1. Consider three locations each separated by 1 km. The wind speed at point \( y + \Delta y \) is 20 m/s, at point \( y \) is 10 m/s, and at point \( y - \Delta y \) is 5 m/s.

(a) Compute the first derivative at point \( y \). Express in m/s per kilometer.

**Ans:** One of several ways, all leading to the same answer:

\[
\frac{f'(y)}{2 \Delta y} = \frac{f(y + \Delta y) - f(y - \Delta y)}{2 \Delta y} = \frac{1}{2} \left[ \frac{(20 - 5) \text{ m/s}}{2 \text{ km}} \right] = 7.5 \text{ m/s/km}
\]

(b) Compute the second derivative at point \( y \). Express in m/s per square kilometer.

**Ans:** One of two ways, leading to the same answer:

\[
\frac{f''(y)}{(\Delta y)^2} = \frac{f(y + \Delta y) - 2f(y) + f(y - \Delta y)}{(\Delta y)^2} = \left[ \frac{(20 - 2(10) + 5) \text{ m/s}}{1 \text{ km}^2} \right] = 5 \text{ m/s/km}^2
\]

2. The horizontal wind at a station is from the west at 10 km/h. Temperature decreases to the east of the station at 1 K per 10 km. The local time rate of change at a point is zero. What is the heating or cooling rate on the air approaching the station? [WATCH YOUR SIGNS. Anticipate the answer.]

**Ans:** There is clearly warm advection going on but the temperature at the station is not changing, so we anticipate the parcel is being subjected to diabatic cooling along the path. Here, \( \frac{\partial T}{\partial t} = 0, v = 0 \) and we’ve made no reference to the vertical direction, so

\[
\frac{dT}{dt} = +u \frac{\partial T}{\partial x} = (+10 \text{ km/h})(-1 \text{ K/10 km}) = -1 \text{ K/h}
\]

3. Sketch a flow field that is diffluent but not clearly divergent. Briefly explain what the difference between the two.

**Ans:** A diffluent or confluent flow is one in which streamlines or isoheights in a horizontal plane appear to diverge or converge but it is an illusion, because the two terms in the horizontal divergence equation oppose each other, and may largely cancel each other out.

In the figure below, a flow is seen slowing and spreading as it travels from left to right. It is diffluent, and may appear to be divergent as well. Note that, along the flow, \( \frac{\partial u}{\partial y} > 0 \), which contributes to divergence. However, \( \frac{\partial v}{\partial x} < 0 \), as the flow is slowing in the +x direction. The two terms in the horizontal divergence equation are in opposition, and may exactly cancel.
4. A baseball pitcher, standing about 18.3 m from home plate (60 ft. 6 in.), delivers a 42.5 m/s (95 mph) fastball to home plate in Wrigley Field. By how much is it deflected horizontally by the Coriolis force? The stadium is located at $\phi = 40^\circ$ N latitude. 

**Hint:** Your choice of horizontal direction is arbitrary. If the pitcher throws the ball eastward, then the Coriolis deflection is to the south, and the acceleration you want is

$$\left[ \frac{dv}{dt} \right]_{\text{Corio}} = -2\Omega u_0 \sin \phi_0,$$

where $u_0$ is the baseball’s initial eastward speed and $\phi_0$ is the initial latitude. In physics, you learned about an equation of the form $x = x_0 + \frac{1}{2}at^2$, right?

**Ans:** We have $u_0 = 42.5$ m/s, and $f \approx 10^{-4}$ per second. The time taken from the mound to home plate is $t = 18.3/u_0 = 0.43$ s. Thus, the Coriolis acceleration, $fu_0 = 0.00425$ m/s$^2$. Using $x = x_0 + \frac{1}{2}at^2$, and recognizing $x_0 = 0$, results in a deflection of 0.00039 m or 0.039 cm, to the right, of course.

5. The equations of horizontal motion involving only the Coriolis accelerations are

$$\frac{du}{dt} = fv,$$

and

$$\frac{dv}{dt} = -fu.$$

Suppose the Earth were a flat disk, rotating counterclockwise. In this case, $f = 2\Omega$, everywhere on the disk. Suppose further the rotation is $|\vec{\Omega}| = 1$ per hour. Shoot a rocket originating at the axis of rotation towards the outer edge at 10 km/h. The motions remain strictly horizontal and there are no other forces acting than Coriolis. You measure a deflection of 100 km towards the right following the motion. How much time did this take? If you encounter a square root, you do not need to solve for it.

**Ans:** It actually does not matter which equation you select, but you don’t need more than one. I’ll select the first one, with the object initially traveling south ($v < 0$). You can solve this one of two ways. First, you can integrate your selected equation twice, solving for position, which gets you to an equation such as that used in the question above. Or, as a shortcut, you can simply recognize that we’re approximating the acceleration as constant, and jump to that same equation:

$$x = x_0 + \frac{1}{2}at^2.$$

You want $\Delta x = x - x_0 = -100$ km; the initial position does not matter. $\Delta x < 0$ since the deflection is westward. $a = \frac{dv}{dt} = fv$, with $v = -10$ km/h (to the south) and $f = 2|\vec{\Omega}| = 2$/h. So, $a = -20$ km/h$^2$. Solve for $t = \sqrt{10}$ h.
6. The figure below depicts a rotating, flat disk being turned counterclockwise. We will use this disk in an attempt to illustrate the Coriolis deflections for parcels moving “southward” (towards the outer edge of the disk) and “northward” (towards the center of spin).

Taking a piece of chalk, start at the axis of rotation and draw the chalk downward towards the outer part of the disk. You see the chalk line curve to the right following the motion. This appears to represent the Coriolis deflection. Now start at the bottom edge of the disk and draw the chalk upward, towards the center of spin. You see the chalk line curve to the left following the motion, and not to the right as would be produced by the Coriolis deflection. This demonstration has failed. Explain what happened and why.

**Ans:** In class, we discussed the Coriolis deflection on north-south motion as resulting from angular momentum (AM) conservation. Consider first the demonstration that appeared to work, when you moved the chalk from the center of spin to the outer edge. Initially, you are holding the chalk over the rotating board. The chalk has no AM, just as a parcel originating at the north pole on Earth has no AM. As you move the chalk south, AM is conserved (it remains zero) and the board is dragged beneath the chalk. You get the correct result, but it isn’t clear it’s actually for the correct reason.

However, when you start at the outer edge of the board, you are still holding the chalk, and it still has no AM. As you push the chalk towards the center of spin, the board does rotate beneath the chalk, resulting in the trace bending to the left. On the real Earth, a parcel starting at rest at the equator has maximum AM, not zero. Actually, in both cases AM is being conserved, so the real problem is the demonstration is not realistic for south to north motion since the chalk is starting with no angular momentum.