ATM 316 Homework #5 Questions. Due Monday, November 4, 2017.
Fall, 2019 – Fovell

1. The mean virtual temperature ($\bar{T}_v$) of the 1000-850 mb layer is 0°C. What is its thickness according to the hypsometric equation? ($R_d = 287 \text{ J kg}^{-1} \text{ K}^{-1}; g_0 = 9.81 \text{ m s}^{-2}$.)

2. The thickness of a certain layer is 5000 gpm (geopotential meters). Layer $\bar{T}_v$ is 0°C. If the layer bottom pressure is 1000 mb, what is the pressure at the top of the layer?
3. In January, the troposphere is 18 km deep at the equator and 8 km deep at the pole. Tropopause pressures at the two locations are 100 and 300 mb, respectively. Estimate the mean $\bar{T}_v$ of the equatorial and arctic tropospheres, using the hypsometric equation and taking the surface pressure to be 1000 mb at both locations. In the arctic case, what difference is caused by presuming the surface pressure is 1030 mb instead?
4. A barometer at the top of Mt. Rainier, at elevation 4.8 km above sea-level, reports a pressure of 550 mb. Station temperature is -10°C. Estimate the station’s sea-level pressure (SLP). The atmosphere is completely dry and you can (and should) presume the atmosphere is hydrostatic. You can also neglect trying to mass-weight the average temperature between station elevation and sea level, should assume the standard atmosphere’s tropospheric lapse rate of 6.5 K/km, and take $g_0 = 9.81 \text{ m/s}^2$. Detail your assumptions and show your work.
5. I could solve the preceding problem using the altimeter equation,

\[ p_{SLP} = p_s \left[ \frac{T_s}{T_s + \Gamma z_s} \right] \left( -\frac{g}{\rho g_0} \right), \]

where \( p_s, z_s, \) and \( T_s \) are the station pressure, elevation, and temperature, respectively; \( \Gamma \) is the tropospheric temperature lapse rate (taken to be \( 6.5^\circ \text{C/km} \)); \( g = g_0 = 9.81 \text{ m/s}^2 \); and \( p_{SLP} \) is the sea-level pressure. Use this equation to verify your answer in the preceding problem.

Then, derive this expression, starting with the hydrostatic equation, using the ideal gas law in the form \( p = \rho R_d T \) where \( R_d = 287 \text{ J kg}^{-1} \text{ K}^{-1} \), and assuming the temperature is linear between elevations \( z = z_s \) and sea-level (\( z = 0 \)). (That is, I'd take \( T = T_s + \Gamma (z_s - z) \).)
6. In the standard troposphere, the temperature lapse rate is about 6.5°C/km and density decreases exponentially with height. Suppose Bizarro World is identical to Earth \((g = 9.81 \text{ m s}^{-2}, \ R_d = 287 \text{ J kg}^{-1} \text{ K}^{-1}, \text{ and hydrostatic})\) except density is constant with height in its atmosphere. What temperature lapse rate \((-\frac{dT}{dz})\) does this require? Hint: Start with the ideal gas law, and differentiate it with respect to height. Evaluate your lapse rate and express it in °C/km.