1. The mean virtual temperature ($\bar{T}_v$) of the 1000-850 mb layer is 0°C. What is its thickness according to the hypsometric equation? ($R_d = 287 \text{ J kg}^{-1} \text{ K}^{-1}$; $g_0 = 9.81 \text{ m s}^{-2}$.)

**Ans:** Simple application of the hypsometric equation; answer is $\approx 1300 \text{ gpm}$.

2. The thickness of a certain layer is 5000 gpm (geopotential meters). Layer $\bar{T}_v$ is 0°C. If the layer bottom pressure is 1000 mb, what is the pressure at the top of the layer?

**Ans:** Use the hypsometric equation to solve for $p_2$; the answer is about 535 mb.

3. In January, the troposphere is 18 km deep at the equator and 8 km deep at the pole. Tropopause pressures at the two locations are 100 and 300 mb, respectively. Estimate the mean $\bar{T}_v$ of the equatorial and arctic tropospheres, using the hypsometric equation and taking the surface pressure to be 1000 mb at both locations. In the arctic case, what difference is caused by presuming the surface pressure is 1030 mb instead?

**Ans:** More straightforward applications of the hypsometric equation. Answers: tropical $\bar{T}_v = 267 \text{ K} = -6^\circ \text{C}$; arctic $\bar{T}_v = 227 \text{ K} = -46^\circ \text{C}$. Increasing the arctic surface pressure alters the mean layer temperature there only by a few degrees, to 221.5 K. But think about this: why did the temperature have to be lower?

4. You are given a situation in which the gradient wind is 10% faster than the geostrophic wind, which is 10 m/s. You are at 40°N. What is the radius of curvature for this flow? Don’t forget to check your sign for reasonableness.

**Ans:** Start with the gradient wind equation in the form

$$\frac{V_g}{V} = 1 + \frac{V}{fR}.$$  

We are given that $V = 1.1V_g$, which yields after some simplification

$$\frac{1.1V_g}{fR} = -\frac{1}{1.1}.$$  

Solving for $R$ yields $R = -12.1 \frac{V_g}{f}$. With $V_g = 10 \text{ m/s}$ and $f = 0.94 \times 10^{-4} \text{ s}^{-1}$, this results in $R = -1287 \text{ km}$. We knew the radius of curvature had to be negative, because $V$ exceeded $V_g$.

5. Say the takeoff weight of a Boeing 747-400 airplane is 400000 kg and its horizontal area is 525 m$^2$. Calculate the pressure increase this plane would cause on the surface if the atmosphere were absolutely and completely hydrostatic and motionless.

**Ans:** Before we start, consider the fact that the hydrostatic equation says that the pressure applied to the surface is based on the weight of the mass that resides directly above. That can be more than just nitrogen, oxygen and other gases. It could also be an airplane, which can be very heavy. So, if a large plane flies directly over your head, why aren’t you crushed?
We’re given the mass and area of the plane, so the mass per unit area is $762 \text{ kg/m}^2$. The hydrostatic equation written in the form

$$\frac{dp}{g} = -\rho dz = \frac{\text{mass}}{\text{volume}} \text{depth} = \frac{\text{mass}}{\text{area}}$$

reveals where mass/area fits in. In this case, $dp$ is the extra pressure owing to adding the plane’s contribution to total mass per area, which is $762 \text{ kg/m}^2$ times $g$, or 7467 Pa, or about 75 mb.

Now, it’s time for a sanity check. This answer is correct, but is it reasonable? If this airplane flew over your head, and created a sudden, 75 mb increase of pressure, I guarantee you would notice it, and while it won’t crush you, your ears or sinuses might actually be injured. Yet, airplanes fly over your head all the time, without disturbing you at all. Why?