1. The horizontal flow field is described by $\vec{U} = -3x^2\hat{i} + xy\hat{j}$. Compute both the horizontal divergence and the vertical component of vorticity for this flow field. Evaluate both for the point $(x,y) = (1,1)$. Show your work.

**Ans:** In this case, $u = -3x^2$ and $v = xy$. Therefore, horizontal divergence is

$$\nabla \cdot \vec{U} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}[-3x^2] + \frac{\partial}{\partial y}[xy] = -6x + x = -5x$$

At point $(x,y) = (1,1)$, the divergence is -5. The vertical vorticity is

$$\hat{k} \cdot (\nabla \times \vec{U}) = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial}{\partial x}[xy] - \frac{\partial}{\partial y}[-3x^2] = y$$

At point $(x,y) = (1,1)$, the vertical vorticity is 1.

2. The 500 mb wind is from the east at 10 m/s. The 1000 mb wind is westerly at 10 m/s. What is the shear between the two layers? Please express the shear as a vector, compute its magnitude, and specify its meteorological direction. In addition, draw a picture and label it completely.

**Ans:** The shear vector magnitude is 20 m/s, from the east (meteorological direction 90°).

![Shear Diagram](image)

3. The wind velocity (m/s) at the location of the star in the figure below is $\vec{U} = 3\hat{i} + 3\hat{j}$. There is no vertical wind component. Compute the temperature advection at the location of the star, in K per second. Show your work. Be sure to specifically state whether this is cold advection, warm advection, or no advection.

**Ans:** First, we try to anticipate the answer. In this case, it’s not immediately clear. The wind is blowing with a component from cold to warm (zonally) and from warm towards cold (meridionally). However, while the wind components are the same magnitude, the meridional gradient is larger, which means we should anticipate the net result being warm advection.
Using finite differencing, we can compute

\[
\begin{align*}
\frac{\partial T}{\partial x} &= \frac{8^\circ C - 3^\circ C}{200 \text{ m}} = \frac{1 \text{ K}}{40 \text{ m}} \\
\frac{\partial T}{\partial y} &= \frac{0^\circ C - 10^\circ C}{200 \text{ m}} = \frac{-2 \text{ K}}{40 \text{ m}}
\end{align*}
\]

Temperature advection is \(-\vec{U} \cdot \nabla T\). Don’t forget the minus sign!

\[
-\vec{U} \cdot \nabla T = -(u \hat{i} + v \hat{j}) \cdot \left( \frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j} \right)
\]

\[
= - \left[ \frac{3 \text{ m/s} \cdot (1 \text{ K/40 m})}{3 \text{ m/s} \cdot (2 \text{ K/40 m})} \right]
\]

\[
= \frac{3 \text{ K}}{40 \text{ s}}
\]

\[
= 0.075 \text{ K/s}
\]

The result is warm advection, as anticipated.
4. Subsaturated air passing over the sea surface is being moistened at a rate of 1 g/kg per hour. At a fixed location, the mixing ratio is decreasing at 2 g/kg per hour. Moisture increases eastward at 0.3 g/kg/km. What must the zonal (west to east) wind component be to explain this difference? Express your answer in km/h, and watch your units.

**Ans:** First, anticipate the answer. The air is being moistened as it travels, but the mixing ratio at a fixed point is declining. Thus, we expect dry (negative moisture) advection. Since moisture increases to the east, a westerly wind is needed to accomplish dry advection.

Let $q_v$ be the water vapor mixing ratio. Following the motion, air is being moistened at 1 g/kg/h:

$$\frac{dq_v}{dt} = 1 \text{ g/kg/h}.$$ At a fixed location, the mixing ratio is decreasing at 2 g/kg/h:

$$\frac{\partial q_v}{\partial t} = -2 \text{ g/kg/h}.$$ The moisture gradient is:

$$\frac{\partial q_v}{\partial x} = 0.3 \text{ g/kg/km}.$$ Thus, we start with

$$\frac{dq_v}{dt} = \frac{\partial q_v}{\partial t} + u \frac{\partial q_v}{\partial x},$$ (1)

in which we have everything but $u$. Solving for that yields $u = 10 \text{ km/h}$, which is positive so it is directed from west to east (westerly), as anticipated.

5. The 1000-500 mb shear vector is directed from S to N. The 500 mb geostrophic wind is westerly. There is cold temperature advection occurring. Where is the cold air located? Answer with: to the N, to the S, to the W, or to the E. Briefly justify your answer. (You DO have sufficient information. I mentioned this concept in class.)

**Ans:** Three ways of approaching this problem:

(a) You know there is cold advection occurring. You know the geostrophic wind component that accomplishes the temperature advection is independent of height. You know the 500 mb geostrophic wind is purely westerly, so here only the zonal wind component matters. This means the cold air is to the W.

(b) You know the shear is parallel to isotherms of layer mean temperature, with cold air to the left. Since the shear is southerly, the cold air is to the W.

(c) Draw the wind vectors. See the 1000 mb wind is directed from NW to SE, and so the situation consists of winds backing with height. Therefore, it is cold advection, and the cold air has to be to the W.