^{-4} \text{ s}^{-1})(2000 \text{ m s}^{-1})(250 \text{ s})^2 = 5875 \text{ m}. \]

The rocket is deflected about 6 km northward over the 250 second flight time.

4. The rocket in the previous problem is seen by a stationary observer looking westward to curve to the south as it flies westward. There is no wind or drag influencing the rocket. Gravity is not a factor. Can you explain how this might be possible?

**Ans:** Coriolis acceleration exists in our reference frame because our coordinate vectors, such as what we call East, vary through the day as seen from space. This is due to Earth rotation. We do not see East or any other coordinate vector change, so we add an apparent acceleration to Newton’s 2nd law to compensate for this.

We will see that another apparent acceleration is caused by Earth’s sphericity. At any given time, what we call East (and other coordinates) varies with location. As an example, as you move westward at a particular latitude, the vector that is locally called East is actually shifting direction. In order to continue traveling along this constantly changing direction, you would need to provide an acceleration. Thus, an object in inertial motion, which moves straight as seen from space, appears to deflect to the south for both westward and eastward travel, as illustrated below, due to the sphericity or curvature deflection.

In this case, notice that the curvature deflection opposes the Coriolis deflection. For very rapidly moving objects, the curvature deflection will exceed the Coriolis deflection in magnitude.
5. The horizontal flow field is described by \( \vec{U} = -\frac{1}{2}y\hat{i} + \frac{1}{2}x\hat{j} \). Compute both the horizontal divergence and the vertical component of vorticity for this flow field. Show your work.

**Ans:** In this case, \( u = -\frac{1}{2}y \) and \( v = \frac{1}{2}x \). Therefore, horizontal divergence is

\[
\nabla \cdot \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 + 0 = 0,
\]

and

\[
\hat{k} \cdot (\nabla \times \vec{U}) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{1}{2} - (-\frac{1}{2}) = 1.
\]

The flow is nondivergent with positive vertical vorticity.

6. The 500 mb wind is from the southwest at 10 m/s. The 1000 mb wind is westerly at 5 m/s. What is the shear between the two layers? Please express the shear as a vector, and compute its magnitude. Also, draw a picture and label it completely.

**Ans:** We have \( \vec{V}_{500} = 7.07\hat{i} + 7.07\hat{j} \) and \( \vec{V}_{1000} = 5\hat{i} \). So the vertical shear from 1000 to 500 mb is

\[
\vec{V}_{500} - \vec{V}_{1000} = (7.07 - 5)\hat{i} + (7.07 - 0)\hat{j} = 2.07\hat{i} + 7.07\hat{j},
\]

with magnitude

\[
|\vec{V}| = \sqrt{(2.07)^2 + (7.07)^2} = 7.37 \text{ m/s}.
\]